CHAPTER 10

Circles

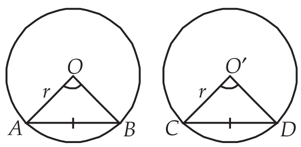
NCERT FOCUS

SOLUTIONS

EXERCISE - 10.2

1. **Given :** Two congruent circles with centres O and O' and radii r having chords AB and CD respectively, such that $AB = CD$.

To Prove : $\angle AOB = \angle CO'D$



Proof : In ΔAOB and $\Delta CO'D$, we have

$AB = CD$ [Given]

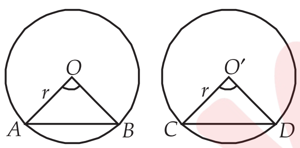
$OA = O'C$ [Each equal to r]

$OB = O'D$ [Each equal to r]

$\therefore \quad \Delta AOB \cong \Delta CO'D$ [By SSS congruency criteria]

$\Rightarrow \quad \angle AOB = \angle CO'D$ [By C.P.C.T.]

2. **Given :** Two congruent circles with centres O and O' and radii r having chords AB and CD respectively, such that $\angle AOB = \angle CO'D$.



To Prove : $AB = CD$

Proof : In ΔAOB and $\Delta CO'D$, we have

$OA = O'C$ [Each equal to r]

$\angle AOB = \angle CO'D$ [Given]

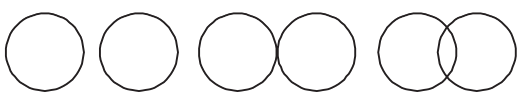
$OB = O'D$ [Each equal to r]

$\therefore \quad \Delta AOB \cong \Delta CO'D$ [By SAS congruency criteria]

Hence, $AB = CD$ [By C.P.C.T.]

EXERCISE - 10.3

1. Let us draw different pairs of circles as shown below:



(i) (ii) (iii)

We have,

In figure	Maximum number of common points
(i)	Zero
(ii)	One
(iii)	Two

Thus, two circles can have at most two points in common.

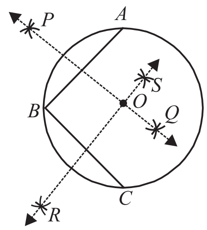
2. **Steps of construction :**

Step I. Take any three points on the given circle. Let these points be A, B and C.

Step II. Join AB and BC.

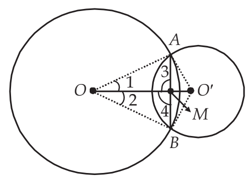
Step III. Draw the perpendicular bisector PQ of AB.

Step IV. Draw the perpendicular bisector RS of BC such that it intersects PQ at O.



Thus, 'O' is the required centre of the given circle.

3. We have two circles with centres O and O', intersecting at A and B.



$\therefore \quad AB$ is the common chord of two circles and OO' is the line segment joining their centres. Let OO' and AB intersect each other at M.

$\therefore \quad$ To prove that OO' is the perpendicular bisector of AB, we join OA, OB, $O'A$ and $O'B$.

Now, in $\Delta OAO'$ and $\Delta OBO'$, we have

$OA = OB$ [Radii of the same circle]

$O'A = O'B$ [Radii of the same circle]

$OO' = OO'$ [Common]

$\therefore \quad \Delta OAO' \cong \Delta OBO'$ [By SSS congruency criteria]

$\Rightarrow \quad \angle 1 = \angle 2$ [By C.P.C.T.]

Now, in ΔAOM and ΔBOM, we have

$OA = OB$ [Radii of the same circle]

$OM = OM$ [Common]

$\angle 1 = \angle 2$ [Proved above]

$\therefore \quad \Delta AOM \cong \Delta BOM$ [By SAS congruency criteria]

$\Rightarrow \quad \angle 3 = \angle 4$ [By C.P.C.T.]

But $\angle 3 + \angle 4 = 180°$ [Linear pair]

$\therefore \quad \angle 3 = \angle 4 = 90° \Rightarrow AM \perp OO'$

Also $AM = BM$ [By C.P.C.T.]

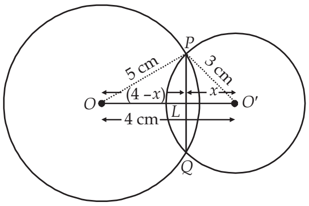
$\Rightarrow \quad M$ is the mid-point of AB.

Thus, OO' is the perpendicular bisector of AB.

EXERCISE - 10.4

1. We have two intersecting circles with centres at O and O' respectively. Let PQ be the common chord.

$\because \quad$ In two intersecting circles, the line joining their centres is perpendicular bisector of the common chord.



$\therefore \quad \angle OLP = \angle OLQ = 90°$ and $PL = LQ$

Now, in right ΔOLP, we have

$PL^2 + OL^2 = OP^2 \Rightarrow PL^2 + (4 - x)^2 = 5^2$

$\Rightarrow \quad PL^2 = 5^2 - (4 - x)^2$

$\Rightarrow \quad PL^2 = 25 - 16 - x^2 + 8x$

$\Rightarrow \quad PL^2 = 9 - x^2 + 8x$ ...(i)

Again, in $\Delta O'LP$, we have

$PL^2 = PO'^2 - LO'^2 = 3^2 - x^2 = 9 - x^2$ ...(ii)

From (i) and (ii), we have

$9 - x^2 + 8x = 9 - x^2$

$\Rightarrow \quad 8x = 0 \Rightarrow x = 0$

$\Rightarrow$ L and O' coincide.

$\therefore$ PQ is a diameter of the smaller circle.

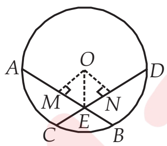
$\Rightarrow \quad PL = 3$ cm

But $PL = LQ \quad \therefore \quad LQ = 3$ cm

$\therefore \quad PQ = PL + LQ = 3 \text{ cm} + 3 \text{ cm} = 6$ cm

$\therefore$ Length of the common chord = 6 cm

2. **Given :** A circle with centre O. Equal chords AB and CD intersect at E.



To prove : $AE = DE$ and $CE = BE$

Construction : Draw $OM \perp AB$ and $ON \perp CD$. Join OE.

Proof : Since $AB = CD$ [Given]

$\therefore OM = ON$ [$\because$ Equal chords of a circle are equidistant from the centre]

Now, in ΔOME and ΔONE, we have

$\angle OME = \angle ONE$ [Each equal to 90°]

$OM = ON$ [Proved above]

$OE = OE$ [Common]

$\therefore \quad \Delta OME \cong \Delta ONE$ [By RHS congruency criteria]

$\Rightarrow \quad ME = NE$ [By C.P.C.T.]

Adding AM on both sides, we get

$AM + ME = AM + NE$

$\Rightarrow \quad AE = DN + NE$

$[\because AB = CD \Rightarrow \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AM = DN]$

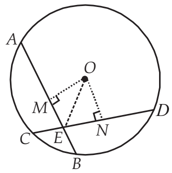
$\Rightarrow \quad AE = DE$ ...(i)

$\Rightarrow \quad AB - BE = CD - CE$

$\Rightarrow \quad BE = CE \quad [\because AB = CD]$ ...(ii)

From (i) and (ii), we have $AE = DE$ and $CE = BE$

3. **Given :** A circle with centre O and equal chords AB and CD are intersecting at E.



To prove : $\angle OEA = \angle OED$

Construction : Draw $OM \perp AB$ and $ON \perp CD$.

Proof : In right ΔOME and right ΔONE,

$OM = ON$ [$\because$ Equal chords of a circle are equidistant from the centre]

$OE = OE$ [Common]

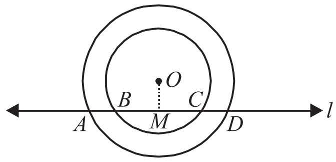
$\angle OME = \angle ONE$ [Each equal to 90°]

$\therefore \quad \Delta OME \cong \Delta ONE$ [By RHS congruency criteria]

$\therefore \quad \angle OEM = \angle OEN$ [By C.P.C.T.]

$\Rightarrow \quad \angle OEA = \angle OED$

4. **Given :** Two concentric circles with centre O. Let a line 'l' intersects the outer circle at A and D and the inner circle at B and C.



To prove : $AB = CD$.

Construction : Draw $OM \perp l$.

Proof : For the outer circle, $OM \perp l$

$\therefore \quad AM = MD$ ...(i)

[$\because$ Perpendicular drawn from the centre of a circle to the chord bisects the chord]

For the inner circle, $OM \perp l$

$\therefore \quad BM = MC$ ...(ii)

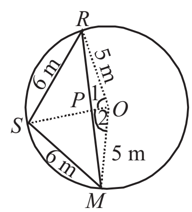
[$\because$ Perpendicular drawn from the centre of a circle to the chord bisects the chord]

Subtracting (ii) from (i), we have

$AM - BM = MD - MC$

$\Rightarrow \quad AB = CD$

5. Let the three girls Reshma, Salma and Mandip be positioned at R, S and M respectively on the circle of radius 5 m.



$RS = SM = 6$ m [Given]

$\because$ Equal chords of a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2$ ...(i)

In ΔPOR and ΔPOM, we have

$OP = OP$ [Common]

$OR = OM$ [Radii of the same circle]

$\angle 1 = \angle 2$ [By (i)]

$\therefore \quad \Delta POR \cong \Delta POM$ [By SAS congruency criteria]

$\therefore \quad PR = PM$ and $\angle OPR = \angle OPM$ [By C.P.C.T.]

$\because \quad \angle OPR + \angle OPM = 180°$ [Linear pair]

$\therefore \quad \angle OPR = \angle OPM = 90°$

$\Rightarrow \quad OP \perp RM$

Now, in ΔRSP and ΔMSP, we have

$RS = MS$ [Given]

$SP = SP$ [Common]

$PR = PM$ [Proved above]

$\therefore \quad \Delta RSP \cong \Delta MSP$ [By SSS congruency criteria]

$\Rightarrow \quad \angle RPS = \angle MPS$ [By C.P.C.T.]

But $\angle RPS + \angle MPS = 180°$

$\Rightarrow \quad \angle RPS = \angle MPS = 90°$

$\therefore$ SP passes through O.

Let $OP = x$ m $\therefore$ $SP = (5 - x)$ m [$\because$ Radius = 5 m]

Now, in right ΔOPR, we have

$x^2 + RP^2 = 5^2 \Rightarrow RP^2 = 5^2 - x^2$ ...(1)

In right ΔSPR, we have

$(5 - x)^2 + RP^2 = 6^2$

$\Rightarrow \quad RP^2 = 6^2 - (5 - x)^2$ ...(2)

From (1) and (2), we have

$5^2 - x^2 = 6^2 - (5 - x)^2$

$\Rightarrow \quad 25 - x^2 = 36 - [25 - 10x + x^2]$

$\Rightarrow \quad -10x + 14 = 0 \Rightarrow 10x = 14$

$\Rightarrow \quad x = \frac{14}{10} = 1.4$

Now, $RP^2 = 5^2 - x^2 \Rightarrow RP^2 = 25 - (1.4)^2$

$\Rightarrow \quad RP^2 = 25 - 1.96 = 23.04$ m

$\therefore \quad RP = \sqrt{23.04} = 4.8$ m

$\therefore \quad RM = 2RP = 2 \times 4.8$ m $= 9.6$ m

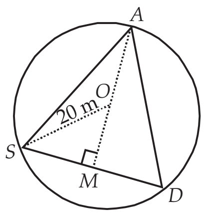
Thus, distance between Reshma and Mandip is 9.6 m.

6. Let Ankur, Syed and David are sitting at A, S and D respectively such that $AS = SD = AD$

i.e. ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle be $2x$ m and O be the centre of circle.

Draw $AM \perp SD$.



Since ΔASD is an equilateral.

$\therefore \quad AM$ passes through O.

$\Rightarrow \quad SM = \frac{1}{2}SD = \frac{1}{2}(2x)$

$\Rightarrow \quad SM = x$ m

Now, in right ΔASM, we have

$AM^2 + SM^2 = AS^2$

$\Rightarrow \quad AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow \quad AM = \sqrt{3}x$ m

Now, $OM = AM - OA = (\sqrt{3}x - 20)$ m

[Given, radius = 20 m]

Again, in right ΔOSM, we have

$OS^2 = SM^2 + OM^2$

$\Rightarrow \quad 20^2 = x^2 + (\sqrt{3}x - 20)^2$

$\Rightarrow \quad 400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$

$\Rightarrow \quad 4x^2 = 40\sqrt{3}x \Rightarrow x(4x - 40\sqrt{3}) = 0$

$\Rightarrow \quad x = 10\sqrt{3}$

Now, $SD = 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m

Thus, the length of the string of each phone $= 20\sqrt{3}$ m

EXERCISE - 10.5

1. We have a circle with centre O, such that $\angle AOB = 60°$ and $\angle BOC = 30°$

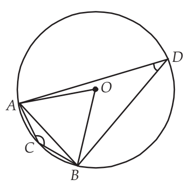
$\because \quad \angle AOC = \angle AOB + \angle BOC$

$\therefore \quad \angle AOC = 60° + 30° = 90°$

We know, angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle ADC = \frac{1}{2}(\angle AOC) = \frac{1}{2}(90°) = 45°$

2. We have a circle having a chord AB equal to radius of the circle.



$\therefore \quad AO = BO = AB$

$\Rightarrow \quad \Delta AOB$ is an equilateral triangle.

Since, each angle of an equilateral triangle is 60°.

$\Rightarrow \quad \angle AOB = 60°$

Since, the arc ACB makes reflex $\angle AOB = 360° - 60° = 300°$ at the centre of the circle and $\angle ACB$ at a point on the minor arc of the circle.

$\therefore \quad \angle ACB = \frac{1}{2}[\text{reflex} \angle AOB] = \frac{1}{2}[300°] = 150°$

Hence, the angle subtended by the chord on the minor arc is 150°.

Similarly, $\angle ADB = \frac{1}{2}[\angle AOB] = \frac{1}{2} \times 60° = 30°$

Hence, the angle subtended by the chord on the major arc is 30°.

3. $\because$ Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad$ Reflex $\angle POR = 2\angle PQR$

But $\angle PQR = 100°$ [Given]

$\therefore \quad$ Reflex $\angle POR = 2 \times 100° = 200°$

Since, $\angle POR +$ reflex $\angle POR = 360°$

$\Rightarrow \quad \angle POR = 360° - 200° \Rightarrow \angle POR = 160°$

In ΔPOR, $OP = OR$ [Radii of the same circle]

$\therefore \quad \angle OPR = \angle ORP$ ...(i)

[$\because$ Angles opposite to equal sides of a triangle are equal.]

Also, $\angle OPR + \angle ORP + \angle POR = 180°$

[$\because$ Sum of the angles of a triangle is 180°]

$\Rightarrow \quad \angle OPR + \angle OPR + 160° = 180°$ [From (i)]

$\Rightarrow \quad 2\angle OPR = 180° - 160° = 20°$

$\Rightarrow \quad \angle OPR = \frac{20°}{2} = 10°$

4. In ΔABC, $\angle ABC + \angle ACB + \angle BAC = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 69° + 31° + \angle BAC = 180°$

$\Rightarrow \quad \angle BAC = 180° - 100° = 80°$

$\because \quad$ Angles in the same segment are equal.

$\therefore \quad \angle BDC = \angle BAC \Rightarrow \angle BDC = 80°$

5. In ΔECD, $\angle BEC = \angle EDC + \angle ECD$

[$\because$ Sum of interior opposite angles of a triangle is equal to exterior angle]

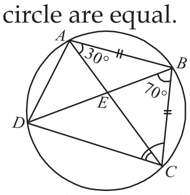
$\Rightarrow \quad 130° = \angle EDC + 20°$

$\Rightarrow \quad \angle EDC = 130° - 20° = 110° \Rightarrow \angle BDC = 110°$

$\because \quad$ Angles in the same segment are equal.

$\therefore \quad \angle BAC = \angle BDC \Rightarrow \angle BAC = 110°$

6. $\because$ Angles in the same segment of a circle are equal.



$\therefore \quad \angle BAC = \angle BDC$

$\Rightarrow \quad \angle BDC = 30°$

Also, $\angle DBC = 70°$ [Given]

In ΔBCD,

$\angle BCD + \angle DBC + \angle CDB = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad \angle BCD + 70° + 30° = 180°$

$\Rightarrow \quad \angle BCD = 180° - 100° = 80°$

Now, in ΔABC, $AB = BC$

$\therefore \quad \angle BCA = \angle BAC$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \angle BCA = 30°$ [$\because \angle BAC = 30°$]

Now, $\angle BCA + \angle ECD = \angle BCD$

$\Rightarrow \quad 30° + \angle ECD = 80°$

$\Rightarrow \quad \angle ECD = 80° - 30° = 50°$

7. Let $ABCD$ is a cyclic quadrilateral and its diagonals AC and BD intersect at O.

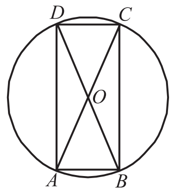
Since, AC and BD are diameters.

$\Rightarrow$ $AC = BD$ ...(i)

[$\because$ All diameters of a circle are equal]

Also, $\angle BAD = 90°$

[$\because$ Angle in a semi-circle is 90°]



Similarly, $\angle ABC = 90°$, $\angle BCD = 90°$ and $\angle CDA = 90°$

Now, in right ΔABC and right ΔBAD, we have

$AC = BD$ [From (i)]

$AB = BA$ [Common]

$\angle ABC = \angle BAD$ [Each equal to 90°]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By RHS congruency criteria]

$\Rightarrow$ $BC = AD$ [By C.P.C.T.]

Similarly, $AB = DC$

Thus, the cyclic quadrilateral $ABCD$ is such that its opposite sides are equal and each of its angle is right angle.

$\therefore$ $ABCD$ is a rectangle.

8. We have, a trapezium $ABCD$ such that $AB \parallel CD$ and $AD = BC$.

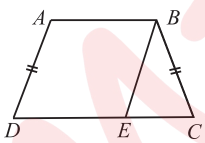
Let us draw $BE \parallel AD$ such that $ABED$ is a parallelogram.

$\because$ The opposite angles of a parallelogram are equal.

$\therefore$ $\angle BAD = \angle BED$ ...(i)

and $AD = BE$ ...(ii)

[Opposite sides of a parallelogram]



But $AD = BC$ [Given] ...(iii)

$\therefore$ From (ii) and (iii), we have

$BE = BC \Rightarrow \angle BEC = \angle BCE$ ...(iv)

[$\because$ Angles opposite to equal sides of a triangle are equal]

Now, $\angle BED + \angle BEC = 180°$ [Linear pair]

$\Rightarrow$ $\angle BAD + \angle BCE = 180°$ [Using (i) and (iv)]

i.e. A pair of opposite angles of quadrilateral $ABCD$ is 180°.

$\Rightarrow$ Trapezium $ABCD$ is cyclic.

9. Since, angles in the same segment of a circle are equal.

$\therefore$ $\angle ACP = \angle ABP$ ...(i)

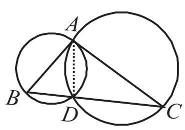
Similarly, $\angle QCD = \angle QBD$ ...(ii)

Since, $\angle ABP = \angle QBD$ [Vertically opposite angles]

$\therefore$ From (i) and (ii), we have

$\angle ACP = \angle QCD$

10. We have, ΔABC and two circles described with diameter as AB and AC respectively. They intersect at a point D, other than A.



Let us join A and D.

$\because$ AB is a diameter and $\angle ADB$ is an angle formed in a semicircle.

$\Rightarrow$ $\angle ADB = 90°$ ...(i)

Similarly, $\angle ADC = 90°$ ...(ii)

Adding (i) and (ii), we have

$\angle ADB + \angle ADC = 90° + 90° = 180°$

i.e., B, D and C are collinear points.

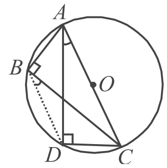
$\Rightarrow$ BC is a straight line. Thus, D lies on BC.

11. We have, ΔABC and ΔADC such that they are having AC as their common hypotenuse.

$\because$ AC is a hypotenuse and $\angle ADC = 90° = \angle ABC$

$\therefore$ Both the triangles are in semicircle.

Case-1 : If both the triangles are in the same semicircle.



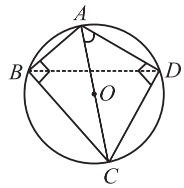
$\Rightarrow$ A, B, C and D are concyclic.

Join BD.

Now, DC is a chord and $\angle CAD$ and $\angle CBD$ are formed in the same segment.

$\Rightarrow$ $\angle CAD = \angle CBD$

Case-2 : If both the triangles are not in same semicircle.



$\Rightarrow$ A, B, C and D are concyclic.

Join BD.

Now, DC is a chord and $\angle CAD$ and $\angle CBD$ are formed in the same segment.

$\Rightarrow$ $\angle CAD = \angle CBD$

12. We have a cyclic parallelogram $ABCD$.

Since, $ABCD$ is a cyclic quadrilateral.

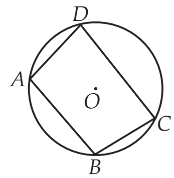
$\therefore$ $\angle A + \angle C = 180°$ ...(i)

But $\angle A = \angle C$ ...(ii)

[$\because$ Opposite angles of a parallelogram are equal]

From (i) and (ii), we have

$\angle A = \angle C = 90°$



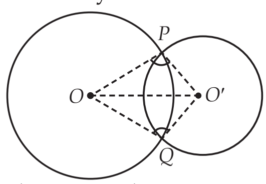
Similarly, $\angle B = \angle D = 90°$

$\Rightarrow$ Each angle of the parallelogram $ABCD$ is 90°.

Thus, $ABCD$ is a rectangle.

EXERCISE - 10.6

1. **Given :** Two circles with centres O and O' respectively such that they intersect each other at P and Q.



To prove : $\angle OPO' = \angle OQO'$.

Construction : Join OP, $O'P$, OQ, $O'Q$ and OO'.

Proof : In $\Delta OPO'$ and $\Delta OQO'$, we have

$OP = OQ$ [Radii of the same circle]

$O'P = O'Q$ [Radii of the same circle]

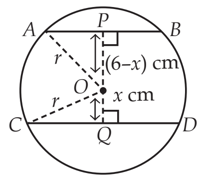
$OO' = OO'$ [Common]

$\therefore$ $\Delta OPO' \cong \Delta OQO'$ [By SSS congruency criteria]

$\Rightarrow$ $\angle OPO' = \angle OQO'$ [By C.P.C.T.]

2. We have a circle with centre O, $AB \parallel CD$ and the perpendicular distance between AB and CD is 6 cm and $AB = 5$ cm, $CD = 11$ cm.

Let 'r' be the radius of the circle.



Let us draw $OP \perp AB$ and $OQ \perp CD$.

Join OA and OC.

Let $OQ = x$ cm

$\therefore$ $OP = (6 - x)$ cm

$\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore \quad AP = \frac{1}{2}AB = \frac{1}{2} \times 5 = \frac{5}{2}$ cm,

$CQ = \frac{1}{2}CD = \frac{1}{2} \times 11 = \frac{11}{2}$ cm

In ΔCQO, we have $CO^2 = CQ^2 + OQ^2$

$\Rightarrow \quad r^2 = \left(\frac{11}{2}\right)^2 + x^2 \quad \Rightarrow \quad r^2 = \frac{121}{4} + x^2$ ...(i)

In ΔAPO, we have $AO^2 = AP^2 + OP^2$

$\Rightarrow \quad r^2 = \left(\frac{5}{2}\right)^2 + (6 - x)^2$

$\Rightarrow \quad r^2 = \frac{25}{4} + [36 - 12x + x^2]$ ...(ii)

From (i) and (ii), we have

$\frac{25}{4} + 36 - 12x + x^2 = \frac{121}{4} + x^2$

$\Rightarrow \quad -12x = \frac{121}{4} - \frac{25}{4} - 36$

$\Rightarrow \quad 12x = 12$

$\Rightarrow \quad x = 1$

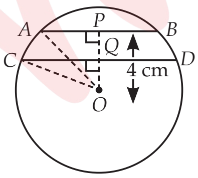
Substituting the value of x in (i), we get

$r^2 = \frac{121}{4} + 1 = \frac{125}{4} \Rightarrow r = \frac{5\sqrt{5}}{2}$ cm

[$\because r \neq -\frac{5\sqrt{5}}{2}$, as radius can't be negative]

Thus, the radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

3. We have a circle with centre O. Parallel chords AB and CD are such that the smaller chord is 4 cm away from the centre.



Let r be the radius of circle.

Draw $OP \perp AB$ and join OA and OC.

We know that, perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore \quad AP = \frac{1}{2}AB = \frac{1}{2}(6 \text{ cm}) = 3$ cm

Similarly, $CQ = \frac{1}{2}CD = \frac{1}{2}(8 \text{ cm}) = 4$ cm

Now in ΔOPA, we have $OA^2 = OP^2 + AP^2$

$\Rightarrow \quad r^2 = 4^2 + 3^2 \Rightarrow r^2 = 16 + 9 = 25$

$\Rightarrow \quad r = \sqrt{25} = 5$ cm

[$\because r \neq -5$, as distance cannot be negative]

Again, in ΔCQO, we have $OC^2 = OQ^2 + CQ^2$

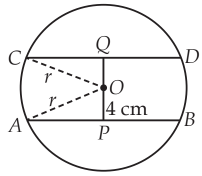
$\Rightarrow \quad r^2 = OQ^2 + 4^2 \Rightarrow OQ^2 = r^2 - 4^2 = 5^2 - 4^2$ [$\because r = 5$ cm]

$\Rightarrow \quad OQ^2 = 25 - 16 = 9$

$\Rightarrow \quad OQ = \sqrt{9} = 3$ cm

The distance of the other chord (CD) from the centre is 3 cm.

Note : In case we take the two parallel chords on either side of the centre, then



In ΔPOA, $OA^2 = OP^2 + PA^2$

$\Rightarrow \quad r^2 = 4^2 + 3^2 = 5^2$

$\Rightarrow \quad r = 5$ cm

In ΔQOC, $OC^2 = CQ^2 + OQ^2$

$\Rightarrow \quad r^2 = 4^2 + OQ^2$

$\Rightarrow \quad OQ^2 = r^2 - 4^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad OQ = 3$ cm

4. **Given :** $\angle ABC$ such that when we produce arms BA and BC, they make two equal chords AD and CE.

To prove : $\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$

Construction : Join AC, DE and AE.

Proof : Since an exterior angle of a triangle is equal to the sum of interior opposite angles.

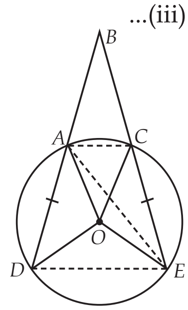
$\therefore$ In ΔBAE, we have

$\angle DAE = \angle ABC + \angle AEC$ ...(i)

The chord DE subtends $\angle DOE$ at the centre and $\angle DAE$ in the remaining part of the circle.

$\therefore \quad \angle DAE = \frac{1}{2}\angle DOE$ ...(ii)

Similarly, $\angle AEC = \frac{1}{2}\angle AOC$ ...(iii)



From (i), (ii) and (iii), we have

$\frac{1}{2}\angle DOE = \angle ABC + \frac{1}{2}\angle AOC$

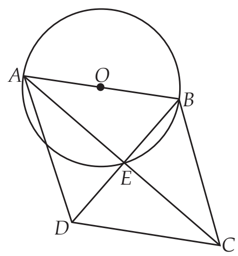
$\Rightarrow \quad \angle ABC = \frac{1}{2}\angle DOE - \frac{1}{2}\angle AOC$

$\Rightarrow \quad \angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$

$\Rightarrow \quad \angle ABC = \frac{1}{2}$ [(Angle subtended by the chord DE at the centre) – (Angle subtended by the chord AC at the centre)]

$\Rightarrow \quad \angle ABC = \frac{1}{2}$ [Difference of the angles subtended by the chords DE and AC at the centre]

5. Let $ABCD$ be a rhombus whose diagonals AC and BD intersect at E. Let O be centre of the circle with diameter AB. We know that the diagonals of a rhombus intersect each other at right angle.



$\Rightarrow \quad \angle AEB = 90°$

i.e., $\angle AEB$ is in semi-circle.

$\Rightarrow$ Circle with AB as diameter passes through E *i.e.*, the point of intersection of its diagonals.

6. **Given :** A circle passing through A, B and C is drawn such that it intersects CD at E.

To prove : $AE = AD$

Construction : Join AE.

Proof : $ABCE$ is a cyclic quadrilateral

$\therefore \quad \angle AEC + \angle B = 180°$ ...(i)

[$\because$ Opposite angles of a cyclic quadrilateral are supplementary]

But $ABCD$ is a parallelogram. [Given]

$\therefore$ $\angle D = \angle B$ ...(ii)

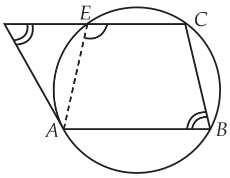
[$\because$ Opposite angles of a parallelogram are equal]

From (i) and (ii), we have

$\angle AEC + \angle D = 180°$ ...(iii)

But $\angle AEC + \angle AED = 180°$ ...(iv)

[Linear pair]



From (iii) and (iv), we have

$\angle D = \angle AED$

i.e., The base angles of ΔADE are equal.

$\therefore$ Opposite sides must be equal.

$\Rightarrow$ $AD = AE$

7. Given : A circle with centre at O. Two chords AC and BD are such that they bisect each other. Let their point of intersection be O.

To prove : (i) AC and BD are diameters.

(ii) $ABCD$ is a rectangle.

Construction : Join AB, BC, CD and DA.

Proof : (i) In ΔAOB and ΔCOD, we have

$AO = CO$ [Radii of same circle]

$BO = DO$ [Radii of same circle]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\therefore$ $\Delta AOB \cong \Delta COD$ [By SAS congruency criteria]

$\Rightarrow$ $AB = CD$ [By C.P.C.T.]

$\Rightarrow$ arc AB = arc CD ...(1) [$\because$ If two chords are equal, then their corresponding arcs are equal (congruent)]

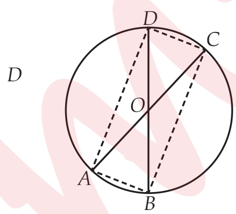
Similarly, arc AD = arc BC ...(2)

Adding (1) and (2), we get

arc AB + arc AD = arc CD + arc BC

$\Rightarrow$ $\overset{\frown}{BAD} = \overset{\frown}{BCD}$

BD divides the circle into two equal parts



$\therefore$ BD is a diameter.

Similarly, AC is a diameter.

(ii) $\Delta AOB \cong \Delta COD$ [From part (i)]

$\Rightarrow$ $\angle OAB = \angle OCD$ [By C.P.C.T.]

$\Rightarrow$ $\angle CAB = \angle ACD \Rightarrow AB \parallel DC$

Similarly, $AD \parallel BC$

$\therefore$ $ABCD$ is a parallelogram

Since, opposite angles of a parallelogram are equal

$\therefore$ $\angle DAB = \angle DCB$

But $\angle DAB + \angle DCB = 180°$

[Sum of the opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow$ $\angle DAB = 90° = \angle DCB$

Thus, $ABCD$ is a rectangle.

8. Given : A triangle ABC inscribed in a circle, such that bisectors of $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at D, E and F respectively.

To prove : Angles of ΔDEF are $90° - \frac{1}{2}\angle A$, $90° - \frac{1}{2}\angle B$ and $90° - \frac{1}{2}\angle C$.

Construction : Join DE, EF and FD.

Proof : Since, angles in the same segment are equal.

$\therefore$ $\angle FDA = \angle FCA$ ...(i)

$\angle EDA = \angle EBA$ ...(ii)

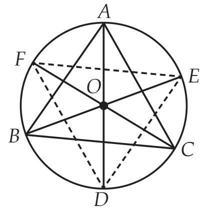
Adding (i) and (ii), we have

$\angle FDA + \angle EDA = \angle FCA + \angle EBA$

$\Rightarrow$ $\angle FDE = \angle FCA + \angle EBA$

$= \frac{1}{2}\angle C + \frac{1}{2}\angle B = \frac{1}{2}[\angle C + \angle B]$

$= \frac{1}{2}[180° - \angle A] = \left(90° - \frac{\angle A}{2}\right)$



Similarly, $\angle FED = \left(90° - \frac{\angle B}{2}\right)$

and $\angle EFD = \left(90° - \frac{\angle C}{2}\right)$

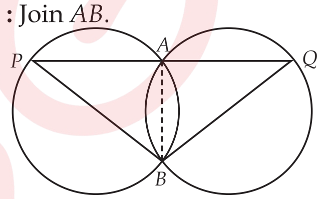
Thus, the angles of ΔDEF are

$$\left(90° - \frac{\angle A}{2}\right), \left(90° - \frac{\angle B}{2}\right) \text{ and } \left(90° - \frac{\angle C}{2}\right).$$

9. Given : Two congruent circles such that they intersect each other at A and B. A line passing through A, meets the circles at P and Q.

To prove : $BP = BQ$

Construction : Join AB.



Proof : Since, angles subtended by equal chords in the congruent circles are equal.

$\Rightarrow$ $\angle APB = \angle AQB$

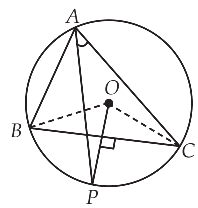
Now, in ΔPBQ, we have

$\angle APB = \angle AQB$

$\Rightarrow$ $PB = BQ$ [Sides opposite to equal angles of a triangle are equal]

10. Given : ΔABC with O as centre of its circumcircle. The perpendicular bisector of BC passes through O. Suppose it cut circumcircle at P.

To prove : The perpendicular bisector of BC and bisector of $\angle A$ of ΔABC intersect at P.



Construction : Join OB and OC.

Proof : In order to prove that the perpendicular bisector of BC and bisector of $\angle A$ of ΔABC intersect at P, it is sufficient to show that AP is bisector of $\angle A$ of ΔABC.

Let arc BC makes angle θ on the circumference

$\therefore$ $\angle BOC = 2\theta$ [Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

Also, in ΔBOC, $OB = OC$ and OP is perpendicular bisector of BC.

So, $\angle BOP = \angle COP = \theta$

Arc CP makes angle θ at O, so it will make angle $\frac{\theta}{2}$ at circumference.

So, $\angle CAP = \frac{\theta}{2}$

Hence, AP is angle bisector of $\angle A$ of ΔABC.