Circles

CHAPTER 10

TRY YOURSELF

SOLUTIONS

1. Let r be the radius of circle.

In ΔAOC,

$OA = OC = r$ [Radii of same circle]

$\therefore \angle OAC = \angle OCA = x$ ...(i) [$\because$ Angles opposite to equal sides of a triangle are equal]

$\because$ BOC is a straight line.

$\therefore \angle AOC + \angle AOB = 180°$

$\Rightarrow \angle AOC = 180° - 70°$

$\Rightarrow \angle AOC = 110°$

Thus, angle subtended by chord AC at centre O, $\angle AOC = 110°$

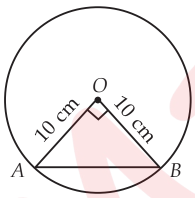
Now, in ΔAOC, $\angle OAC + \angle OCA + \angle AOC = 180°$

$x + x + \angle AOC = 180°$ [Using (i)]

$\Rightarrow 2x = 180° - 110°$

$\Rightarrow 2x = 70° \Rightarrow x = 35°$

2. Let AB be the chord of a circle which makes a right angle at centre O.



Radius of circle = 10 cm [Given]

$\therefore OA = OB = 10$ cm ...(i)

Now, in right ΔOAB, we have

$AB^2 = OA^2 + OB^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = (10)^2 + (10)^2$ [Using (i)]

$\Rightarrow AB^2 = 100 + 100$

$\Rightarrow AB = \sqrt{200} \Rightarrow AB = 10\sqrt{2}$ cm

Hence, length of chord of circle is $10\sqrt{2}$ cm.

3. Given, $AB = BC = CA$

We know that, equal chords of a circle subtend equal angles at the centre.

$\therefore \angle AOB = \angle BOC = \angle AOC$ ...(i)

Now, $\angle AOB + \angle BOC + \angle AOC = 360°$ [$\because$ Sum of angles at a point is 360°]

$\Rightarrow 3\angle AOB = 360°$ [Using (i)]

$\Rightarrow \angle AOB = \dfrac{360°}{3} = 120°$

Hence, angle subtended by the chords AB, BC and CA at the centre O is 120°.

4. Given, $CD = DE = EF = FG$

We know that, equal chords of a circle subtend equal angles at the centre.

$\therefore \angle COD = \angle DOE = \angle EOF = \angle FOG = 40°$ ...(i)

Now, $\angle COG = \angle COD + \angle DOE + \angle EOF + \angle FOG$

$\Rightarrow \angle COG = 4 \times \angle COD = 4 \times 40°$

$\Rightarrow \angle COG = 160°$

$\therefore$ Reflex $\angle COG = 360° - 160° = 200°$

5. Given, radius $(OB) = 5$ cm, $OC = 3$ cm and $OC \perp AB$.

Now, in right angled ΔOCB,

$OB^2 = OC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow (5)^2 = (3)^2 + BC^2$

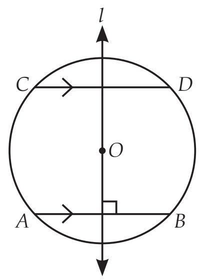
$\Rightarrow BC^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow BC = 4$ cm [$\because BC \neq -4$, as length can't be negative]

We know that, the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore AB = 2BC = 2 \times 4 = 8$ cm

6. We know that the perpendicular bisector of any chord of a circle always passes through the centre of the circle. Since, l is the perpendicular bisector of AB. Therefore, l passes through the centre, O of the circle.



But, $l \perp AB$ and $AB \parallel CD \Rightarrow l \perp CD$.

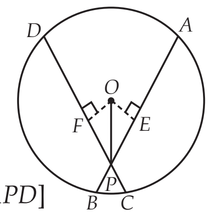
Thus, $l \perp CD$ and passes through the centre, O of the circle. So, l is the perpendicular bisector of CD also.

7. Given, PQ and RS are two chords of a circle having centre at O and $ON = 4$ cm.

Since, equal chords of a circle are equidistant from the centre.

$\therefore OM = ON = 4$ cm

8. Draw $OE \perp AB$ and $OF \perp CD$.



In ΔOEP and ΔOFP, we have

$\angle OEP = \angle OFP$ [Each equal 90°]

$OP = OP$ [Common]

and $\angle OPE = \angle OPF$ [$\because OP$ bisects $\angle APD$]

$\therefore \Delta OEP \cong \Delta OFP$ [By AAS congruency criteria]

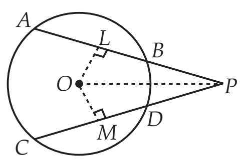
$\Rightarrow OE = OF$ [By C.P.C.T.]

Thus, chords AB and CD are equidistant from the centre O of the circle.

But, chords of a circle which are equidistant from the centre are equal.

$\therefore AB = CD$

9. **Given :** AB and CD are two equal chords of a circle intersecting at a point P.



To prove : $PB = PD$

Construction : Join OP, draw $OL \perp AB$ and $OM \perp CD$

Proof : We have, $AB = CD$

$\Rightarrow OL = OM$ ...(i)

[$\because$ Equal chords of a circle are equidistant from the centre]

Now, in ΔOLP and ΔOMP,

$OL = OM$ [From (i)]

$\angle OLP = \angle OMP$ [Each equal to 90°]

$OP = OP$ [Common]

$\therefore$ $\Delta OLP \cong \Delta OMP$ [By RHS congruency criteria]

$\Rightarrow$ $LP = MP$ [By C.P.C.T.]... (ii)

Also, $AB = CD$ [Given]

$\Rightarrow \frac{1}{2}(AB) = \frac{1}{2}(CD)$

$\Rightarrow$ $BL = DM$ ... (iii)

[$\because$ The perpendicular drawn from the centre of a circle bisects the chord.]

On subtracting (iii) from (ii), we get

$LP - BL = MP - DM$

$\Rightarrow$ $PB = PD$

10. In ΔOAB, $OA = OB$ [Radii of same circle]

$\therefore$ $\angle OBA = \angle OAB = 40°$

[$\because$ Angles opposite to equal sides of a triangle are equal]

Also, $\angle AOB + \angle OBA + \angle OAB = 180°$

[$\because$ Sum of angles of a triangle is 180°]

$\therefore$ $\angle AOB + 40° + 40° = 180°$

$\Rightarrow$ $\angle AOB = 180° - 80° = 100°$

Since, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.

$\therefore$ $\angle AOB = 2\angle ACB \Rightarrow 100° = 2\angle ACB$

$\therefore$ $\angle ACB = 50°$

11. Given, $\angle AOC = 130°$

Reflex $\angle AOC = 360° - \angle AOC$

$\Rightarrow$ Reflex $\angle AOC = 360° - 130° = 230°$

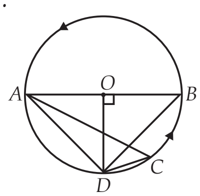
We know that, the angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.

$\therefore$ $\angle ABC = \frac{1}{2}$ (Reflex $\angle AOC$)

$= \frac{1}{2} \times 230° = 115°$

12. Given, circle $C(O, r)$ and $OD \perp AB$.

$\therefore$ $\angle AOD = \angle BOD = 90°$



We know, angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of circle.

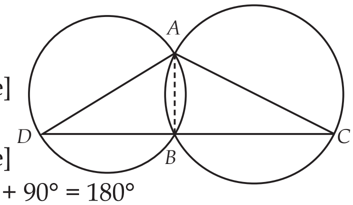
So, $\angle BOD = 2\angle BAD$

$\Rightarrow$ $\angle BAD = \frac{1}{2}\angle BOD = \frac{1}{2} \times 90° = 45°$

Similarly, $\angle AOD = 2\angle ACD$

$\Rightarrow$ $\angle ACD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 90° = 45°$

13. Join AB.



$\angle ABD = 90°$

[Angle in a semi-circle]

$\angle ABC = 90°$

[Angle in a semi-circle]

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

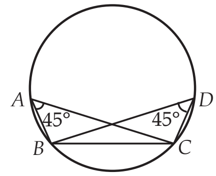
Therefore, DBC is a straight line. Thus, B lies on the line segment DC.

14. $\angle ACB = \angle BDA$ [$\because$ Angles in the same segment.]

But, $\angle ACB = 40°$ [Given]

$\Rightarrow$ $y = 40°$

15. True



Given, $\angle BAC = 45°$ and $\angle BDC = 45°$, which shows that angles in the same segment of a circle are equal. Thus, A, B, C and D are concyclic.

16. Given, $ED \parallel AC$, $\angle CBE = 50°$

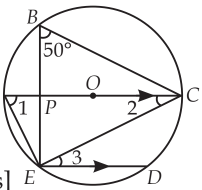
$\angle CBE = \angle 1$ [Angles in the same segment]

$\therefore$ $\angle 1 = 50°$ ...(i) ($\because$ $\angle CBE = 50°$)

$\angle AEC = 90°$ ...(ii) [Angle in a semi-circle]

Now, in ΔAEC,

$\angle 1 + \angle AEC + \angle 2 = 180°$ [By Angle sum property of a triangle]



$\Rightarrow$ $50° + 90° + \angle 2 = 180°$

[From (i) and (ii)]

$\Rightarrow$ $\angle 2 = 180° - 140°$

$\Rightarrow$ $\angle 2 = 40°$ ...(iii)

Now, $ED \parallel AC$ [Given]

$\therefore$ $\angle 2 = \angle 3$ [Alternate interior angles]

$\therefore$ $\angle 3 = 40°$ *i.e.*, $\angle CED = 40°$

17. Since, MAB is a straight line.

$\therefore$ $\angle MAD + \angle DAB = 180°$

$\Rightarrow$ $\angle DAB = 180° - \angle MAD = 180° - 110°$

$\therefore$ $\angle DAB = 70°$

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore$ $\angle BAD + \angle BCD = 180°$

$\Rightarrow$ $\angle BCD = 180° - \angle BAD = 180° - 70° \Rightarrow \angle BCD = 110°$

Now, DCN is a straight line.

$\therefore$ $\angle DCB + \angle BCN = 180°$

$\Rightarrow$ $\angle BCN = 180° - \angle DCB = 180° - 110°$

$\therefore$ $\angle BCN = 70°$

18. Since, PSY is a straight line.

$\therefore$ $\angle PSR + \angle RSY = 180°$

$\Rightarrow$ $\angle PSR = 180° - \angle RSY = 180° - 74°$

$\therefore$ $\angle PSR = 106°$ ...(i)

Now, Reflex $\angle POR = 2 \times \angle PSR$

[$\because$Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\therefore$ Reflex $\angle POR = 2 \times 106°$ [Using (i)]

$= 212°$

19. Given : $ABCD$ is a parallelogram. A circle, whose centre O, passes through A, B is so drawn that it intersects AD at P and BC at Q.

To prove : P, Q, C and D are concyclic.

Construction : Join PQ.

Proof : $\because$ A, P, Q and B are four points lying on a circle.

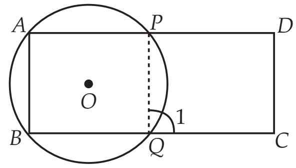
$\therefore$ $APQB$ is a cyclic quadrilateral.

$\angle 1 = \angle A$ [Exterior angle property of a cyclic quadrilateral]

But $\angle A = \angle C$ [Opposite angles of parallelogram $ABCD$]

$\therefore$ $\angle 1 = \angle C$ ... (i)

But $\angle C + \angle D = 180°$ [Sum of co-interior angles is 180°]



$\Rightarrow$ $\angle 1 + \angle D = 180°$ [From (i)]

Thus, the quadrilateral $QCDP$ is cyclic.

So, the points P, Q, C and D are concyclic.