Number Systems

CHAPTER 1

NCERT FOCUS

SOLUTIONS

EXERCISE - 1.1

1. Yes, zero is a rational number. Because 0 can be written in the form of p/q.

$0 = 0/1 = \frac{0}{2} = \frac{0}{3}$ etc. Denominator q can also be taken as negative integer.

2. We have, $q_1 = \frac{3+4}{2} = \frac{7}{2}; 3 < \frac{7}{2} < 4$

$$q_2 = \frac{3+\frac{7}{2}}{2} = \frac{\frac{13}{2}}{2} = \frac{13}{4} \quad \therefore\ 3 < \frac{13}{4} < \frac{7}{2} < 4$$

$$q_3 = \frac{4+\frac{7}{2}}{2} = \frac{\frac{15}{2}}{2} = \frac{15}{4} \quad \therefore\ 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$$

$$q_4 = \frac{\frac{7}{2}+\frac{13}{4}}{2} = \frac{\frac{14+13}{4}}{2} = \frac{\frac{27}{4}}{2} = \frac{27}{8}$$

$$\therefore\quad 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2} < \frac{15}{4} < 4$$

$$q_5 = \frac{1}{2}\left(\frac{7}{2}+\frac{15}{4}\right) = \frac{1}{2}\left(\frac{14+15}{4}\right) = \frac{29}{8}$$

$$\therefore\quad 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2} < \frac{29}{8} < \frac{15}{4} < 4$$

$$q_6 = \frac{1}{2}\left(\frac{13}{4}+\frac{27}{8}\right) = \frac{1}{2}\left(\frac{26+27}{8}\right) = \frac{53}{16}$$

$$\therefore\quad 3 < \frac{13}{4} < \frac{53}{16} < \frac{27}{8} < \frac{7}{2} < \frac{29}{8} < \frac{15}{4} < 4$$

Thus, the six rational numbers between 3 and 4 are

$\frac{7}{2}, \frac{13}{4}, \frac{15}{4}, \frac{27}{8}, \frac{29}{8}$ and $\frac{53}{16}$.

3. Since, we need to find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, therefore, multiply the numerator and denominator of $\frac{3}{5}$ and $\frac{4}{5}$ by 6.

$$\therefore\quad \frac{3}{5} = \frac{3\times 6}{5\times 6} = \frac{18}{30} \text{ and } \frac{4}{5} = \frac{4\times 6}{5\times 6} = \frac{24}{30}$$

$\therefore$ Five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30}$, $\frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

i.e., $\frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$.

4. (i) True, as the collection of all natural numbers and 0 is called whole numbers.

(ii) False, as negative integers are not whole numbers.

(iii) False, as rational numbers of the form p/q, where $q \neq 0$ and q does not divide p completely, are not whole numbers.

EXERCISE - 1.2

1. (i) True; because all irrational numbers can be represented on numbers line. And we know that numbers which can be represented on number line are known as real numbers.

(ii) False; because negative numbers cannot be the square root of any natural number.

(iii) False; because rational numbers are also a part of real numbers.

2. No, if we take a positive integer say 9, its square root is 3, which is a rational number.

3. Draw a line $X'OX$ and take point A on it such that $OA = 1$ unit. Draw $BA \perp OA$ such that $BA = 1$ unit. Join OB. We get, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2+1^2} = \sqrt{2}$ units.

Now, draw $BB_1 \perp OB$ such that $BB_1 = 1$ unit. Join OB_1.

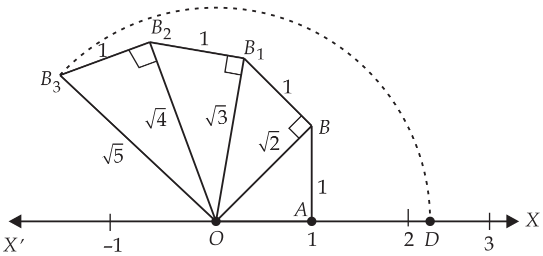
We get, $OB_1 = \sqrt{OB^2 + BB_1^2} = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$ units.

Next, draw $B_1B_2 \perp OB_1$ such that $B_1B_2 = 1$ unit. Join OB_2.

We get, $OB_2 = \sqrt{OB_1^2 + B_1B_2^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4}$ units.

Again, draw $B_2B_3 \perp OB_2$ such that $B_2B_3 = 1$ unit. Join OB_3.

We get, $OB_3 = \sqrt{OB_2^2 + B_2B_3^2} = \sqrt{(\sqrt{4})^2 + 1^2} = \sqrt{5}$ units.



Take O as centre and OB_3 as radius, draw an arc which cuts OX at D. Point D represents the number $\sqrt{5}$ on number line.

4. Do it yourself.

EXERCISE - 1.3

1. (i) We have, $\frac{36}{100} = 0.36$

$\therefore$ The decimal expansion of $\frac{36}{100}$ is terminating.

(ii) On dividing 1 by 11, we have

$$\begin{array}{l} 11\,\overline{)\,1.000000\,}(\,0.090909... \\ \quad\;\; \underline{-99} \\ \qquad\;\; 100 \\ \qquad\; \underline{-99} \\ \qquad\quad\; 100 \\ \qquad\quad \underline{-99} \\ \qquad\qquad 1 \end{array}$$

$\therefore \quad \frac{1}{11} = 0.090909... = 0.\overline{09}$

Thus, the given decimal expansion is non-terminating repeating.

(iii) We have, $4\frac{1}{8} = \frac{33}{8}$

Now,

$$\begin{array}{l} 8\,\overline{)\,33.000\,}(\,4.125 \\ \; \underline{-32} \\ \quad\; 10 \\ \quad \underline{-8} \\ \qquad 20 \\ \quad \underline{-16} \\ \qquad\; 40 \\ \qquad \underline{-40} \\ \qquad\quad 0 \end{array}$$

$\therefore \quad 4\frac{1}{8} = 4.125$. Thus, the decimal expansion is terminating.

(iv) On dividing 3 by 13, we have

$$\begin{array}{l} 13\,\overline{)\,3.00000000\,}(\,0.23076923... \\ \; \underline{-26} \\ \quad\; 40 \\ \quad \underline{-39} \\ \qquad 100 \\ \qquad \underline{-91} \\ \qquad\quad 90 \\ \qquad \underline{-78} \\ \qquad\quad 120 \\ \qquad \underline{-117} \\ \qquad\qquad 30 \\ \qquad\quad \underline{-26} \\ \qquad\qquad\; 40 \\ \qquad\qquad \underline{-39} \\ \qquad\qquad\quad 1 \end{array}$$

$\therefore \quad 3/13 = 0.23076923... = 0.\overline{230769}$

Thus, the decimal expansion of 3/13 is non-terminating repeating.

(v) On dividing 2 by 11, we have

$$\begin{array}{l} 11\,\overline{)\,2.0000\,}(\,0.1818... \\ \;\; \underline{-11} \\ \qquad 90 \\ \quad\; \underline{-88} \\ \qquad\; 20 \\ \qquad \underline{-11} \\ \qquad\quad 90 \\ \qquad \underline{-88} \\ \qquad\quad\; 2 \end{array}$$

$\therefore \quad \frac{2}{11} = 0.1818... = 0.\overline{18}$

Thus, the decimal expansion of 2/11 is non-terminating repeating.

(vi) Dividing 329 by 400, we have

$$\begin{array}{l} 400\,\overline{)\,329.0000\,}(\,0.8225 \\ \;\; \underline{-3200} \\ \qquad 900 \\ \quad\; \underline{-800} \\ \qquad 1000 \\ \qquad \underline{-800} \\ \qquad\; 2000 \\ \qquad \underline{-2000} \\ \qquad\quad\; 0 \end{array}$$

$\therefore \quad \frac{329}{400} = 0.8225$

Thus, the decimal expansion of 329/400 is terminating.

2. We are given that $\frac{1}{7} = 0.\overline{142857}$

$\therefore \quad \frac{2}{7} = 2 \times \frac{1}{7} = 2 \times (0.\overline{142857}) = 0.\overline{285714}$,

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times (0.\overline{142857}) = 0.\overline{428571}$,

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times (0.\overline{142857}) = 0.\overline{571428}$,

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times (0.\overline{142857}) = 0.\overline{714285}$ and

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times (0.\overline{142857}) = 0.\overline{857142}$

Thus, without actually doing the long division we can predict the decimal expansions of the given rational numbers.

3. (i) Let $x = 0.\overline{6} = 0.6666...$ ... (1)

Multiplying (1) by 10, we get

$10x = 6.6666...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = 6.6666... - 0.6666...$

$\Rightarrow \quad 9x = 6 \;\Rightarrow\; x = \frac{6}{9} = \frac{2}{3}$. Thus, $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.4\overline{7} = 0.4777...$ ... (1)

Multiplying (1) by 10, we get

$10x = 4.777...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = 4.777... - 0.4777...$

$\Rightarrow \quad 9x = 4.3 \;\Rightarrow\; x = \frac{43}{90}$. Thus, $0.4\overline{7} = \frac{43}{90}$

(iii) Let $x = 0.\overline{001} = 0.001001...$ ... (1)

Multiplying (1) by 1000, we get

$\Rightarrow \quad 1000x = 1.001001...$ ... (2)

Subtracting (1) from (2), we get

$1000x - x = (1.001...) - (0.001...)$

$\Rightarrow \quad 999x = 1 \;\Rightarrow\; x = \frac{1}{999}$. Thus, $0.\overline{001} = \frac{1}{999}$

4. Let $x = 0.99999...$ ... (1)

Multiplying (1) by 10, we get

$10x = 9.9999...$ ... (2)

Subtracting (1) from (2), we get

$10x - x = (9.9999...) - (0.9999...)$

$\Rightarrow \quad 9x = 9 \;\Rightarrow\; x = \frac{9}{9} = 1$. Thus, $0.9999... = 1$

As, 0.9999... goes on forever, there is no gap between 1 and 0.9999... . Hence, both are equal.

5. In 1/17, the number of entries in the repeating block of digits is less than the divisor *i.e.*, 17.

$\therefore$ The maximum number of digits in the repeating block is 16. To perform the long division, we have

$$
\begin{array}{r}
0.0588235294117647... \\
17\overline{)1.0000000000000000} \\
-85 \\
\hline 150 \\
-136 \\
\hline 140 \\
-136 \\
\hline 40 \\
-34 \\
\hline 60 \\
-51 \\
\hline 90 \\
-85 \\
\hline 50 \\
-34 \\
\hline 160 \\
-153 \\
\hline 70 \\
-68 \\
\hline 20 \\
-17 \\
\hline 30 \\
-17 \\
\hline 130 \\
-119 \\
\hline 110 \\
-102 \\
\hline 80 \\
-68 \\
\hline 120 \\
-119 \\
\hline 1
\end{array}
$$

The remainder 1 is the same digit from which we started the division.

$\therefore \quad \dfrac{1}{17} = 0.\overline{0588235294117647}$

Thus, there are 16 digits in the repeating block in the decimal expansion of 1/17. Hence, our answer is verified.

6. Let us look decimal expansion of the following rational numbers:

$\dfrac{3}{2} = \dfrac{3\times5}{2\times5} = \dfrac{15}{10} = 1.5$ [Denominator = 2 = 2^1]

$\dfrac{1}{5} = \dfrac{1\times2}{5\times2} = \dfrac{2}{10} = 0.2$ [Denominator = 5 = 5^1]

$\dfrac{7}{8} = \dfrac{7\times125}{8\times125} = \dfrac{875}{1000} = 0.875$ [Denominator = 8 = 2^3]

$\dfrac{8}{125} = \dfrac{8\times8}{125\times8} = \dfrac{64}{1000} = 0.064$ [Denominator = 125 = 5^3]

$\dfrac{13}{20} = \dfrac{13\times5}{20\times5} = \dfrac{65}{100} = 0.65$ [Denominator = 20 = $2^2 \times 5^1$]

$\dfrac{17}{16} = \dfrac{17\times625}{16\times625} = \dfrac{10625}{10000} = 1.0625$ [Denominator = 16 = 2^4]

We observe that the prime factorisation of q (*i.e.*, denominator) has only powers of 2 or powers of 5 or powers of both.

7. $\sqrt{2} = 1.414213562...$; $\sqrt{3} = 1.732050807...$;

$\sqrt{5} = 2.236067977...$

8. To find irrational numbers, firstly we will divide 5 by 7 and 9 by 11.

Now,

$$
\begin{array}{r}
0.714285... \\
7\overline{)\;5.000000} \\
-49 \\
\hline 10 \\
-7 \\
\hline 30 \\
-28 \\
\hline 20 \\
-14 \\
\hline 60 \\
-56 \\
\hline 40 \\
-35 \\
\hline 5
\end{array}
\qquad \text{and} \qquad
\begin{array}{r}
0.8181... \\
11\overline{)\;9.0000} \\
-88 \\
\hline 20 \\
-11 \\
\hline 90 \\
-88 \\
\hline 20 \\
-11 \\
\hline 9
\end{array}
$$

$\therefore \quad \dfrac{5}{7} = 0.\overline{714285}$ and $\dfrac{9}{11} = 0.\overline{81}$

Thus, three irrational numbers between $0.\overline{714285}$ and $0.\overline{81}$ are 0.750750075000750..., 0.767076700767000767..., 0.78080078008000780...

9. (i) $\because$ 23 is not a perfect square.

$\therefore \quad \sqrt{23}$ is an irrational number.

(ii) $\because$ 225 = 15 × 15 = 15^2 $\therefore$ 225 is a perfect square.

Thus, $\sqrt{225}$ is a rational number.

(iii) $\because$ 0.3796 is a terminating decimal.

$\therefore$ It is a rational number.

(iv) $7.478478... = 7.\overline{478}$. Since, $7.\overline{478}$ is a non-terminating recurring (repeating) decimal.

$\therefore$ It is a rational number.

(v) Since, 1.101001000100001... is a non-terminating, non-repeating decimal number.

$\therefore$ It is an irrational number.

EXERCISE - 1.5

1. (i) We know that difference of a rational and an irrational number is always irrational.

$\therefore \quad 2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23} = 3+\sqrt{23}-\sqrt{23} = 3$, which is a rational number.

(iii) Since, $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$, which is a rational number.

(iv) $\because$ The quotient of rational and irrational number is an irrational number.

$\therefore \quad \dfrac{1}{\sqrt{2}}$ is an irrational number.

(v) $\because$ Product of a rational and an irrational number is an irrational number.

$\therefore \quad 2\pi$ is an irrational number.

2. (i) We have, $(3+\sqrt{3})(2+\sqrt{2}) = 2(3+\sqrt{3})+\sqrt{2}(3+\sqrt{3})$

$= 6+2\sqrt{3}+3\sqrt{2}+\sqrt{6}$

(ii) We have, $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$

$= 3^2 - 3 = 9 - 3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2})$

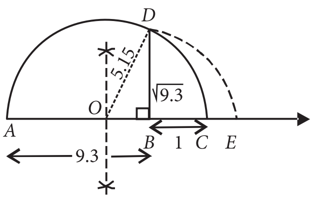
$= 5 + 2 + 2\sqrt{10} = 7 + 2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$

3. When we measure the length of a line with a scale or with any other device, we only get an approximate rational value, *i.e.*, c and d both are irrational.

$\therefore$ c/d is irrational and hence π is irrational. Thus, there is no contradiction in saying that π is irrational.

4. Draw a line segment AB = 9.3 units and extend it to C such that BC = 1 unit and AC = 10.3 units.



Find mid-point of AC and mark it as O. Draw a semicircle taking O as centre and AO as radius, where $AO = \frac{AC}{2} = 5.15$ units.

Draw $BD \perp AC$ and intersecting the semicircle at D.

In ΔOBD, $BD^2 = OD^2 - OB^2$

$\Rightarrow$ $BD^2 = (5.15)^2 - (4.15)^2 = (5.15 + 4.15)(5.15 - 4.15)$

$\Rightarrow$ $BD = \sqrt{9.3}$ units.

To represent $\sqrt{9.3}$ units on the number line, let us treat the line BC as the number line, with B as zero, C as 1, and so on.

Draw an arc with centre B and radius $BD = \sqrt{9.3}$ units, which intersects the number line BC (produced) at E.

$BD = BE = \sqrt{9.3}$ units

$\therefore$ E represents $\sqrt{9.3}$

5. (i) $\frac{1}{\sqrt{7}} = \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$= \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7})^2 - (\sqrt{6})^2} = \frac{(\sqrt{7}+\sqrt{6})}{7-6} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{(\sqrt{5}-\sqrt{2})}{5-2} = \frac{(\sqrt{5}-\sqrt{2})}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$= \frac{(\sqrt{7}+2)}{(\sqrt{7})^2 - (2)^2} = \frac{(\sqrt{7}+2)}{7-4} = \frac{\sqrt{7}+2}{3}$

EXERCISE - 1.6

1. (i) $\because$ $64 = 8 \times 8 = 8^2$

$\therefore$ $(64)^{1/2} = (8^2)^{1/2} = 8^{2 \times 1/2}$ $[\because (a^m)^n = a^{m \times n}]$

$= 8$

(ii) $\because$ $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore$ $(32)^{1/5} = (2^5)^{1/5} = 2^{5 \times 1/5}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2$

(iii) $\because$ $125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $(125)^{1/3} = (5^3)^{1/3} = 5^{3 \times 1/3}$ $[\because (a^m)^n = a^{m \times n}]$

$= 5$

2. (i) $\because$ $9 = 3 \times 3 = 3^2$

$\therefore$ $(9)^{3/2} = (3^2)^{3/2} = 3^{2 \times 3/2}$ $[\because (a^m)^n = a^{m \times n}]$

$= 3^3 = 27$

(ii) $\because$ $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$\therefore$ $(32)^{2/5} = (2^5)^{2/5} = 2^{5 \times 2/5}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2^2 = 4$

(iii) $\because$ $16 = 2 \times 2 \times 2 \times 2 = 2^4$

$\therefore$ $(16)^{3/4} = (2^4)^{3/4} = 2^{4 \times 3/4}$ $[\because (a^m)^n = a^{m \times n}]$

$= 2^3 = 8$

(iv) $\because$ $125 = 5 \times 5 \times 5 = 5^3$

$\therefore$ $(125)^{-1/3} = (5^3)^{-1/3} = 5^{3 \times (-1/3)}$ $[\because (a^m)^n = a^{m \times n}]$

$= 5^{-1} = 1/5$ $[\because a^{-n} = 1/a^n]$

3. (i) Since, $2^{2/3} \cdot 2^{1/5} = 2^{2/3 + 1/5}$ $[\because a^m \cdot a^n = a^{m+n}]$

$= 2^{13/15}$

(ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = 3^{-3 \times 7} = 3^{-21} = \frac{1}{3^{21}}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

(iii) $\frac{11^{1/2}}{11^{1/4}} = 11^{\frac{1}{2}} \div 11^{\frac{1}{4}} = 11^{\frac{1}{2}-\frac{1}{4}}$ $[\because a^m \div a^n = a^{m-n}]$

$= 11^{\frac{1}{4}}$

(iv) $7^{1/2} \cdot 8^{1/2} = (7 \times 8)^{1/2}$ $[\because a^m \times b^m = (ab)^m]$

$= (56)^{1/2}$

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