CHAPTER
2

# POLYNOMIALS

## TRY YOURSELF SOLUTIONS

**1.** (i) $2\sqrt{2}y^5 + 3y^4$ is a polynomial in $y$, since exponent of $y$ in each term is a whole number.

(ii) $2\sqrt{x} + x\sqrt{2}$ can be written as $2x^{1/2} + \sqrt{2}x$. Here, the exponent of variable in $2x^{1/2}$ is 1/2, which is not a whole number. Therefore, the given expression is not a polynomial.

(iii) $x^2 + \dfrac{3}{x^2} + 4$ can be written as $x^2 + 3x^{-2} + 4$. Here, the exponent of variable in term $3x^{-2}$ is – 2, which is not a whole number. Therefore, the given expression is not a polynomial.

**2.** There are three terms in the given polynomial, namely, $x^2, \dfrac{\pi}{2}x$ and – 7.

**3.** The given polynomial, $p(x) = 13x^{15} + 12x^{14} + 11x^{13} + 10x^{12} + 9x^{11}$ can be written as $13x^{15} + 12x^{14} + 11x^{13} + 10x^{12} + 9x^{11} + 0x^{10}$.

$\therefore$ Coefficient of $x^{10}$ is 0.

**4.** Monomial = $4x^8$ and binomial = $2x + 7$

**5.** We have, $(x^3 + 5)(4 - x^5) = 4x^3 - x^8 + 20 - 5x^5$

Clearly, the highest power of the variable is 8. Therefore, the degree of the given polynomial is 8.

**6.** The only term here is 100, which can be written as $100x^0$. And exponent of $x$ is 0. Therefore, the degree of the given polynomial is 0.

**7.** (i) $5x^{28} + 3$ (ii) $2x^9 + 6x + 7$

**8.** (i) Clearly, $5x^2 + 8x$ is a polynomial of degree 2. So, it is a quadratic polynomial.

(ii) Clearly, $2x - x^3$ is a polynomial of degree 3. So, it is a cubic polynomial.

(iii) Clearly, $3 + 2x$ is a polynomial of degree 1. So, it is a linear polynomial.

(iv) Clearly, $5x^3$ is a polynomial of degree 3. So, it is a cubic polynomial.

**9.** (i) Clearly, $5 - x - x^2$ is a polynomial of degree 2. So, it is a quadratic polynomial.

(ii) Clearly, $p^4$ is a polynomial of degree 4. So, it is a biquadratic polynomial.

**10.** We have, $p(x) = \sqrt{2}x^2 + \sqrt{2}x + 6$

$p(\sqrt{2}) = \sqrt{2}(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) + 6$

$= 2\sqrt{2} + 2 + 6 = 2\sqrt{2} + 8$

**11.** We have, $p(y) = 3y^4 - 2y^3 + 15y + k$

$p(1) = -1 \Rightarrow 3(1)^4 - 2(1)^3 + 15(1) + k = -1$

$\Rightarrow 3 - 2 + 15 + k = -1 \Rightarrow 16 + k = -1 \Rightarrow k = -17$

**12.** We have, $p(t) = 3t^4 + 1$

$\therefore$ $p(0) = 3(0)^4 + 1 = 3 \times 0 + 1 = 1,$

$p(1) = 3(1)^4 + 1 = 3 \times 1 + 1 = 4,$

$p(-1) = 3(-1)^4 + 1 = 3 \times 1 + 1 = 4,$

$p(3) = 3(3)^4 + 1 = 3 \times 81 + 1 = 244$ and

$p(-3) = 3(-3)^4 + 1 = 3 \times 81 + 1 = 244$

**13.** Let $p(x) = x + 2$. Then $p(2) = 2 + 2 = 4$, $p(-2) = -2 + 2 = 0$. Therefore, –2 is a zero of the polynomial $x + 2$, but 2 is not.

**14.** Let $p(x) = x^2 - 2x - 3$

Now, $p(3) = 3^2 - 2(3) - 3 = 9 - 6 - 3 = 0$

Since, $p(3) = 0$, therefore $x = 3$ is a root of the polynomial equation $x^2 - 2x - 3 = 0$.

**15.** We have, $q(x) = 2x - 7$. To find its zero, put $q(x) = 0$.

$\therefore$ $2x - 7 = 0 \Rightarrow 2x = 7 \Rightarrow x = \dfrac{7}{2}$.

Thus, zero of polynomial $q(x)$ is $\dfrac{7}{2}$.

**16.** We have, $p(y) = ly - m$ ; $l \neq 0$. To find its zero, put $p(y) = 0$.

$\therefore$ $ly - m = 0 \Rightarrow ly = m \Rightarrow y = \dfrac{m}{l}$.

Thus, $y = \dfrac{m}{l}$ is the zero of the polynomial $p(y) = ly - m$.

**17.** Here, degree of $p(x) = 4$ and degree of $g(x) = 1$. So degree of $g(x) <$ degree of $p(x)$. By long division method, we get

$$
\begin{array}{r|l}
x+3 & 7x^4 + 3x^3 - 2x^2 + x - 4 \;\big(\; 7x^3 - 18x^2 + 52x - 155 \\
 & \underline{\underset{-}{7x^4} \underset{-}{+} 21x^3} \\
 & \quad -18x^3 - 2x^2 \\
 & \quad \underline{\underset{+}{-18x^3} \underset{+}{-} 54x^2} \\
 & \qquad 52x^2 + x \\
 & \qquad \underline{\underset{-}{52x^2} \underset{-}{+} 156x} \\
 & \qquad\quad -155x - 4 \\
 & \qquad\quad \underline{\underset{+}{-155x} \underset{+}{-} 465} \\
 & \qquad\qquad 461
\end{array}
$$

Thus, quotient is $7x^3 - 18x^2 + 52x - 155$ and remainder is 461.

**18.** To check if $x + 1$ is a factor of $x^3 + 1$, we divide $x^3 + 1$ by $x + 1$. Therefore, by long division method, we get

$$x+1\,\overline{)\,x^3+1\,}\,(x^2-x+1$$
$$\underline{\begin{array}{r} x^3+x^2 \\ -\quad - \end{array}}$$
$$-x^2+1$$
$$\underline{\begin{array}{r} -x^2-x \\ +\quad + \end{array}}$$
$$x+1$$
$$\underline{\begin{array}{r} x+1 \\ -\quad - \end{array}}$$
$$0$$

So, we find that the remainder is 0. Therefore, $x + 1$ is a factor of $x^3 + 1$.

**19.** $$x+1\,\overline{)\,5x^2+x-1\,}\,(5x-4$$
$$\underline{\begin{array}{r} 5x^2+5x \\ -\quad - \end{array}}$$
$$-4x-1$$
$$\underline{\begin{array}{r} -4x-4 \\ +\quad + \end{array}}$$
$$3$$

Here, Dividend $p(x) = 5x^2 + x - 1$, Divisor $g(x) = x + 1$, Quotient $q(x) = 5x - 4$, Remainder $r(x) = 3$

$\therefore$ R.H.S. $= g(x)\,q(x) + r(x) = (x + 1)(5x - 4) + 3$

$= 5x^2 - 4x + 5x - 4 + 3 = 5x^2 + x - 1 =$ L.H.S.

**20.** Let $p(x) = ax^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + a$ be the given polynomials.

Since zero of $(x - 3)$ is 3, therefore remainders when $p(x)$ and $q(x)$ are divided by $(x - 3)$ are given by $p(3)$ and $q(3)$ respectively.

By the given condition, we have $p(3) = q(3)$

$\Rightarrow$ $a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 3^3 - 4 \times 3 + a$

$\Rightarrow$ $27a + 36 + 9 - 4 = 27 - 12 + a$

$\Rightarrow$ $26a + 26 = 0 \Rightarrow 26a = -26 \Rightarrow a = -1.$

**21.** Clearly, $q(t)$ will be a multiple of $2t - 1$ only if $2t - 1$ divides $q(t)$ leaving remainder zero.

Now, zero of $2t - 1$ is $\frac{1}{2}$.

$$\therefore \quad \text{Remainder } = q\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 2$$
$$= \frac{1}{2} + \frac{3}{4} + \frac{1}{2} - 2 = \frac{-1}{4} \neq 0$$

Since the remainder obtained on dividing $q(t)$ by $2t - 1$ is not 0. Therefore, $q(t)$ is not a multiple of $2t - 1$.

**22.** Let $p(x) = x^4 - k^2x^2 + 2x - k$

$\because$ Zero of the polynomial $x - k$ is $k$, therefore by remainder theorem, reminder when $p(x)$ is divided by $(x - k)$ is given by $p(k)$.

$\therefore$ $p(k) = k^4 - k^2(k)^2 + 2k - k = k^4 - k^4 + k = k$

So, the remainder is $k$.

**23.** Let $p(x) = 3x^2 + kx + 6$ be the given polynomial. As, $(x + 3)$ is a factor of $p(x)$, therefore $p(-3) = 0$.

$\Rightarrow$ $3(-3)^2 + k(-3) + 6 = 0 \Rightarrow 27 - 3k + 6 = 0$

$\Rightarrow$ $33 - 3k = 0 \Rightarrow k = 11$

**24.** Let $p(x) = 4x^2 - bx - ca$.

As $(x - a)$ is a factor of $p(x)$, therefore $p(a) = 0$.

$\therefore$ $4a^2 - ba - ca = 0$

$\Rightarrow$ $a(4a - b - c) = 0$

$\Rightarrow$ $4a - b - c = 0$ $\qquad [\because a \neq 0]$

$\Rightarrow$ $4a = b + c$

$\Rightarrow$ $a = \dfrac{b+c}{4}$

**25.** Let $p(x) = 2x^3 + 6x + 8$.

In order to prove that $(x + 1)$ is a factor of $p(x)$, it is sufficient to show that $p(-1) = 0$.

Now, $p(-1) = 2(-1)^3 + 6(-1) + 8 = -2 - 6 + 8 = 0$

Hence, $(x + 1)$ is a factor of the given polynomial.

**26.** Let $p(x) = x^3 - 3x^2 - 10x + 24$

In order to prove that $(x - 2)$, $(x + 3)$ and $(x - 4)$ are factors of $p(x)$, it is sufficient to show that $p(2)$, $p(-3)$ and $p(4)$ are equal to zero.

Now, $p(2) = 2^3 - 3(2)^2 - 10(2) + 24 = 8 - 12 - 20 + 24 = 0$,

$p(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24$

$\quad = -27 - 27 + 30 + 24 = 0$

And $p(4) = 4^3 - 3(4)^2 - 10(4) + 24 = 64 - 48 - 40 + 24 = 0$

Hence, $(x - 2)$, $(x + 3)$ and $(x - 4)$ are factors of the given polynomial.

**27.** Given polynomial is $2r^2 + 5r - 3$

On comparing it with $ax^2 + bx + c$, we get

$\quad a = 2, b = 5$ and $c = -3$

Here, $ac = -6$

So, we need to find two numbers whose sum is 5 and product is $-6$. One such pair is 6 and $(-1)$.

So, $2r^2 + 5r - 3 = 2r^2 + (6 - 1)r - 3 = 2r^2 + 6r - r - 3$

$= 2r(r + 3) - 1(r + 3) = (2r - 1)(r + 3)$

**28.** Given polynomial is $x^2 + 3\sqrt{3}x + 6$

On comparing it with $ax^2 + bx + c$, we get

$\quad a = 1,\ b = 3\sqrt{3}$ and $c = 6$

Here, $ac = 6$.

So, we need to find two numbers whose sum is $3\sqrt{3}$ and product is 6. One such pair is $\sqrt{3}$ and $2\sqrt{3}$.

So, $x^2 + 3\sqrt{3}x + 6 = x^2 + (\sqrt{3} + 2\sqrt{3})x + 6$

$= x^2 + \sqrt{3}x + 2\sqrt{3}x + 6 = x(x + \sqrt{3}) + 2\sqrt{3}(x + \sqrt{3})$

$= (x + 2\sqrt{3})(x + \sqrt{3})$

So, factors of $x^2 + 3\sqrt{3}x + 6$ are $(x + 2\sqrt{3})$ and $(x + \sqrt{3})$.

**29.** Area $= 16a^2 - 32a + 15$

$= 16a^2 - 20a - 12a + 15$ $\qquad$ (By splitting the middle term)

$= 4a(4a - 5) - 3(4a - 5) = (4a - 3)(4a - 5)$

$\qquad$ [Here, $4a - 3 > 0$ and $4a - 5 > 0$ because $a > \frac{5}{4}$.]

$=$ (Length) $\times$ (Breadth)

$\therefore$ Length $= 4a - 3$

and Breadth $= 4a - 5$ $\qquad$ $(\because 4a - 3 > 4a - 5)$.

**30.** Let $p(y) = 15y^2 - 8y + 1$

$$= 15\left(y^2 - \frac{8}{15}y + \frac{1}{15}\right) = 15\,q(y), \text{ (say)}$$

where $q(y) = y^2 - \dfrac{8}{15}y + \dfrac{1}{15}$.

Now, factors of 1 are ±1 and factors of 15 are ±1, ±3, ±5, ±15. So, some possibilities for the zeroes of $q(y)$ are $\pm 1, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm\frac{1}{15}$.

Now, we find that

$$q\left(\frac{1}{3}\right)=\left(\frac{1}{3}\right)^2-\frac{8}{15}\left(\frac{1}{3}\right)+\frac{1}{15}=\frac{1}{9}-\frac{8}{45}+\frac{1}{15}=\frac{5-8+3}{45}=0$$

and $q\left(\frac{1}{5}\right)=\left(\frac{1}{5}\right)^2-\frac{8}{15}\left(\frac{1}{5}\right)+\frac{1}{15}$

$$=\frac{1}{25}-\frac{8}{75}+\frac{1}{15}=\frac{3-8+5}{75}=0$$

$\therefore$ By factor theorem, $\left(y-\frac{1}{3}\right)$ and $\left(y-\frac{1}{5}\right)$ are the factors of $q(y)$

Hence, $p(y)=15\left(y-\frac{1}{3}\right)\left(y-\frac{1}{5}\right)$

$$=15\left(\frac{3y-1}{3}\right)\left(\frac{5y-1}{5}\right)=(3y-1)(5y-1)$$

**31.** Let $p(x) = x^2 - 22x + 120$

Now, if $p(x) = (x - \alpha)(x - \beta)$, we know that constant term will be $\alpha\beta = 120$. So, look for the factors of 120.

Some of these are ± 1, ± 2, ± 3, ± 4, ± 5, ± 6, ± 10, ± 12, ± 20

Now, $p(10) = (10)^2 - 22(10) + 120 = 0$

$\therefore$ $(x - 10)$ is a factor of $p(x)$.

Now to find, the other factor, divide $p(x)$ by $(x - 10)$.

$$\begin{array}{r|l} x-10 & x^2-22x+120 \;(\, x-12 \\ & \underline{\overset{-}{x^2}\;\overset{+}{-\,10x}} \\ & \quad -12x+120 \\ & \quad \underline{\overset{+}{-12x}\;\overset{-}{+\,120}} \\ & \qquad 0 \end{array}$$

So, other factor of $p(x) = (x - 12)$

Hence $p(x) = (x - 12)(x - 10)$.

**32.** Since, $(2x + 3)$ is a factor of the given polynomial, therefore let us divide $4x^3 + 12x^2 + 5x - 6$ by $2x + 3$ to get the other factors.

$$\begin{array}{r|l} 2x+3 & 4x^3+12x^2+5x-6 \;(\, 2x^2+3x-2 \\ & \underline{\overset{-}{4x^3}+\overset{-}{6x^2}} \\ & \quad 6x^2+5x \\ & \quad \underline{\overset{-}{6x^2}+\overset{-}{9x}} \\ & \qquad -4x-6 \\ & \qquad \underline{\overset{+}{-4x}\overset{+}{-6}} \\ & \qquad\quad 0 \end{array}$$

$\therefore$ $4x^3 + 12x^2 + 5x - 6 = (2x + 3)(2x^2 + 3x - 2)$

Now, we will factorise $2x^2 + 3x - 2$ to find the other two factors, by splitting its middle term.

$\therefore$ $2x^2 + 3x - 2 = 2x^2 + 4x - x - 2$

$= 2x(x + 2) - 1(x + 2) = (2x - 1)(x + 2)$

Hence, $4x^3 + 12x^2 + 5x - 6 = (2x + 3)(2x - 1)(x + 2)$

**33.** Let $p(x) = x^3 - 6x^2 + 3x + 10$

All possible factors of 10 are ± 1, ± 2, ± 5 and ± 10.

Now, we find that $p(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$

$= -1 - 6 - 3 + 10 = 0$,

$p(2) = 2^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$ and

$p(5) = 5^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0$

So, by factor theorem, $(x + 1)$, $(x - 2)$ and $(x - 5)$ are the factors of $p(x)$.

**34.** Let $p(t) = 2t^3 - 5t^2 - 19t + 42$

$= 2\left[t^3-\frac{5}{2}t^2-\frac{19}{2}t+21\right] = 2q(t)$, (say)

where $q(t) = t^3-\frac{5}{2}t^2-\frac{19}{2}t+21$

Factors of 21 are ± 1, ± 3, ± 7 and ± 21

Now, we can find that $q(-3)=(-3)^3-\frac{5}{2}(-3)^2-\frac{19}{2}(-3)+21$

$=-27-\frac{45}{2}+\frac{57}{2}+21=0$

So, $(t + 3)$ is one of the factor of $q(t)$. Let us divide $q(t)$ by $(t + 3)$ to find other factors.

$$\begin{array}{r|l} t+3 & t^3-\frac{5}{2}t^2-\frac{19}{2}t+21 \;(\, t^2-\frac{11}{2}t+7 \\ & \underline{\overset{-}{t^3}+\overset{-}{3t^2}} \\ & \quad -\frac{11}{2}t^2-\frac{19}{2}t \\ & \quad \underline{\overset{+}{-\frac{11}{2}t^2}\overset{+}{-\frac{33}{2}t}} \\ & \qquad 7t+21 \\ & \qquad \underline{\overset{-}{7t}+\overset{-}{21}} \\ & \qquad\quad 0 \end{array}$$

So, $q(t) = (t+3)\left(t^2-\frac{11}{2}t+7\right)$

Now, $t^2-\frac{11}{2}t+7=t^2-\frac{7}{2}t-2t+7$

$=t\left(t-\frac{7}{2}\right)-2\left(t-\frac{7}{2}\right)=\left(t-\frac{7}{2}\right)(t-2)$

So, $p(t)=2(t+3)\left(t-\frac{7}{2}\right)(t-2)$

$= (t + 3)(2t - 7)(t - 2)$

**Short cut method :** We have $p(t) = 2t^3 - 5t^2 - 19t + 42$

By hit and trial method, we have

$p(2) = 2(2)^3 - 5(2)^2 - 19(2) + 42 = 16 - 20 - 38 + 42 = 0$

$\therefore$ $(t - 2)$ is one of the factors of $p(t)$.

So, $2t^3 - 5t^2 - 19t + 42 = 2t^2(t - 2) - t(t - 2) - 21(t - 2)$

$= (t - 2)[2t^2 - t - 21] = (t - 2)(2t^2 - 7t + 6t - 21)$

$= (t - 2)[t(2t - 7) + 3(2t - 7)] = (t - 2)(2t - 7)(t + 3)$

**35.** $(2x - 3y)(2x - 3y) = (2x - 3y)^2$

$= (2x)^2 - 2(2x)(3y) + (3y)^2$ $\quad [\because (x - y)^2 = x^2 - 2xy + y^2]$

$= 4x^2 - 12xy + 9y^2$

**36.** We have, $101 \times 103 = (100 + 1) \times (100 + 3)$

$= (100)^2 + (1 + 3)(100) + (1)(3)$

$[\because (x + a)(x + b) = x^2 + (a + b)x + ab]$

$= 10000 + 400 + 3 = 10403$

**37.** (i) We have, $0.54 \times 0.54 - 0.46 \times 0.46$

$= (0.54)^2 - (0.46)^2 = (0.54 + 0.46)(0.54 - 0.46) = 1 \times 0.08 = 0.08.$

$[\because x^2 - y^2 = (x - y)(x + y)]$

(ii) We have, $(0.99)^2 = (1 - 0.01)^2$

$= (1)^2 - 2 \times 1 \times 0.01 + (0.01)^2 = 1 - 0.02 + 0.0001$

$[\because (x - y)^2 = x^2 - 2xy + y^2]$

$= 1.0001 - 0.02 = 0.9801$

**38.** We have, $(3x + 2y)^2 = (3x)^2 + (2y)^2 + 2 \times 3x \times 2y$

$[\because (x + y)^2 = x^2 + 2xy + y^2]$

$\Rightarrow (3x + 2y)^2 = 9x^2 + 4y^2 + 12xy$

$\Rightarrow 12^2 = 9x^2 + 4y^2 + 12 \times 6$

$[\because 3x + 2y = 12$ and $xy = 6$ (Given)$]$

$\Rightarrow 144 = 9x^2 + 4y^2 + 72$

$\Rightarrow 144 - 72 = 9x^2 + 4y^2 \Rightarrow 9x^2 + 4y^2 = 72$

**39.** We have, $x - \frac{1}{x} = -1$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (-1)^2$ [On squaring both sides]

$\Rightarrow x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 1$ $[\because (x - y)^2 = x^2 - 2xy + y^2]$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 1 \Rightarrow x^2 + \frac{1}{x^2} = 1 + 2 = 3$

**40.** $49a^2 + 70ab + 25b^2 = (7a)^2 + 2(7a)(5b) + (5b)^2$

$= (7a + 5b)^2$ $[\because x^2 + 2xy + y^2 = (x + y)^2]$

$= (7a + 5b)(7a + 5b)$

**41.** (i) We have, $(x - 2y - 3z)^2$

$= \{x + (-2y) + (-3z)\}^2$

$= x^2 + (-2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx]$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

(ii) We have, $(-x + 2y + z)^2 = \{(-x) + 2y + z\}^2$

$= (-x)^2 + (2y)^2 + z^2 + 2 \times (-x)(2y) + 2 \times 2y \times z + 2 \times (-x) \times z$

$= x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$

**42.** We have, $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$

$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$

**43.** (i) We have, $21^3 - 15^3$

$= (21 - 15)\{(21)^2 + 21 \times 15 + (15)^2\}$

$[\because x^3 - y^3 = (x - y)(x^2 + xy + y^2)]$

$= 6 \times (441 + 315 + 225) = 6 \times 981 = 5886$

(ii) We have, $(999)^3 = (1000 - 1)^3$

$= (1000)^3 - (1)^3 - 3(1000)(1)(1000 - 1)$

$[\because (x - y)^3 = x^3 - y^3 - 3xy(x - y)]$

$= 1000000000 - 1 - 3000000 + 3000 = 997002999$

**44.** We have, $8x^3 + y^3 + 27z^3 - 18xyz$

$= (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)\{(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (3z)(2x)\}$

$[\because x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6zx)$

**45.** We have, $8x^3 - (2x - y)^3 = (2x)^3 - (2x - y)^3$

$= [2x - (2x - y)][(2x)^2 + 2x \times (2x - y) + (2x - y)^2]$

$= y[4x^2 + 4x^2 - 2xy + 4x^2 + y^2 - 4xy]$

$= y[12x^2 - 6xy + y^2]$