Quadrilaterals

CHAPTER 8

NCERT FOCUS SOLUTIONS

EXERCISE - 8.1

1. Let the angles of a quadrilateral be $3x$, $5x$, $9x$ and $13x$.

$\therefore$ $3x + 5x + 9x + 13x = 360°$

[Angle sum property of a quadrilateral]

$\Rightarrow$ $30x = 360° \Rightarrow x = 12°$

$\therefore$ $3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

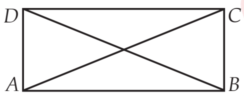
$9x = 9 \times 12° = 108°$

$13x = 13 \times 12° = 156°$

$\therefore$ Required angles of the quadrilateral are 36°, 60°, 108° and 156°.

2. Let $ABCD$ be a parallelogram such that $AC = BD$.

In ΔABC and ΔDCB, we have



$AC = DB$ [Given]

$AB = DC$

[Opposite sides of a parallelogram]

$BC = CB$ [Common]

$\therefore$ $\Delta ABC \cong \Delta DCB$ [By SSS congruence]

$\Rightarrow$ $\angle ABC = \angle DCB$ [By C.P.C.T.] ...(i)

Now, $AB \parallel DC$ and BC is a transversal.

[$\because$ $ABCD$ is a parallelogram]

$\therefore$ $\angle ABC + \angle DCB = 180°$ ...(ii)

[Angles on the same side of a transversal]

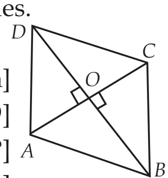
From (i) and (ii), we have $\angle ABC = \angle DCB = 90°$

$\Rightarrow$ $ABCD$ is a parallelogram having an angle equal to 90°.

$\therefore$ $ABCD$ is a rectangle.

3. Let $ABCD$ be a quadrilateral such that the diagonals AC and BD bisect each other at right angles.

In ΔAOB and ΔAOD, we have



$AO = OA$ [Common]

$OB = OD$ [O is the mid-point of BD]

$\angle AOB = \angle AOD$ [Each equals 90°]

$\therefore$ $\Delta AOB \cong \Delta AOD$ [By SAS congruence]

$\therefore$ $AB = AD$ [By C.P.C.T.] ...(i)

Similarly, $AB = BC$ ...(ii)

$BC = CD$ ...(iii)

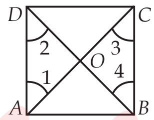
And $CD = DA$ ...(iv)

$\therefore$ From (i), (ii), (iii) and (iv), we have $AB = BC = CD = DA$

Thus, the given quadrilateral is a rhombus.

Alternative solution : $ABCD$ can be first proved a parallelogram. Then proving one pair of adjacent sides equal will result in rhombus.

4. Let $ABCD$ be a square such that its diagonals AC and BD intersect at O.



(i) To prove that the diagonals are equal, *i.e.*, $AC = BD$.

In ΔABC and ΔBAD, we have

$AB = BA$ [Common]

$BC = AD$ [Sides of a square]

$\angle ABC = \angle BAD$ [Each equals 90°]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruence]

$\Rightarrow$ $AC = BD$ [By C.P.C.T.] ...(1)

(ii) To prove diagonals bisect each other.

$\because$ $AD \parallel BC$ and AC is a transversal.

[A square is a parallelogram]

$\therefore$ $\angle 1 = \angle 3$ [Alternate interior angles]

Similarly, $\angle 2 = \angle 4$

Now, in ΔOAD and ΔOCB, we have

$AD = CB$ [Sides of a square]

$\angle 1 = \angle 3$ [Proved]

$\angle 2 = \angle 4$ [Proved]

$\therefore$ $\Delta OAD \cong \Delta OCB$ [By ASA congruence]

$\Rightarrow$ $OA = OC$ and $OD = OB$ [By C.P.C.T.]

$\Rightarrow$ Diagonals AC and BD bisect each other at O. ...(2)

(iii) To prove diagonals intersect at right angles.

In ΔOBA and ΔODA, we have

$OB = OD$ [Proved]

$BA = DA$ [Sides of a square]

$OA = AO$ [Common]

$\therefore$ $\Delta OBA \cong \Delta ODA$ [By SSS congruence]

$\Rightarrow$ $\angle AOB = \angle AOD$ [By C.P.C.T.] ...(3)

$\because$ $\angle AOB$ and $\angle AOD$ form a linear pair.

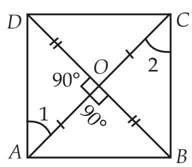
$\therefore$ $\angle AOB + \angle AOD = 180°$

$\therefore$ $\angle AOB = \angle AOD = 90°$ [By (3)]

$\Rightarrow$ $AC \perp BD$ ...(4)

From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles.

5. Let $ABCD$ be a quadrilateral such that diagonals AC and BD are equal and bisect each other at right angles.



Now, in ΔAOD and ΔAOB, we have

$\angle AOD = \angle AOB$ [Each equals 90°]

$AO = OA$ [Common]

$OD = OB$ [$\because$ AC bisects BD]

$\therefore$ $\Delta AOD \cong \Delta AOB$ [By SAS congruence]

$\Rightarrow$ $AD = AB$ [By C.P.C.T.] ...(i)

Similarly, we have $AB = BC$ ...(ii)

$BC = CD$ ...(iii)

$CD = DA$ ...(iv)

From (i), (ii), (iii) and (iv), we have

$AB = BC = CD = DA$

$\therefore$ Quadrilateral $ABCD$ has all sides equal.

In ΔAOD and ΔCOB, we have

$AO = CO$ [Given]

$OD = OB$ [Given]

$\angle AOD = \angle COB$ [Vertically opposite angles]

So, $\Delta AOD \cong \Delta COB$ [By SAS congruence]

$\therefore$ $\angle 1 = \angle 2$ [By C.P.C.T.]

But they form a pair of alternate interior angles.

$\therefore$ $AD \parallel BC$

Similarly, $AB \parallel DC$

$\therefore$ $ABCD$ is a parallelogram.

And a parallelogram having its all sides equal is a rhombus.

$\therefore$ $ABCD$ is a rhombus.

Now, in ΔABC and ΔBAD, we have

$AC = BD$ [Given]

$BC = AD$ [Proved]

$AB = BA$ [Common]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SSS congruence]

$\therefore$ $\angle ABC = \angle BAD$ [By C.P.C.T.] ...(v)

Now, since $AD \parallel BC$ and AB is a transversal.

$\therefore$ $\angle ABC + \angle BAD = 180°$ ...(vi)

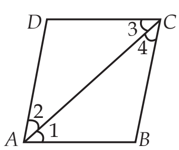
[Interior angles on the same side of the transversal]

$\Rightarrow$ $\angle ABC = \angle BAD = 90°$ [By (v) and (vi)]

So, rhombus $ABCD$ is having one angle equal to 90°.

Thus, $ABCD$ is a square.

6. We have a parallelogram $ABCD$ in which diagonal AC bisects $\angle A$ $\Rightarrow \angle 1 = \angle 2$



(i) Since $ABCD$ is a parallelogram.

$\therefore$ $AB \parallel DC$ and AC is a transversal.

$\therefore$ $\angle 1 = \angle 3$ [Alternate interior angles] ...(1)

Also, $BC \parallel AD$ and AC is a transversal.

$\therefore$ $\angle 2 = \angle 4$ [Alternate interior angles] ...(2)

Also, $\angle 1 = \angle 2$ [$\because$ AC bisects $\angle A$] ...(3)

From (1), (2) and (3), we have

$\angle 3 = \angle 4$ $\Rightarrow AC$ bisects $\angle C$.

(ii) In ΔABC, we have

$\angle 1 = \angle 4$ [From (2) and (3)]

$\Rightarrow$ $BC = AB$ ...(4)

[$\because$ Sides opposite to equal angles of a triangle are equal]

Similarly, $AD = DC$ ...(5)

Also, $ABCD$ is a parallelogram [Given]

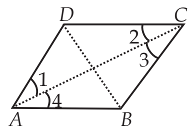
$\therefore$ $AB = DC$ ...(6)

From (4), (5) and (6), we have

$AB = BC = CD = DA$

Thus, $ABCD$ is a rhombus.

7. We have, a rhombus $ABCD$



$\therefore$ $AB = BC = CD = DA$

Also, $AB \parallel CD$ and $AD \parallel BC$

Now, in ΔADC,

$AD = CD \Rightarrow \angle 1 = \angle 2$ ...(i)

[Angles opposite to equal sides of a triangle are equal]

Also, since $AD \parallel BC$ and AC is the transversal.

$\therefore$ $\angle 1 = \angle 3$ [Alternate interior angles] ...(ii)

From (i) and (ii), we have

$\angle 2 = \angle 3$ ...(iii)

Since, $AB \parallel DC$ and AC is transversal.

$\therefore$ $\angle 2 = \angle 4$ [Alternate interior angles] ...(iv)

From (i) and (iv), we have

$\angle 1 = \angle 4$ ...(v)

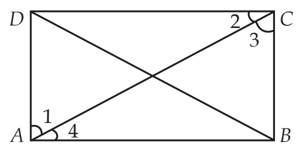
From (iii) and (v), we have

AC bisects $\angle C$ as well as $\angle A$.

Similarly, we can prove that BD bisects $\angle B$ as well as $\angle D$.

8. We have a rectangle $ABCD$ such that AC bisects $\angle A$ as well as $\angle C$.

$\Rightarrow$ $\angle 1 = \angle 4$ and $\angle 2 = \angle 3$ ...(1)



(i) Since every rectangle is a parallelogram.

$\therefore$ $ABCD$ is a parallelogram.

$\Rightarrow$ $AB \parallel CD$ and AC is a transversal.

$\therefore$ $\angle 2 = \angle 4$ [Alternate interior angles] ...(2)

From (1) and (2), we have

$\angle 3 = \angle 4$

In ΔABC, $\angle 3 = \angle 4$

$\Rightarrow$ $AB = BC$

[Sides opposite to equal angles of a triangle are equal]

$\Rightarrow$ $ABCD$ is a rectangle having adjacent sides equal.

$\Rightarrow$ $ABCD$ is a square.

(ii) Since $ABCD$ is a square and diagonals of a square bisect the opposite angles.

So, BD bisects $\angle B$ as well as $\angle D$.

9. We have parallelogram $ABCD$, BD is the diagonal and points P and Q are such that

$DP = BQ$ [Given]

(i) Since $AD \parallel BC$ and BD is a transversal.

$\therefore$ $\angle ADB = \angle CBD$ [Alternate interior angles]

$\Rightarrow$ $\angle ADP = \angle CBQ$

Now, in ΔAPD and ΔCQB, we have

$AD = CB$ [Opposite sides of parallelogram $ABCD$]

$PD = QB$ [Given]

$\angle ADP = \angle CBQ$ [Proved]

$\therefore$ $\Delta APD \cong \Delta CQB$ [By SAS congruence]

(ii) Since $\Delta APD \cong \Delta CQB$ [Proved]

$\therefore$ $AP = CQ$ [By C.P.C.T.]

(iii) Since $AB \parallel CD$ and BD is a transversal.

$\therefore$ $\angle ABD = \angle CDB$ [Alternate interior angles]

$\Rightarrow$ $\angle ABQ = \angle CDP$

Now, in ΔAQB and ΔCPD, we have

$QB = PD$ [Given]

$\angle ABQ = \angle CDP$ [Proved]

$AB = CD$ [Opposite sides of a parallelogram]

$\therefore$ $\Delta AQB \cong \Delta CPD$ [By SAS congruence]

(iv) Since $\Delta AQB \cong \Delta CPD$ [Proved]

$\therefore$ $AQ = CP$ [By C.P.C.T.]

(v) In quadrilateral $APCQ$,

$AP = CQ$ and $AQ = CP$ [Proved]

$\therefore$ $APCQ$ is a parallelogram.

10. (i) In ΔAPB and ΔCQD, we have

$\angle APB = \angle CQD$ [Each equals 90°]

$AB = CD$ [Opposite sides of a parallelogram]

$\angle ABP = \angle CDQ$ [$\because$ $AB \parallel CD$ and BD is a transversal

$\Rightarrow$ Alternate angles are equal]

$\therefore$ $\Delta APB \cong \Delta CQD$ [By AAS congruence]

(ii) Since $\Delta APB \cong \Delta CQD$ [Proved]

$\therefore$ $AP = CQ$ [By C.P.C.T.]

11. (i) In quadrilateral $ABED$, we have

$AB = DE$ [Given]

$AB \parallel DE$ [Given]

So, $ABED$ is a quadrilateral in which a pair of opposite sides (AB and DE) are parallel and of equal length.

$\therefore$ $ABED$ is a parallelogram.

(ii) In quadrilateral $BEFC$, we have

$BC = EF$ [Given]

$BC \parallel EF$ [Given]

So, $BEFC$ is a quadrilateral in which a pair of opposite sides (BC and EF) are parallel and of equal length.

$\therefore$ $BEFC$ is a parallelogram.

(iii) $\because$ $ABED$ is a parallelogram [Proved]

$\therefore$ $AD \parallel BE$ and $AD = BE$

[Opposite sides of a parallelogram] ...(1)

Also, $BEFC$ is a parallelogram. [Proved]

$\therefore$ $BE \parallel CF$ and $BE = CF$

[Opposite sides of a parallelogram] ...(2)

From (1) and (2), we have

$AD \parallel CF$ and $AD = CF$

(iv) Since $AD \parallel CF$ and $AD = CF$ [Proved]

So, in quadrilateral $ACFD$, one pair of opposite sides (AD and CF) are parallel and equal in length.

$\therefore$ Quadrilateral $ACFD$ is a parallelogram.

(v) Since $ACFD$ is a parallelogram. [Proved]

$\therefore$ $AC = DF$ [Opposite sides of a parallelogram]

(vi) In ΔABC and ΔDEF, we have

$AB = DE$ [Given]

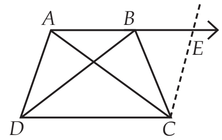
$BC = EF$ [Given]

$AC = DF$ [Proved]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SSS congruence]

12. Produce AB to E and draw $CE \parallel AD$.

Join AC and BD.



(i) $\because$ $AB \parallel DC \Rightarrow AE \parallel DC$

Also, $AD \parallel CE$ [By construction]

$\therefore$ $AECD$ is a parallelogram.

$\Rightarrow$ $AD = CE$ ...(1)

[Opposite sides of a parallelogram]

But $AD = BC$ [Given] ...(2)

By (1) and (2), we have, $BC = CE$

Now, in ΔBCE, we have $BC = CE$

$\Rightarrow$ $\angle CEB = \angle CBE$ ...(3)

[Angles opposite to equal sides of a triangle are equal]

Also, $\angle ABC + \angle CBE = 180°$ [Linear pair] ...(4)

and $\angle A + \angle CEB = 180°$ ...(5)

[Interior angles on same side of transversal]

From (4) and (5), we get

$\angle ABC + \angle CBE = \angle A + \angle CEB$

$\Rightarrow$ $\angle ABC = \angle A$ [From (3)]

$\Rightarrow$ $\angle B = \angle A$ ...(6)

(ii) $\because$ $AB \parallel CD$ and AD is a transversal.

$\therefore$ $\angle A + \angle D = 180°$ ...(7)

[Interior angles on same side of transversal]

Similarly, $\angle B + \angle C = 180°$ ...(8)

From (7) and (8), we get

$\angle A + \angle D = \angle B + \angle C$

$\Rightarrow$ $\angle C = \angle D$ [From (6)]

(iii) In ΔABC and ΔBAD, we have

$AB = BA$ [Common]

$BC = AD$ [Given]

$\angle ABC = \angle BAD$ [Proved]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruence]

(iv) Since $\Delta ABC \cong \Delta BAD$ [Proved]

$\therefore$ $AC = BD$ [By C.P.C.T.]

EXERCISE - 8.2

1. (i) In ΔACD, we have

S is the mid-point of AD and R is the mid-point of CD.

$\therefore$ $SR = \frac{1}{2}AC$ and $SR \parallel AC$ ...(1)

[By mid-point theorem]

(ii) In ΔABC,

P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ $PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ ...(2)

[By mid-point theorem]

From (1) and (2), we get

$PQ = \frac{1}{2}AC = SR$ and $PQ \parallel AC \parallel SR$

$\Rightarrow$ $PQ = SR$ and $PQ \parallel SR$

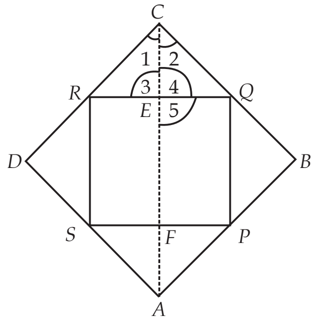
(iii) In quadrilateral $PQRS$, we have

$PQ = SR$ and $PQ \parallel SR$ [Proved]

$\therefore$ $PQRS$ is a parallelogram.

2. Join AC.

In ΔABC, P and Q are the mid-points of AB and BC respectively.



$\therefore \quad PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ ...(i)

[By mid-point theorem]

In ΔADC, R and S are the mid-points of CD and DA respectively.

$\therefore \quad SR = \frac{1}{2}AC$ and $SR \parallel AC$ ...(ii)

[By mid-point theorem]

From (i) and (ii), we get

$PQ = \frac{1}{2}AC = SR$ and $PQ \parallel AC \parallel SR$

$\Rightarrow \quad PQ = SR$ and $PQ \parallel SR$

$\therefore \quad PQRS$ is a parallelogram. ...(iii)

Now, in ΔERC and ΔEQC,

$\angle 1 = \angle 2$ [The diagonals of a rhombus bisect the opposite angles]

$CR = CQ$ $\left[\because CD = CB \Rightarrow \frac{CD}{2} = \frac{CB}{2}\right]$

$CE = EC$ [Common]

$\therefore \quad \Delta ERC \cong \Delta EQC$ [By SAS congruence]

$\Rightarrow \quad \angle 3 = \angle 4$ [By C.P.C.T.] ...(iv)

But $\angle 3 + \angle 4 = 180°$ [Linear pair] ...(v)

From (iv) and (v), we get

$\Rightarrow \quad \angle 3 = \angle 4 = 90°$

Now, since $PQ \parallel AC$ and EQ is transversal.

$\therefore \quad \angle RQP + \angle 5 = 180°$

[Interior angles on the same side of transversal]

$\Rightarrow \quad \angle RQP = 180° - \angle 5$

But $\angle 5 = \angle 3$ [Vertically opposite angles]

$\therefore \quad \angle 5 = 90°$

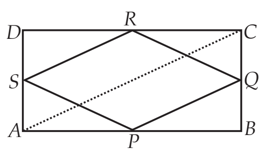
So, $\angle RQP = 180° - \angle 5 = 90°$

$\therefore$ One angle of parallelogram $PQRS$ is 90°.

Thus, $PQRS$ is a rectangle.

3. Join AC.

In ΔABC, P and Q are mid-points of AB and BC respectively.



$\therefore \quad PQ = \frac{1}{2}AC$ and $PQ \parallel AC$

...(i) [By mid-point theorem]

Similarly, in ΔADC, we have

$SR = \frac{1}{2}AC$ and $SR \parallel AC$ ...(ii)

From (i) and (ii), we get

$PQ = SR$ and $PQ \parallel SR$

$\therefore \quad PQRS$ is a parallelogram.

Now, in ΔPAS and ΔPBQ, we have

$\angle A = \angle B$ [Each equals 90°]

$AP = BP$ [$\because P$ is the mid-point of AB]

$AS = BQ$ $\left[\because AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC\right]$

$\therefore \quad \Delta PAS \cong \Delta PBQ$ [By SAS congruence]

$\Rightarrow \quad PS = PQ$ [By C.P.C.T.]

Also, $PS = QR$ and $PQ = SR$.

[Opposite sides of a parallelogram]

So, $PQ = QR = RS = SP$

i.e., $PQRS$ is a parallelogram having all of its sides equal.

Hence, $PQRS$ is a rhombus.

4. Let G be the point where EF intersect diagonal BD.

In ΔDAB, E is the mid-point of AD and $EG \parallel AB$

[$\because EF \parallel AB$]

$\therefore \quad G$ is the mid-point of BD

[By converse of mid-point theorem]

Again in ΔBDC, we have

G is the mid-point of BD and $GF \parallel DC$.

[$\because AB \parallel DC$ and $EF \parallel AB \Rightarrow EF \parallel DC$]

$\therefore \quad F$ is the mid-point of BC.

[By converse of mid-point theorem]

5. Since, the opposite sides of a parallelogram are parallel and equal.

$\therefore \quad AB \parallel DC \Rightarrow AE \parallel FC$ ...(i)

and $AB = DC$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AE = FC$ ...(ii)

From (i) and (ii), we have

$AECF$ is a parallelogram.

Now, in ΔDQC, we have

F is the mid-point of DC and $FP \parallel CQ$ [$\because AF \parallel CE$]

$\therefore \quad P$ is the mid-point of DQ.

[By converse of mid-point theorem]

$\Rightarrow \quad DP = PQ$ ...(iii)

Similarly, in ΔBAP, E is the mid-point of AB and $EQ \parallel AP$.

[$\because AF \parallel CE$]

$\therefore \quad Q$ is the mid-point of BP.

[By converse of mid-point theorem]

$\Rightarrow \quad BQ = PQ$ ...(iv)

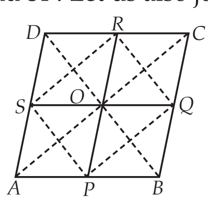
$\therefore$ From (iii) and (iv), we have

$DP = PQ = BQ$

So, the line segments AF and EC trisect the diagonal BD.

6. Let $ABCD$ be a quadrilateral and P, Q, R and S be the mid-points of AB, BC, CD and DA.

Join PQ, QR, RS and SP. Let us also join PR and SQ.



Now, in ΔABC, P and Q are the mid-points of AB and BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(i)

[By mid-point theorem]

Similarly, $RS \parallel AC$ and $RS = \frac{1}{2}AC$ ...(ii)

By (i) and (ii), we get

$PQ \parallel RS$ and $PQ = RS$

$\therefore \quad PQRS$ is a parallelogram.

Since, the diagonals of a parallelogram bisect each other, *i.e.*, PR and SQ bisect each other.

Thus, the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

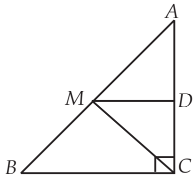
7. (i) In ΔACB,

M is the mid-point of AB. [Given]

$MD \parallel BC$ [Given]

$\therefore \quad D$ is the mid-point of AC.

[By converse of mid-point theorem]



(ii) Since, $MD \parallel BC$ and AC is a transversal.

$\therefore \quad \angle MDA = \angle BCA$ [Corresponding angles]

But $\angle BCA = 90°$ [Given]

$\therefore \quad \angle MDA = 90°$

$\Rightarrow \quad MD \perp AC.$

(iii) In ΔADM and ΔCDM, we have

$\angle ADM = \angle CDM$ [Each equals 90°]

$MD = DM$ [Common]

$AD = CD$ [$\because D$ is the mid-point of AC]

$\therefore \quad \Delta ADM \cong \Delta CDM$ [By SAS congruence]

$\Rightarrow \quad MA = MC$ [By C.P.C.T.] ...(i)

Now, since M is the mid-point of AB. [Given]

$\therefore \quad MA = \frac{1}{2}AB$ ...(ii)

From (i) and (ii), we have

$CM = MA = \frac{1}{2}AB$