CHAPTER 13

Surface Areas and Volumes

TRY YOURSELF — SOLUTIONS

1. Curved surface area = circumference of base × height
$= 2\pi rh = 180 \times 30 = 5400 \text{ cm}^2$

2. Radius of the base of the cylindrical kaleidoscope, $(r) = 3.5$ cm.
Height (length) of kaleidoscope $(h) = 25$ cm
Area of chart paper required = curved surface area of the kaleidoscope $= 2\pi rh = 2\times\frac{22}{7}\times 3.5\times 25 = 550 \text{ cm}^2$

3. Curved surface area of a cylinder $= 2\pi rh$

$\Rightarrow \quad 8.8 = 2\times\frac{22}{7}\times 0.7\times h \Rightarrow h = \frac{8.8\times 7}{44\times 0.7} = 2$ m

Hence, the height of the cylinder is 2 m.

Total surface area $= 2\pi r(h + r) = 2\times\frac{22}{7}\times 0.7\,(2+0.7)$

$= 2\times\frac{22}{7}\times 0.7\times 2.7 = 11.88 \text{ m}^2$

4. Here, radius $(r) = 1\frac{3}{4}\text{cm} = \frac{7}{4}\text{cm}$

Height $(h) = 3$ cm
Now, area of cardboard used = Curved surface area of cylinder

$= 2\pi rh = 2\times\frac{22}{7}\times\frac{7}{4}\times 3 = 33 \text{ cm}^2$

5. External radius, $R = \frac{25}{2} = 12.5$ cm

Thickness of pipe = 1 cm [Given]
$\therefore$ Internal radius, r = (External radius – thickness)
= (12.5 – 1) cm = 11.5 cm
Given, height of the pipe = 20 m = 2000 cm
Total surface area of the pipe = External curved surface area + internal curved surface area + 2 (surface area of each base)
$= 2\pi(R + r)h + 2\pi(R^2 - r^2)$

$= 2\times\frac{22}{7}(12.5 + 11.5)2000 + 2\times\frac{22}{7}[(12.5)^2 - (11.5)^2]$

$= 2\times\frac{22}{7}\times 24\times 2000 + 2\times\frac{22}{7}\times 24 = 2\times\frac{22}{7}\times 24\times(2000+1)$

$= 2\times\frac{22}{7}\times 24\times 2001 = 301865.14 \text{ cm}^2$

6. Here external radius $(R) = 8$ cm
Internal radius $(r) = 6$ cm
and height $(h) = 2.1$ m = (2.1 × 100) cm = 210 cm
Now, CSA of hollow cylinder (pipe) $= 2\pi h(R + r)$

$= 2\times\frac{22}{7}\times 210(8+6) = 18480 \text{ cm}^2 = \frac{18480}{10000}\text{m}^2 = 1.848 \text{ m}^2$

7. Here, radius $(r) = 7$ cm and
Height $(h) = 13\sqrt{2}$ cm
Let l be the slant height of the cone.
$\therefore \quad l^2 = r^2 + h^2 = 7^2 + (13\sqrt{2})^2$

$\Rightarrow \quad l = \sqrt{49+338} = \sqrt{387} = 3\sqrt{43}$ cm

8. We have, $r = 3$ cm and $h = 4$ cm
Let l be the slant height of the cone. Then,
$l^2 = r^2 + h^2$
$\Rightarrow \quad l^2 = 3^2 + 4^2 \Rightarrow l = \sqrt{25} = 5$ cm
$\therefore$ Area of the curved surface $= \pi rl$

$= \left(\frac{22}{7}\times 3\times 5\right)\text{cm}^2 = 47.14 \text{ cm}^2$

9. Here, radius $(r) = 21$ cm and
Slant height $(l) = 60$ cm

$\therefore$ CSA of cone $= \pi rl = \frac{22}{7}\times 21\times 60 = 3960 \text{ cm}^2$

10. We have, radius $(r) = 6$ cm height $(h) = 8$ cm

$\therefore$ Slant height $(l) = \sqrt{r^2+h^2} = \sqrt{6^2+8^2} = \sqrt{100} = 10$ cm

So, TSA of cone $= \pi r(r + l) = \frac{22}{7}\times 6(6+10) = \frac{22}{7}\times 6\times 16$

$= 301.71 \text{ cm}^2$

11. Height of conical tent $(h) = 3.5$ m and radius of the base of conical tent $(r) = 12$ m

Slant height $(l) = \sqrt{h^2+r^2} = \sqrt{(3.5)^2+(12)^2}$

$= \sqrt{12.25+144} = \sqrt{156.25}$ m $= 12.5$ m

$\therefore$ Canvas required = Curved surface area of the conical tent $= \pi rl = \left(\frac{22}{7}\times 12\times 12.5\right)\text{m}^2 = 471.42 \text{ m}^2$

Hence, the canvas required to make the conical tent is 471.42 m^2.

12. let r be the radius of the sphere.

$\therefore$ Surface area of sphere $= 4\pi r^2 = 11880$ [Given]

$\Rightarrow r^2 = \frac{11880\times 7}{4\times 22} = 945 \Rightarrow r = 3\sqrt{105}$ cm

Now, diameter of sphere $= 2r = 2\times 3\sqrt{105} = 6\sqrt{105}$ cm

13. Let r_1 and r_2 be the radii of two spheres.

Then, $\frac{\text{Surface area of first sphere}}{\text{Surface area of second sphere}} = \frac{16}{49}$ (Given)

$\Rightarrow \quad \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{16}{49} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{16}{49} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{16}{49}} = \frac{4}{7}$

$\therefore$ Required ratio = 4 : 7

14. Let r units be the radius of sphere.

$\therefore$ Total surface area of sphere = $4\pi r^2$ sq. units

Radius of hemisphere (r_1) = $4r$ units

$\therefore$ Curved surface area of hemisphere = $2\pi r_1^2$

$= 2\pi(4r)^2 = 32\pi r^2$ sq. units

$\therefore$ Required ratio = $\dfrac{4\pi r^2}{32\pi r^2} = 1:8$

15. Let r cm be the radius of hemispherical dome.

Then, circumference = $2\pi r$

$\Rightarrow$ $2\pi r = 17.6$

$\Rightarrow$ $2\times\dfrac{22}{7}\times r = 17.6 \Rightarrow r = \dfrac{17.6\times 7}{2\times 22} = 2.8$ m

Surface area of inside of dome = $2\pi r^2$

$= 2\times\dfrac{22}{7}\times(2.8)^2 = 49.28\ \text{m}^2 = 49.28 \times 10000\ \text{cm}^2$

$= 492800\ \text{cm}^2$

Cost of painting 100 cm^2 = ₹ 5

Cost of painting 1 cm^2 = ₹$\left(\dfrac{5}{100}\right)$

Cost of painting 492800 cm^2 = ₹ $\left(\dfrac{5}{100}\times 492800\right)$ = ₹ 24,640

16. Here, height (h) = 1.5 m = 150 cm

Radius (r) = $\dfrac{70}{2}$ cm

$\therefore$ Volume of cylindrical rod

$= \pi r^2 h = \dfrac{22}{7}\times\dfrac{70}{2}\times\dfrac{70}{2}\times 150 = 577500\ \text{cm}^3$

17. Radius of cylindrical vessel (r) = 20 cm

Quantity of rain water in the vessel = 105600 cm^3

Let h be the height of water in the vessel.

Quantity of rain water in the vessel = volume of rainfall

$\therefore$ $\pi r^2 h = 105600$

$\Rightarrow$ $\dfrac{22}{7}\times 20\times 20\times h = 105600$

$\Rightarrow$ $h = \dfrac{105600\times 7}{22\times 20\times 20} = 84$ cm

18. Let r and h be the radius and height of cylinder.

Circumference of base of cylinder = 220 cm [Given]

$\Rightarrow$ $2\pi r = 220$

And C.S.A. = 2200 cm^2

$\Rightarrow$ $2\pi rh = 2200$

$\Rightarrow$ $220 \times h = 2200$ [$\because$ $2\pi r = 220$]

$\Rightarrow$ $h = 10$ cm

Now, $2\pi r = 220$

$\Rightarrow$ $2\times\dfrac{22}{7}\times r = 220 \Rightarrow r = 35$ cm

$\therefore$ Volume of cylinder = $\pi r^2 h$

$= \dfrac{22}{7}\times 35\times 35\times 10 = 38500\ \text{cm}^3$

19. External diameter of the pipe = 2.4 cm

External radius of the pipe, (R) = $\dfrac{2.4}{2}$ cm = 1.2 cm

Thickness of the pipe = 3 mm = 0.3 cm

Internal radius, (r) = External radius – thickness

= 1.2 cm – 0.3 cm = 0.9 cm

Length of the pipe (h) = 3.5 m = 350 cm

Volume of lead = $\pi(R^2 - r^2)h$

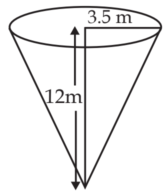
$= \dfrac{22}{7}\times[(1.2)^2 - (0.9)^2]\times 350 = \dfrac{22}{7}\times 0.63\times 350 = 693\ \text{cm}^3$

$\because$ Mass = volume × density

= 693 × 12 [$\because$ Weight of 1 cm^3 = 12 g (Given)]

$= 8316\ \text{g} = \dfrac{8316}{1000}$ kg = 8.316 kg

20. Given, diameter = 7 m $\Rightarrow$ r = 3.5 m and h = 12 m



Capacity = Volume = $\dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\times\dfrac{22}{7}\times 3.5\times 3.5\times 12$

$= 154\ \text{m}^3$

$= 154000\ l$ [1 m^3 = 1000 l]

21. Radius of base of conical granary (r) = 8.4 m

Height of base of conical granary (h) = 87.5 m

$\therefore$ Volume of conical granary = $\dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\times\dfrac{22}{7}\times(8.4)^2\times 87.5$

$= \dfrac{1}{3}\times\dfrac{22}{7}\times 8.4\times 8.4\times 87.5 = 6468\ \text{m}^3$

Volume of rice in each bag = 196 m^3 [Given]

$\therefore$ Required number of bags

$= \dfrac{\text{Volume of conical granary}}{\text{Volume of each bag}} = \dfrac{6468}{196} = 33$

22. Let r and h be the radius and height of the original right circular cone.

$\therefore$ Volume of original cone = $\dfrac{1}{3}\pi r^2 h = V$ (Given)

If radius is halved and height is doubled, then volume of new cone $= \dfrac{1}{3}\pi\times\left(\dfrac{r}{2}\right)^2\times(2h)$

$= \dfrac{1}{3}\pi\times\dfrac{r^2}{4}\times 2h = \dfrac{1}{2}\left(\dfrac{1}{3}\pi r^2 h\right) = \dfrac{1}{2}V$

23. Radius of bigger sphere (R) = 2.1 cm

$\therefore$ Volume of bigger sphere = $\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi\times(2.1)^3$ cm^3

Let radius of smaller sphere be x cm.

$\therefore$ Volume of small sphere = $\dfrac{4}{3}\pi x^3$ cm^3

Also, it is given that

Volume of bigger sphere = 8 × volume of small sphere

$\Rightarrow$ $\dfrac{4}{3}\pi(2.1)^3 = 8\times\dfrac{4}{3}\pi x^3$

$\Rightarrow$ $x^3 = \dfrac{(2.1)^3}{8} = \left(\dfrac{2.1}{2}\right)^3 \Rightarrow x = \dfrac{2.1}{2} = 1.05$ cm

24. Original radius of sphere (r_1) = 10 cm

$\therefore$ Original volume of sphere = $\dfrac{4}{3}\pi r_1^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 = 4190.5 \text{ cm}^3$ (approx.)

New radius (r_2) = 10 + 2 = 12 cm

$\therefore$ New volume of sphere = $\frac{4}{3}\pi r_2^3 = \frac{4}{3} \times \frac{22}{7} \times 12 \times 12 \times 12$

= 7241.1 cm^3 (approx.)

$\therefore$ Percentage increase in volume

$= \frac{7241.1 - 4190.5}{4190.5} \times 100 = 72.8\%$ (approx.)

25. Cost of painting 100 m^2 = ₹ 20

Cost of painting 1 m^2 = ₹ $\frac{20}{100}$

Total cost of painting inner surface of hemispherical bowl = ₹ 30.80 [Given]

$\therefore$ Inner surface area of bowl

$= \frac{\text{Total cost}}{\text{Cost per m}^2} = \frac{30.80}{20/100} = 154 \text{ m}^2$

$\Rightarrow$ $2\pi r^2 = 154$ (r is radius of hemisphere)

$\Rightarrow$ $2 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow$ $r^2 = \frac{154 \times 7}{2 \times 22} = 24.5$

$\Rightarrow$ $r = 4.9$ m (approx.)

Now, volume of air inside the bowl = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (4.9)^3 = \frac{2}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9$

= 246.5 m^3 (approx.)