Surface Areas and Volumes

CHAPTER 13

NCERT FOCUS **SOLUTIONS**

EXERCISE - 13.2

1. Let r be the radius of the cylinder.
Here, height (h) = 14 cm and curved surface area = 88 cm^2
Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow$ $2\pi rh = 88 \Rightarrow 2\times\frac{22}{7}\times r\times 14 = 88$

$\Rightarrow$ $r = \frac{88\times 7}{2\times 22\times 14} = 1$ cm

$\therefore$ Diameter = 2 × r = (2 × 1) cm = 2 cm

2. Here, height (h) = 1 m
Diameter of the base = 140 cm = 1.40 m

$\therefore$ Radius (r) $= \frac{1.40}{2} = 0.70$ m

Total surface area of the cylinder = $2\pi r(h + r)$

$= 2\times\frac{22}{7}\times 0.70(1+0.70)$ = 2 × 22 × 0.10 (1.70)

$= 44\times\frac{17}{100} = \frac{748}{100} = 7.48\text{m}^2$

Hence, area of the required sheet is 7.48 m^2

3. Length of the metal pipe = 77 cm
It is in the form of a cylinder
$\therefore$ Height (h) of the cylinder = 77 cm
Inner diameter = 4 cm

$\Rightarrow$ Inner radius (r) $= \frac{4}{2}$ = 2 cm

Outer diameter = 4.4 cm

$\Rightarrow$ Outer radius (R) $= \frac{4.4}{2}$ = 2.2 cm

(i) Inner curved surface area = $2\pi rh$

$= 2\times\frac{22}{7}\times 2\times 77 = 968$ cm^2

(ii) Outer curved surface area = $2\pi Rh$

$= 2\times\frac{22}{7}\times 2.2\times 77 = \frac{10648}{10} = 1064.8$ cm^2

(iii) Total surface area = [Inner curved surface area] + [Outer curved surface area] + [Surface area of two circular bases] = $(2\pi rh) + (2\pi Rh) + [2\pi(R^2 - r^2)]$

$= 968 + 1064.8 + 2\times\frac{22}{7}\left[(2.2)^2 - (2)^2\right]$

$= 2032.8 + \frac{2\times 22\times 0.84}{7}$

= 2032.8 + 5.28 = 2038.08 cm^2

4. Diameter of roller = 84 cm

$\Rightarrow$ Radius of roller $= \frac{84}{2} = 42$ cm

Length of the roller = 120 cm
Curved surface area of the roller = $2\pi rh$

$= 2\times\frac{22}{7}\times 42\times 120$ = 2 × 22 × 6 × 120 = 31680 cm^2

$\therefore$ Area of the playground levelled in one revolution by the roller = 31680 cm^2 $= \frac{31680}{10000}$ m^2

$\therefore$ Area levelled in 500 revolutions

$= 500\times\frac{31680}{10000} = \frac{5\times 3168}{10} = 1584$ m^2

5. Diameter of the pillar = 50 cm

$\therefore$ Radius (r) $= \frac{50}{2}$ = 25 cm $= \frac{1}{4}$ m

and height (h) = 3.5 m
Now, curved surface area of pillar = $2\pi rh$

$= 2\times\frac{22}{7}\times\frac{1}{4}\times 3.50 = \frac{44\times 350}{7\times 4\times 100} = \frac{11}{2}$ m^2

Cost of painting of 1 m^2 area = ₹ 12.50

$\therefore$ Cost of painting of $\frac{11}{2}$ m^2 area = ₹ $\left(\frac{11}{2}\times 12.50\right)$ = ₹ 68.75.

6. Radius (r) = 0.7 m
Let height of the cylinder be h m.
Curved surface area of a cylinder = $2\pi rh$

$\Rightarrow$ $2\times\frac{22}{7}\times\frac{7}{10}\times h = 4.4 \Rightarrow h = \frac{44}{10}\times\frac{7}{22}\times\frac{10}{7}\times\frac{1}{2} = 1$ m

Thus, the required height is 1 m.

7. Inner diameter of the well = 3.5 m

$\therefore$ Radius of the well = $\frac{3.5}{2}$ m

and height (h) of the well = 10 m

(i) Inner curved surface area = $2\pi rh = 2\times\frac{22}{7}\times\frac{3.5}{2}\times 10$

$= \frac{2\times 22\times 35\times 10}{7\times 2\times 10} = 110$ m^2

(ii) Cost of plastering per m^2 = ₹ 40
$\therefore$ Total cost of plastering the area of 110 m^2 = ₹ (110 × 40)
= ₹ 4400

8. Length of the cylindrical pipe = 28 m
i.e., h = 28 m
Diameter of the pipe = 5 cm

$\therefore$ Radius (r) = $\frac{5}{2}$ cm = $\frac{5}{200}$ m

Curved surface area of a cylinder = $2\pi rh$

$= 2\times\frac{22}{7}\times\frac{5}{200}\times 28 = \frac{22\times5\times4}{100} = \frac{440}{100} = 4.4\text{ m}^2$

Thus, the total radiating surface is 4.40 m^2.

9. The storage tank is in the form of a cylinder

$\therefore$ Diameter of the tank = 4.2 m

$\Rightarrow$ Radius $(r) = \frac{4.2}{2} = 2.1\text{ m}$ and height (h) = 4.5 m

Now,

(i) Lateral (or curved) surface area = $2\pi rh$

$= 2\times\frac{22}{7}\times 2.1\times 4.5 = 2\times 22\times 0.3\times 4.5 = 59.4\text{ m}^2$

(ii) Total surface area of the tank = $2\pi r(r + h)$

$= 2\times\frac{22}{7}\times 2.1(2.1+4.5) = 44\times 0.3\times 6.6 = 87.12\text{ m}^2$

Let actual area of the steel used be x m^2.

$\therefore$ Area of steel that wasted $= \frac{1}{12}\times x = \frac{x}{12}\text{ m}^2$

$\Rightarrow$ Area of steel used $= x - \frac{x}{12} = \frac{12x - x}{12} = \frac{11x}{12}\text{ m}^2$

$\Rightarrow \frac{11x}{12} = 87.12 \Rightarrow x = \frac{8712}{100}\times\frac{12}{11}$

$\Rightarrow x = \frac{104544}{1100} \Rightarrow x = 95.04\text{ m}^2$

Thus, the required area of the steel that was actually used is 95.04 m^2.

10. The lampshade is in the form of a cylinder, where radius $= \frac{20}{2} = 10\text{ cm}$ and height = 30 cm.

A margin of 2.5 cm is to be added to top and bottom

$\therefore$ Total height of the cylinder, h

= 30 cm + 2.5 cm + 2.5 cm = 35 cm

Now, curved surface area = $2\pi rh$

$= 2\times\frac{22}{7}\times 10\times(35) = 2\times 22\times 10\times 5 = 2200\text{ cm}^2$

Thus, the required area of the cloth is 2200 cm^2.

11. Here, the penholders are in the form of cylinders

Radius of penholder (r) = 3 cm

Height of penholder (h) = 10.5 cm

Since, a penholder must be open from the top

Now, Surface area of a penholder (cylinder)

= [Lateral surface area] + [Base area] = $[2\pi rh] + \pi r^2$

$= \left(2\times\frac{22}{7}\times 3\times 10.5\right) + \left(\frac{22}{7}\times 3\times 3\right)$

$= (44\times 3\times 1.5) + \frac{198}{7}$

$= 198 + \frac{198}{7} = \frac{1386+198}{7} = \frac{1584}{7}\text{ cm}^2$

$\therefore$ Surface area of 35 penholders

$= 35\times\frac{1584}{7} = 5\times 1584 = 7920\text{ cm}^2$

Thus, 7920 cm^2 of cardboard was required.

EXERCISE - 13.3

1. Here, diameter of the base of a cone = 10.5 cm

$\Rightarrow$ Radius $(r) = \frac{10.5}{2}$ cm

and Slant height (l) = 10 cm

$\therefore$ Curved surface area of the cone = πrl

$= \frac{22}{7}\times\frac{10.5}{2}\times 10 = (11\times 15\times 1)\text{ cm}^2 = 165\text{ cm}^2$

2. Here, diameter = 24 m and slant height (l) = 21 m

$\therefore$ Radius $(r) = \frac{24}{2} = 12$ m

$\therefore$ Total surface area = $\pi r(r + l)$

$= \frac{22}{7}\times 12\times(12+21) = \frac{22}{7}\times 12\times 33$

$= \frac{8712}{7} = 1244.57\text{ m}^2$ (approx.)

3. Here, curved surface area = 308 cm^2

Slant height (l) = 14 cm

(i) Let the radius of the base be 'r' cm.

$\therefore \pi rl = 308 \Rightarrow \frac{22}{7}\times r\times 14 = 308$

$\Rightarrow r = \frac{308\times 7}{22\times 14} = 7\text{ cm}$

Thus, radius of the cone is 7 cm.

(ii) Base area $= \pi r^2 = \frac{22}{7}\times 7^2 = 154\text{ cm}^2$

and curved surface area = 308 cm^2 [Given]

$\therefore$ Total surface area = [Curved surface area] + [Base area]

$= (308 + 154)\text{ cm}^2 = 462\text{ cm}^2$

4. Here, height of the tent (h) = 10 m

Radius of the base (r) = 24 m

(i) The slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{24^2 + 10^2} = \sqrt{576+100} = \sqrt{676} = 26$ m

Thus, the required slant height of the tent is 26 m.

(ii) Curved surface area of the cone = πrl

$\therefore$ Area of the canvas required

$= \frac{22}{7}\times 24\times 26 = \frac{13728}{7}\text{ m}^2$

Cost of 1 m^2 canvas = ₹ 70

$\therefore$ Cost of $\frac{13728}{7}\text{ m}^2$ canvas = ₹$\left(70\times\frac{13728}{7}\right)$ = ₹ 137280

5. Here, Base radius (r) = 6 m; Height (h) = 8 m

$\therefore$ Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ m

Now, curved surface area = πrl = 3.14 × 6 × 10 = 188.4 m^2

Thus, area of the canvas (tarpaulin) required to make the tent = 188.4 m^2

Let the length of the tarpaulin be L m.

$\therefore$ Length × breadth = 188.4

$\Rightarrow$ $L \times 3 = 188.4 \Rightarrow L = \frac{188.4}{3} = 62.8\,\text{m}$

Extra length of tarpaulin for margins = 20 cm

$= \frac{20}{100}\,\text{m} = 0.2\,\text{m}$

Thus, total length of tarpaulin required

= (62.8 + 0.2) m = 63 m

6. Here, base radius (r) = $\frac{14}{2}$ = 7 m and

Slant height (l) = 25 m

$\therefore$ Curved surface area = πrl

$= \frac{22}{7} \times 7 \times 25 = 22 \times 25 = 550\ \text{m}^2$

Cost of white washing 100 m^2 area = ₹ 210

$\therefore$ Cost of whitewashing 550 m^2 area

= ₹ $\left(\frac{210}{100} \times 550\right)$ = ₹ 1155

7. Radius of the base (r) = 7 cm and

height (h) = 24 cm

So, slant height $(l) = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ cm

Now, Lateral surface area = $\pi rl = \frac{22}{7} \times 7 \times 25 = 550\,\text{cm}^2$

So, sheet required to make 1 cap = 550 cm^2

$\therefore$ Sheet required to make 10 such caps = (10 × 550) cm^2

= 5500 cm^2

8. Diameter of the base = 40 cm

$\Rightarrow$ Radius (r) = $\frac{40}{2}$ cm = 20 cm = $\frac{20}{100}$ m = 0.2 m

Height (h) = 1 m

$\Rightarrow$ Slant height (l) $= \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2} = \sqrt{1.04}$

= 1.02 m [$\sqrt{1.04} = 1.02$ (Given)]

Now, curved surface area = πrl

$\therefore$ Curved surface area of 1 cone

$= 3.14 \times 0.2 \times 1.02 = \left(\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right)\text{m}^2$

$\Rightarrow$ Curved surface area of 50 cones

$= 50 \times \left[\frac{314}{100} \times \frac{2}{10} \times \frac{102}{100}\right] = \left(\frac{314 \times 102}{10 \times 100}\right)\text{m}^2$

Cost of painting 1 m^2 area = ₹ 12

$\therefore$ Cost of painting $\left[\frac{314 \times 102}{1000}\right]\text{m}^2$ area

= ₹ $\left(\frac{12 \times 314 \times 102}{1000}\right)$ = ₹ $\frac{384336}{1000}$

= ₹ 384.336 = ₹ 384.34 (approx)

Thus, the required cost of painting is ₹ 384.34 (approx).

EXERCISE - 13.4

1. (i) Here, r = 10.5 cm

$\therefore$ Surface area of a sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2 = 4 \times \frac{22}{7} \times \frac{105}{10} \times \frac{105}{10} = 1386\ \text{cm}^2$

(ii) Here, r = 5.6 cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (5.6)^2$

$= 4 \times \frac{22}{7} \times \frac{56}{10} \times \frac{56}{10} = 394.24\ \text{cm}^2$

(iii) Here, r = 14 cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times (14)^2$

$= 4 \times \frac{22}{7} \times 14 \times 14 = 2464\ \text{cm}^2$

2. (i) Here, Diameter = 14 cm

$\Rightarrow$ Radius (r) $= \frac{14}{2} = 7\,\text{cm}$

$\therefore$ Surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2$

$= 4 \times \frac{22}{7} \times 7 \times 7 = 88 \times 7 = 616\ \text{cm}^2$

(ii) Here, Diameter = 21 cm

$\Rightarrow$ Radius (r) $= \frac{21}{2}\,\text{cm}$

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386\ \text{cm}^2$

(iii) Here, Diameter = 3.5 m

$\Rightarrow$ Radius (r) $= \frac{3.5}{2}\,\text{m} = \frac{35}{20}\,\text{m}$

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{35}{20}\right)^2$

$= 4 \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} = 38.5\,\text{m}^2$

3. Here, radius (r) = 10 cm

Total surface area of hemisphere = $3\pi r^2$

= 3 × 3.14 × 10 × 10 = 942 cm^2

4. **Case I :** When radius (r_1) = 7 cm

Surface area $= 4\pi r_1^2 = 4 \times \frac{22}{7} \times (7)^2 = 616\ \text{cm}^2$

Case II : When radius (r_2) = 14 cm

Surface area $= 4\pi r_2^2 = 4 \times \frac{22}{7} \times 14 \times 14 = 2464\ \text{cm}^2$

$\therefore$ Required ratio $= \frac{616}{2464} = \frac{1}{4}$

Hence, the required ratio is 1 : 4.

5. Inner diameter of the hemisphere = 10.5 cm

$\therefore$ Radius $(r) = \frac{10.5}{2}\text{cm} = \frac{105}{20}\text{cm}$

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Inner curved surface area of hemispherical bowl

$= 2\times\frac{22}{7}\times\left(\frac{105}{20}\right)^2 = \frac{17325}{100}\text{cm}^2$

Cost of tin-plating 100 cm^2 area = ₹ 16

$\therefore$ Cost of tin-plating $\frac{17325}{100}\text{cm}^2$ area

$= ₹\left(\frac{16}{100}\times\frac{17325}{100}\right) = ₹\ 27.72$

6. Let the radius of the sphere be r cm.

Surface area = $4\pi r^2$

$\Rightarrow 4\pi r^2 = 154 \Rightarrow 4\times\frac{22}{7}\times r^2 = 154$

$\Rightarrow r^2 = \frac{154\times 7}{4\times 22} = \left(\frac{7}{2}\right)^2 \Rightarrow r = \frac{7}{2} = 3.5$

Thus, the required radius of the sphere is 3.5 cm.

7. Let the radius of the earth be r.

$\therefore$ Radius of the moon = $\frac{r}{4}$

Surface area of a sphere = $4\pi r^2$

Since, the earth as well as the moon are considered to be spheres.

$\therefore$ Surface area of the earth = $4\pi r^2$

and surface area of the moon $= 4\pi\left(\frac{r}{4}\right)^2$

$\therefore \frac{\text{Surface area of moon}}{\text{Surface area of earth}} = \frac{4\pi\left(\frac{r}{4}\right)^2}{4\pi r^2} = \frac{\left(\frac{r}{4}\right)^2}{r^2} = \frac{r^2}{16r^2} = \frac{1}{16}$

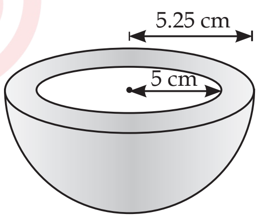
Thus, the required ratio = 1 : 16.

8. Inner radius (r) = 5 cm

Thickness = 0.25 cm

$\therefore$ Outer radius (R)

= (5.00 + 0.25) cm = 5.25 cm



$\therefore$ Outer curved surface area of the bowl = $2\pi R^2$

$= 2\times\frac{22}{7}\times(5.25)^2 = 173.25\ \text{cm}^2$

9. (i) For the sphere radius = r

$\therefore$ Surface area of the sphere = $4\pi r^2$

(ii) For the right circular cylinder :

Radius of the cylinder = Radius of the sphere

$\therefore$ Radius of the cylinder = r

Height of the cylinder = Diameter of the sphere

$\therefore$ Height of the cylinder $(h) = 2r$

Since, curved surface area of the cylinder = $2\pi rh$

$= 2\pi r(2r) = 4\pi r^2$

(iii) $\frac{\text{Surface area of the sphere}}{\text{Surface area of the cylinder}} = \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$

Thus, the required ratio is 1 : 1.

EXERCISE - 13.6

1. Let the base radius of the cylindrical vessel be r cm.
We have,
Circumference = $2\pi r$

$\Rightarrow 2\pi r = 132$ [$\because$ Circumference = 132 cm]

$\Rightarrow 2\times\frac{22}{7}\times r = 132 \Rightarrow r = \frac{132\times 7}{2\times 22} = 21$ cm

Since height of the vessel (h) = 25 cm

$\therefore$ Volume of cylinder = $\pi r^2 h = \frac{22}{7}\times 21\times 21\times 25$

$= 34650\ \text{cm}^3$

Capacity of the vessel = volume of the vessel = 34650 cm^3
Since, 1000 cm^3 = 1 litre

$\Rightarrow 34650\ \text{cm}^3 = \frac{34650}{1000} l = 34.65\ l$

Thus, the vessel can hold 34.65 l of water.

2. Inner diameter of the cylindrical pipe = 24 cm

$\Rightarrow$ Inner radius of the pipe $(r) = \frac{24}{2}\text{cm} = 12\,\text{cm}$

Outer diameter of the pipe = 28 cm

$\Rightarrow$ Outer radius of the pipe $(R) = \frac{28}{2}\text{cm} = 14\,\text{cm}$

Length of the pipe (h) = 35 cm

$\therefore$ Amount of wood (volume) in the pipe

= Outer volume – Inner volume = $\pi R^2 h - \pi r^2 h$

$= \pi h\,(R + r)\,(R - r)$

$= \frac{22}{7}\times 35\times(14+12)\times(14-12) = 22\times 5\times 26\times 2 = 5720\ \text{cm}^3$

Mass of 1 cm^3 of wood = 0.6 g [Given]

$\Rightarrow$ Mass of 5720 cm^3 of wood = 5720 × 0.6 g = 3432 g

$= \frac{3432}{1000}\text{kg} = 3.432\,\text{kg}$ [$\because$ 1000 g = 1 kg]

Thus, the required mass of the pipe is 3.432 kg.

3. For rectangular pack

Length (l) = 5 cm, Breadth (b) = 4 cm
Height (h) = 15 cm
Volume = $l\times b\times h = 5\times 4\times 15 = 300\ \text{cm}^3$

$\therefore$ Capacity of the rectangular pack = 300 cm^3

For cylindrical pack, base diameter = 7 cm

$\therefore$ Radius of the base $(r) = \frac{7}{2}$cm

Height (h) = 10 cm

$\therefore$ Volume $= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 10 = 11\times 7\times 5 = 385\ \text{cm}^3$

$\therefore$ Volume of the cylindrical pack = 385 cm^3

So, the cylindrical pack has greater capacity by 85 cm^3.

4. Height of the cylinder (h) = 5 cm

Let the base radius of the cylinder be r cm.

(i) Lateral surface of the cylinder = 94.2 cm^2

$\Rightarrow$ $2\pi rh = 94.2 \Rightarrow 2 \times 3.14 \times r \times 5 = \dfrac{942}{10}$

$\Rightarrow \dfrac{10 \times 314}{100} \times r = \dfrac{942}{10} \Rightarrow r = \dfrac{471}{157} = 3\,\text{cm}$

Thus, the radius of the cylinder = 3 cm

(ii) Volume of a cylinder = $\pi r^2 h$

$\Rightarrow$ Volume of the given cylinder = $3.14 \times (3)^2 \times 5$

$= 141.3\ \text{cm}^3$

Thus, the required volume is 141.3 cm^3.

5. (i) Total cost of painting = ₹ 2200

Cost of painting 1 m^2 = ₹ 20

$\therefore$ Area $= \dfrac{\text{Total cost}}{\text{Cost of } 1\text{m}^2} = \dfrac{2200}{20} = 110\,\text{m}^2$

$\Rightarrow$ Inner curved surface of the vessel = 110 m^2

(ii) Let r and h be the base radius and height of the cylindrical vessel.

Curved surface area of a cylinder = $2\pi rh$

$\therefore$ $2\pi rh = 110 \Rightarrow 2 \times \dfrac{22}{7} \times r \times 10 = 110$

[Given, height = 10 m]

$\Rightarrow r = \dfrac{110 \times 7}{2 \times 22 \times 10} = \dfrac{7}{4} \Rightarrow r = 1.75$ m

$\therefore$ The required radius of the base = 1.75 m

(iii) Since, volume of a cylinder = $\pi r^2 h$

$\Rightarrow$ Volume (capacity) of the vessel

$= \dfrac{22}{7} \times \left(\dfrac{7}{4}\right)^2 \times 10 = \dfrac{385}{4} = 96.25\,\text{m}^3$

Since, 1 m^3 = 1000000 cm^3 = 1000 l = 1 kl

$\therefore$ 96.25 m^3 = 96.25 kl

Thus, the required volume = 96.25 kl

6. Capacity of the cylindrical vessel

= 15.4 l = 15.4 × 1000 cm^3 [$\because$ 1 l = 1000 cm^3]

$= \dfrac{15.4 \times 1000}{1000000}\,\text{m}^3 = \dfrac{15.4}{1000}\,\text{m}^3$ [$\because$ 1000000 cm^3 = 1m^3]

Now, volume of the vessel $= \dfrac{15.4}{1000}\,\text{m}^3$

and height of the vessel = 1 m

Let radius of the base of the vessel be r m.

Now, Volume = $\pi r^2 h \Rightarrow \pi r^2 h = \dfrac{15.4}{1000}$

$\Rightarrow \dfrac{22}{7} \times r^2 \times 1 = \dfrac{154}{10000} \Rightarrow r^2 = \dfrac{154}{10000} \times \dfrac{7}{22} = \dfrac{49}{10000}$

$\Rightarrow r^2 = \left(\dfrac{7}{100}\right)^2 \Rightarrow r = \dfrac{7}{100}\,\text{m}$

Now, total surface area of the cylindrical vessel

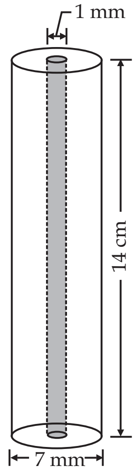
$= 2\pi r(h + r) = 2 \times \dfrac{22}{7} \times \dfrac{7}{100}\left(1 + \dfrac{7}{100}\right)$

$= \dfrac{44}{100} \times \left(1 + \dfrac{7}{100}\right) = 0.4708$

Thus, 0.4708 m^2 sheet is required.

7. Since, 10 mm = 1 cm

$\therefore$ $1\,\text{mm} = \dfrac{1}{10}\,\text{cm}$



For graphite cylinder,

Diameter = $1\,\text{mm} = \dfrac{1}{10}\,\text{cm}$

$\Rightarrow$ Radius (r) = $\dfrac{1}{10} \times \dfrac{1}{2} = \dfrac{1}{20}\,\text{cm}$

Length (h) = 14 cm

$\therefore$ Volume = $\pi r^2 h$

$= \dfrac{22}{7} \times \dfrac{1}{20} \times \dfrac{1}{20} \times 14 = 0.11$

Thus, the required volume of the graphite = 0.11 cm^3

Now, diameter of the pencil = 7 mm $= \dfrac{7}{10}\,\text{cm}$

$\therefore$ Radius of the pencil (R) $= \dfrac{7}{20}\,\text{cm}$

Height of the pencil (h) = 14 cm

Volume of the pencil = $\pi R^2 h$

$= \dfrac{22}{7} \times \left(\dfrac{7}{20}\right)^2 \times 14 = \dfrac{22}{7} \times \dfrac{7}{20} \times \dfrac{7}{20} \times 14$

$= \dfrac{11 \times 7 \times 7}{100} = 5.39\,\text{cm}^3$

Volume of the wood = Volume of the pencil – Volume of the graphite

= 5.39 cm^3 – 0.11 cm^3 = 5.28 cm^3

Thus, the required volume of the wood is 5.28 cm^3

8. Diameter of the base of cylindrical bowl = 7 cm

$\Rightarrow$ Radius of the base (r) = $\dfrac{7}{2}$cm and height (h) = 4 cm

$\therefore$ Volume of one bowl = $\pi r^2 h$

$= \dfrac{22}{7} \times \left(\dfrac{7}{2}\right)^2 \times 4 = \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times 4$

= 11 × 7 × 2 = 154 cm^3

i.e., Volume of soup in a bowl = 154 cm^3

$\Rightarrow$ Volume of soup in 250 bowls = 250 × 154 cm^3

= 38500 cm^3 $= \dfrac{38500}{1000}\,l = 38.5\,l$ $\left[\because 1\,\text{cm}^3 = \dfrac{1}{1000}\,l\right]$

Thus, the hospital needs to prepare 38.5 litres of soup daily for 250 patients.

EXERCISE - 13.7

1. (i) Radius of the cone (r) = 6 cm

Height (h) = 7 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 6\times 6\times 7$

= 22 × 2 × 6 = 264 cm^3

(ii) Here, radius of the cone (r) = 3.5 cm $= \frac{35}{10}$ cm

Height (h) = 12 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times\left(\frac{35}{10}\right)^2\times 12 = \frac{1}{3}\times\frac{22}{7}\times\frac{35}{10}\times\frac{35}{10}\times 12$ = 154 cm^3

2. (i) Here, r = 7 cm and l = 25 cm

$\therefore$ $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = 24$ cm

Now, volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(7)^2\times 24 = \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24$

= 22 × 7 × 8 = 1232 cm^3

$= \frac{1232}{1000} l = 1.232\ l$ [$\because$ 1000 cm^3 = 1 l]

Thus, the required capacity of the conical vessel is 1.232 l.

(ii) Here, height (h) = 12 cm and l = 13 cm

$\therefore$ $r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144}$ = 5 cm

Now, volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times(5)^2\times 12$

$= \frac{22\times 5\times 5\times 4}{7} = \frac{2200}{7}\text{cm}^3 = \frac{2200}{7\times 1000} l$ [$\because$ 1000 cm^3 = 1 l]

Thus, the required capacity of the conical vessel is $\frac{11}{35} l$.

3. Here, height of the cone (h) = 15 cm

Volume of the cone = 1570 cm^3 [Given]

Let the radius of the base be r cm.

$\therefore$ $\frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow$ $\frac{1}{3}\times 3.14\times r^2\times 15 = 1570$ $\Rightarrow$ $\frac{1}{3}\times\frac{314}{100}\times r^2\times 15 = 1570$

$\Rightarrow$ $r^2 = \frac{1570\times 3\times 100}{314\times 15} = \frac{5\times 3\times 100}{15} = 100$

$\Rightarrow$ $r^2 = 10^2 \Rightarrow r = \sqrt{10^2}$ = 10 cm

Thus, the required radius of the base is 10 cm.

4. Volume of the cone = 48π cm^3 [Given]

Height of the cone (h) = 9 cm

Let r be its base radius.

$\therefore$ $\frac{1}{3}\pi r^2 h = 48\pi$ $\Rightarrow$ $\frac{1}{3}\pi r^2\times 9 = 48\pi$

$\Rightarrow$ $r^2 = \frac{48\times\pi\times 3}{9\times\pi} = 16 = 4^2$ $\Rightarrow$ $r = \sqrt{4^2}$ = 4 cm

$\therefore$ Diameter of the base of the cone = 2 × 4 = 8 cm

5. Here, diameter of the conical pit = 3.5 m

$\therefore$ Radius (r) $= \frac{3.5}{2} = \frac{35}{20}$ m, Depth (h) = 12 m

$\therefore$ Volume (capacity) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times\left(\frac{35}{20}\right)^2\times 12 = \frac{1}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times 12$

$= \frac{11\times 35}{10} = \frac{385}{10}$ = 38.5 m^3

$\because$ 1 m^3 = 1 kl $\Rightarrow$ 38.5 m^3 = 38.5 kl

Thus, the capacity of the conical pit is 38.5 kl.

6. Volume of the cone = 9856 cm^3

Diameter of the base = 28 cm

$\Rightarrow$ Radius of the base $= \frac{28}{2}$ = 14 cm

(i) Let the height of the cone be h cm.

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(14)^2\times h$

$= \frac{1}{3}\times\frac{22}{7}\times 14\times 14\times h$

$\Rightarrow$ $\frac{1}{3}\times 22\times 2\times 14\times h = 9856$

$\Rightarrow$ $h = \frac{9856\times 3}{22\times 2\times 14}$ = 16 × 3 = 48

Thus, the required height is 48 cm.

(ii) Let the slant height be l cm.

$\Rightarrow$ $l^2 = r^2 + h^2 \Rightarrow l^2 = 14^2 + 48^2 = 196 + 2304 = 2500 = (50)^2$

$\therefore$ l = 50

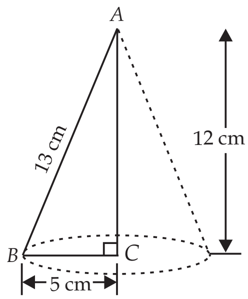
Thus, the required slant height = 50 cm.

(iii) The curved surface area of a cone is given by πrl

$\therefore$ Curved surface area $= \frac{22}{7}\times 14\times 50$ = 22 × 2 × 50 = 2200

Thus, the curved surface area of the cone is 2200 cm^2.

7. Sides of the right triangle are 5 cm, 12 cm and 13 cm. When this triangle is revolved about the side of 12 cm, we get a cone as shown in the figure.



Thus, radius of the base of the cone so formed (r) = 5 cm

Height (h) = 12 cm

$\therefore$ Volume of the cone so obtained

$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\pi\times(5)^2\times 12$

$= \frac{1}{3}\times\pi\times 5\times 5\times 12$ = 100 π

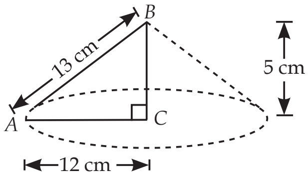
Thus, the required volume of the cone is 100 π cm^3.

8. Since the right triangle is revolved about the side of 5 cm.

∴ Height of the cone so obtained (h) = 5 cm

Radius of the cone (r) = 12 cm

∴ Volume $= \frac{1}{3}\pi r^2 h$



$= \frac{1}{3} \times \pi \times (12)^2 \times 5$

$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5$

= π × 240 = 240 π cm^3

Now, ratio of two volumes $= \frac{100\pi\, cm^3}{240\pi\, cm^3} = \frac{5}{12} = 5:12$

Thus, the required ratio is 5 : 12.

9. Here, the heap of wheat is in the form of a cone with base diameter = 10.5 m

∴ Base radius (r) $= \frac{10.5}{2} m = \frac{105}{20} m$

Height (h) = 3 m

∴ Volume of the heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{105}{20}\right)^2 \times 3 = 86.625$

Thus, the required volume = 86.625 m^3

Now the area of the canvas to cover the heap must be equal to the curved surface area of the conical heap.

∴ Area of the canvas = $\pi r l$, where $l = \sqrt{r^2 + h^2}$

$\therefore\ l = \sqrt{\left(\frac{105}{20}\right)^2 + 3^2} = \sqrt{\frac{11025}{400} + 9}$

$= \sqrt{36.5625} = 6.05$ m (approx.)

Now, $\pi r l = \frac{22}{7} \times \frac{105}{20} \times 6.05$ = 11 × 1.5 × 6.05 = 99.825 m^2

Thus, the required area of the canvas is 99.825 m^2.

EXERCISE - 13.8

1. (i) Here, radius (r) = 7 cm

∴ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (7)^3 = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = \frac{4312}{3} = 1437\frac{1}{3} cm^3$

Thus, the required volume = $1437\frac{1}{3} cm^3$

(ii) Here, radius (r) = 0.63 m

∴ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{63}{100}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{63}{100} \times \frac{63}{100} \times \frac{63}{100}$

$= \frac{1047816}{1000000}$ = 1.047816 m^3

Thus, the required volume is 1.05 m^3 (approx.)

2. (i) Diameter of the ball = 28 cm

⇒ Radius (r) $= \frac{28}{2}$ = 14 cm

∴ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = \frac{34496}{3} = 11498\frac{2}{3} cm^3$

Thus, the amount of water displaced $= 11498\frac{2}{3} cm^3$

(ii) Diameter of the ball = 0.21 m

⇒ Radius (r) $= \frac{0.21}{2} = \frac{21}{200} m$

∴ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{200}\right)^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{200} \times \frac{21}{200} \times \frac{21}{200}$

$= \frac{11 \times 21 \times 21}{1000000} = \frac{4851}{1000000}$ = 0.004851 m^3

3. Diameter of a metallic ball = 4.2 cm

⇒ Radius (r) = $\frac{4.2}{2}$ = 2.1 cm

∴ Volume of the metallic ball

$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10} = \frac{4 \times 22 \times 21 \times 21}{10 \times 10 \times 10} cm^3$

Also, density of the metal = 8.9 g per cm^3 [Given]

∴ Mass of the ball = 8.9 × [Volume of the ball]

$= \frac{89}{10} \times \frac{4 \times 22 \times 21 \times 21}{10 \times 10 \times 10} = \frac{3453912}{10000} = 345.3912 g$

= 345.39 g (approx.)

Thus, the mass of the ball is 345.39 g (approx.)

4. Let radius of the earth = r

Diameter of the moon $= \frac{1}{4}$(Diameter of the earth) [Given]

⇒ Radius of the moon $= \frac{1}{4}$(Radius of the earth)

⇒ Radius of the moon $= \frac{1}{4}(r) = \frac{r}{4}$

Volume of the earth $= \frac{4}{3}\pi r^3$

Volume of the moon $= \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = \frac{4}{3} \times \pi \times \frac{r \times r \times r}{4 \times 4 \times 4} = \frac{\pi r^3}{48}$

Now, $\dfrac{\text{Volume of the earth}}{\text{Volume of the moon}} = \dfrac{\frac{4}{3}\pi r^3}{\frac{\pi r^3}{48}} = \dfrac{4}{3}\pi r^3 \times \dfrac{48}{\pi r^3} = \dfrac{64}{1}$

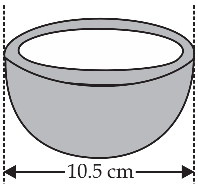
or Volume of the moon $= \dfrac{1}{64} \times$ Volume of the earth

$\therefore$ The required fraction is $\dfrac{1}{64}$.

5. Diameter of the hemisphere = 10.5 cm

$\Rightarrow$ Radius $(r) = \dfrac{10.5}{2} = \dfrac{105}{20}$ cm

Volume of the hemispherical bowl $= \dfrac{2}{3}\pi r^3$



$= \dfrac{2}{3} \times \dfrac{22}{7} \times \dfrac{105}{20} \times \dfrac{105}{20} \times \dfrac{105}{20}$

$= \dfrac{11 \times 105 \times 105}{20 \times 20} = 303.1875 \text{ cm}^3$

$\therefore$ Capacity of the hemispherical bowl = 303.1875 cm^3

$= \dfrac{3031875}{10000 \times 1000}\, l = 0.3031875\, l$ [$\because$ 1000 cm^3 = 1 l]

= 0.303 l (approx.)

Thus, the capacity of the bowl is 0.303 l (approx.).

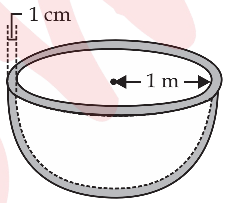
6. Inner radius (r) = 1 m

$\because$ Thickness = 1 cm = $\dfrac{1}{100}$ m = 0.01 m

$\therefore$ Outer radius (R) = 1 + 0.01 = 1.01 m

Now, volume of outer hemispherical bowl

$= \dfrac{2}{3}\pi R^3 = \dfrac{2}{3} \times \dfrac{22}{7} \times (1.01)^3$



Volume of inner hemispherical bowl

$= \dfrac{2}{3}\pi r^3 = \dfrac{2}{3} \times \dfrac{22}{7} \times (1)^3$

$\therefore$ Volume of the iron used = [Outer volume] – [Inner volume]

$= \dfrac{2}{3} \times \dfrac{22}{7} \times (1.01)^3 - \dfrac{2}{3} \times \dfrac{22}{7} \times (1)^3$

$= \dfrac{2}{3} \times \dfrac{22}{7} \times [(1.01)^3 - (1)^3]$

$= \dfrac{44}{21}(1.030301 - 1)$ = 0.06348 m^3 (approx.)

Thus, the required volume of the iron is 0.06348 m^3.

7. Let r be the radius of the sphere.

$\therefore$ Its surface area = $4\pi r^2$ $\Rightarrow$ $4\pi r^2 = 154$

$\Rightarrow$ $r^2 = \dfrac{154}{4\pi} = \dfrac{154 \times 7}{4 \times 22} = \dfrac{7 \times 7}{4} \Rightarrow r^2 = \left(\dfrac{7}{2}\right)^2 \Rightarrow r = \dfrac{7}{2}$ cm

Now, volume of the sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{7}{2}\right)^3 = \dfrac{4}{3} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= \dfrac{11 \times 7 \times 7}{3} = \dfrac{539}{3} = 179\dfrac{2}{3} \text{cm}^3$

Thus, the required volume of the sphere is $179\dfrac{2}{3}\text{cm}^3$.

8. (i) Total cost of white washing = ₹ 4989.60

Cost of white washing of 1 m^2 area = ₹ 20

$\therefore$ Inside surface area of the dome

$= \dfrac{\text{Total cost}}{\text{Cost of } 1\text{m}^2 \text{ area}} = \dfrac{4989.6}{20} = 249.48 \text{m}^2$

Thus, the required inside surface area of the dome is 249.48 m^2

(ii) Let r be the radius of the hemispherical dome

$\therefore$ Surface area = $2\pi r^2$

$\Rightarrow$ $2\pi r^2 = 249.48$ $\Rightarrow$ $2 \times \dfrac{22}{7} \times r^2 = \dfrac{24948}{100}$

$\Rightarrow$ $r^2 = \dfrac{24948}{100} \times \dfrac{7}{2 \times 22} = \dfrac{3969}{100}$

$\Rightarrow$ $r^2 = \left(\dfrac{63}{10}\right)^2$ $\Rightarrow$ $r = \dfrac{63}{10} = 6.3 \text{m}$

Volume of hemisphere $= \dfrac{2}{3}\pi r^3$

$\therefore$ Volume of air in the dome $= \dfrac{2}{3} \times \dfrac{22}{7} \times (6.3)^3$

$= \dfrac{2}{3} \times \dfrac{22}{7} \times \dfrac{63}{10} \times \dfrac{63}{10} \times \dfrac{63}{10} = \dfrac{2 \times 22 \times 3 \times 63 \times 63}{1000}$

$= \dfrac{523908}{1000} = 523.9 \text{m}^3$ (approx.)

Thus, the required volume of air inside the dome is 523.9 m^3 (approx).

9. (i) Radius of small sphere = r

$\therefore$ Its volume $= \dfrac{4}{3}\pi r^3$

Volume of 27 small spheres $= 27 \times \left(\dfrac{4}{3}\pi r^3\right)$

Radius of the new sphere = r'

$\therefore$ Volume of the new sphere $= \dfrac{4}{3}\pi (r')^3$

Since, $\dfrac{4}{3}\pi (r')^3 = 27 \times \dfrac{4}{3}\pi r^3 \Rightarrow (r')^3 = \dfrac{27 \times \frac{4}{3}\pi r^3}{\frac{4}{3}\pi} = 27r^3$

$\Rightarrow$ $(r')^3 = (3r)^3$ $\Rightarrow$ $r' = 3r$

(ii) Surface area of small sphere = $4\pi r^2$

$\Rightarrow$ $S = 4\pi r^2$ and $S' = 4\pi(3r)^2$ [$\because$ $r' = 3r$]

Now, $\dfrac{S}{S'} = \dfrac{4\pi r^2}{4\pi (3r)^2} = \dfrac{4\pi r^2}{4\pi (9r^2)} = \dfrac{1}{9}$

Thus, $S : S' = 1 : 9$

10. Diameter of the spherical capsule = 3.5 mm

$\Rightarrow$ Radius (r) $= \dfrac{3.5}{2}$ mm

$\therefore$ Volume of the spherical capsule

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\left(\frac{35}{20}\right)^3$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{35}{20}\times\frac{35}{20}\times\frac{35}{20} = \frac{22\times 35\times 35}{3\times 20\times 20}$$

$$= \frac{26950}{1200} = 22.45833 \text{ mm}^3 = 22.46 \text{ mm}^3 \text{ (approx.)}$$

Thus, the required quantity of medicine is 22.46 mm^3 (approx.).

EXERCISE - 13.9

1. Here, diameter of sphere = 21 cm

$\Rightarrow$ Radius of sphere (r) $= \dfrac{21}{2}$ cm

$\therefore$ Surface area of sphere = $4\pi r^2$

$\therefore$ Surface area of 8 spheres $= 8\times 4\times\dfrac{22}{7}\times\left(\dfrac{21}{2}\right)^2$

= 8 × 22 × 3 × 21 = 11088 cm^2

Now, radius of cylinder (r_1) = 1.5 cm

Height of cylinder (h) = 7 cm

Curved surface area of a cylinder = $2\pi r_1 h$

$\therefore$ Curved surface area of 8 cylinders

$$= 8\times 2\times\frac{22}{7}\times 1.5\times 7 = 528 \text{ cm}^2$$

Surface area of the top of 8 cylinders = $8\pi r^2$

$$= 8\times\frac{22}{7}\times(1.5)^2 = 56.57 \text{ cm}^2 \text{ (approx.)}$$

$\therefore$ Cost of painting

$$= ₹\left[(11088 - 56.57)\times\frac{25}{100} + 528\times\frac{5}{100}\right]$$

$$= ₹\left[(11031.43)\times\frac{25}{100} + \frac{2640}{100}\right] = ₹\left[\frac{275785.75}{100} + \frac{2640}{100}\right]$$

$$= ₹\left[\frac{278425.75}{100}\right] = ₹\ 2784.25$$

Hence, the cost of paint required = ₹ 2784.25 (approx.)

2. Let the diameter of a sphere be d.

After decreasing, diameter of the sphere

$$= d - \frac{25}{100}\times d = d - \frac{1}{4}d = \frac{3}{4}d$$

Since, surface area of a sphere = $4\pi r^2 = \pi(2r)^2 = \pi d^2$

Surface area of sphere, when diameter of the sphere is $\frac{3}{4}d = \pi\left(\frac{3}{4}d\right)^2 = \frac{9\pi}{16}d^2$

Now, percentage decrease in curved surface area

$$= \frac{\pi d^2 - \frac{9\pi}{16}d^2}{\pi d^2}\times 100 = \frac{16-9}{16}\times 100 = \frac{7}{16}\times 100 = 43.75\%$$