CHAPTER 8

Quadrilaterals

TRY YOURSELF — SOLUTIONS

1. Let the angles of the quadrilateral be x, $2x$, $4x$ and $5x$.

Since sum of the angles of a quadrilateral is 360°.

$\therefore$ $x + 2x + 4x + 5x = 360°$

$\Rightarrow$ $12x = 360° \Rightarrow x = 30°$

$\therefore$ $2x = 2 \times 30° = 60°$,

$4x = 4 \times 30° = 120°$ and $5x = 5 \times 30° = 150°$

$\therefore$ Required angles are 30°, 60°, 120° and 150°.

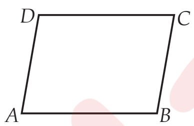
2. Since sum of the angles of a quadrilateral is 360°.

$\therefore$ $(x + 20)° + (x - 20)° + (2x + 5)° + (2x - 5)° = 360°$

$\Rightarrow$ $6x = 360 \Rightarrow x = 60$

3. No. As we know, the sum of all the angles of a quadrilateral is 360°, so a quadrilateral can have maximum of three obtuse angles.

4. Since, $ABCD$ is a parallelogram.



Therefore, $AD \parallel BC$.

Now, $AD \parallel BC$ and transversal AB intersects them at A and B respectively.

$\therefore$ $\angle A + \angle B = 180°$

[Interior angles on the same side of the transversal]

Similarly, we can prove that

$\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$.

5. In quadrilateral $ABCD$,

$AB \parallel DC$ [$l \parallel m$ (Given)] ...(i)

$AD \parallel BC$ [$p \parallel q$ (Given)] ...(ii)

$\therefore$ From (i) and (ii), $ABCD$ is a parallelogram.

$\Rightarrow$ $\angle ABC + \angle BCD = 180°$

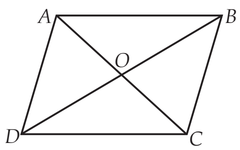
[Sum of consecutive angles of a parallelogram is 180°]

$\Rightarrow$ $\angle ABC + 108° = 180°$ [$\angle BCD = 108°$ (Given)]

$\Rightarrow$ $\angle ABC = 180° - 108° = 72°$

Now, $\angle DAB = \angle BCD = 108°$ and $\angle ADC = \angle ABC = 72°$

6. Since, diagonals of a parallelogram bisect each other. Therefore, O is mid-point of AC and BD.

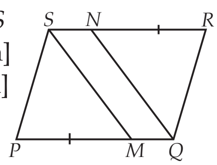


$\therefore$ $OC = \frac{1}{2}AC = \frac{1}{2} \times 6.8 = 3.4\,\text{cm}$ [$AC = 6.8$ cm (Given)]

And $OD = \frac{1}{2}BD = \frac{1}{2} \times 5.6 = 2.8\,\text{cm}$ [$BD = 5.6$ cm (Given)]

7. In parallelogram $PQRS$, $PQ = RS$

[Opposite sides of a parallelogram]



Also, we have $PM = RN$ [Given]

$\therefore$ $PQ - PM = RS - RN$

$\Rightarrow$ $MQ = SN$ ...(i)

Now, $MQ \parallel SN$ ...(ii)

[$\because$ $PQRS$ is a parallelogram $\Rightarrow PQ \parallel SR$]

$\therefore$ From (i) and (ii), we have

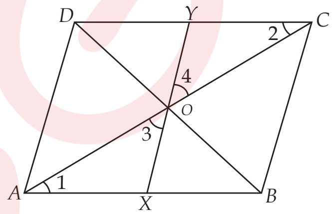
$MQNS$ is a parallelogram

$\therefore$ $MS \parallel NQ$ [Opposite sides of a parallelogram]

8. Since $ABCD$ is a parallelogram.

$\therefore$ $AB \parallel DC$

$\because$ $AB \parallel DC$ and transversal AC intersects them at A and C.



$\therefore$ $\angle 1 = \angle 2$ [Alternate interior angles] ...(i)

Now, in ΔOAX and ΔOCY, we have

$\angle 1 = \angle 2$ [From (i)]

$OA = OC$ [Diagonals of a parallelogram bisect each other]

$\angle 3 = \angle 4$ [Vertically opposite angles]

$\therefore$ $\Delta OAX \cong \Delta OCY$ [By ASA congruence]

$\therefore$ $OX = OY$ [By C.P.C.T.]

9. Since $\angle SRQ + \angle QRT = 180°$ [Linear pair]

$\Rightarrow$ $\angle SRQ + 72° = 180°$ [$\because \angle QRT = 72°$ (Given)]

$\Rightarrow$ $\angle SRQ = 180° - 72° = 108°$

But $\angle SRP = \frac{1}{2}\angle SRQ$

[$\because$ Diagonal bisects angle at vertex]

$\Rightarrow$ $\angle SRP = \frac{1}{2} \times 108° = 54°$ ...(i)

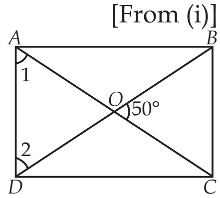
In ΔSPR, $SP = SR$

$\Rightarrow$ $\angle SPR = \angle SRP$

[Angles opposite to equal sides are equal]

$\Rightarrow$ $\angle SPR = 54°$ [From (i)]

10. In rectangle $ABCD$, $\angle BOC = 50°$



And $\angle AOD = \angle BOC$

[Vertically opposite angles]

$\therefore$ $\angle AOD = 50°$ ...(i)

Now, we know that diagonals of a rectangle are equal and bisect each other.

$\therefore$ $OA = OD$

$\Rightarrow$ $\angle 1 = \angle 2$ ...(ii)

[Angles opposite to equal sides are equal]

Now, in ΔODA, we have

$\angle 1 + \angle 2 + \angle AOD = 180°$

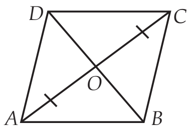
$\Rightarrow \angle 2 + \angle 2 + 50° = 180°$ [Using (i) and (ii)]

$\Rightarrow 2\angle 2 = 180° - 50° = 130°$

$\Rightarrow \angle 2 = 65°$

$\therefore \angle ODA = 65°$

11. Let $ABCD$ be a rhombus.



$\therefore AB = BC = CD = DA$

[Sides of a rhombus are equal]

Now, in ΔAOD and ΔCOD,

$OA = OC$

[Diagonals of a parallelogram bisect each other and rhombus is a parallelogram]

$OD = OD$ [Common]

$AD = CD$

Therefore, $\Delta AOD \cong \Delta COD$ [By SSS congruence]

$\therefore \angle AOD = \angle COD$ [By C.P.C.T.]

But, $\angle AOD + \angle COD = 180°$ [Linear pair]

$\therefore 2\angle AOD = 180°$

$\Rightarrow \angle AOD = 90°$

So, the diagonals of a rhombus are perpendicular to each other.

12. In ΔPQR, since X and Y are the mid-points of sides PQ and PR respectively. Therefore, by mid-point theorem, we have

$XY \parallel QR$ and $XY = \frac{1}{2}QR$

$\Rightarrow XY = \frac{1}{2} \times 10$ [$QR = 10$ cm (Given)]

$\Rightarrow XY = 5$ cm

13. In ΔXYZ, L and M are mid-points of XY and XZ respectively. Therefore, by mid-point theorem, we have

$LM = \frac{1}{2}YZ$ ...(i)

Now, since M and N are mid-points of XZ and YZ respectively.

$\therefore MN = \frac{1}{2}XY$ ...(ii)

And N and L are mid-points of YZ and XY respectively.

$\therefore NL = \frac{1}{2}XZ$ ...(iii)

Now, perimeter of $\Delta LMN = LM + MN + NL$

$= \frac{1}{2}YZ + \frac{1}{2}XY + \frac{1}{2}XZ$ [Using (i), (ii) and (iii)]

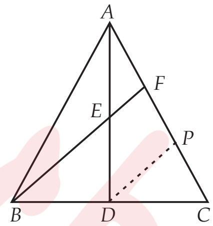
$= \frac{1}{2} \times 5.6 + \frac{1}{2} \times 4.4 + \frac{1}{2} \times 4.8$

[$\because YZ = 5.6$ cm, $XY = 4.4$ cm, $XZ = 4.8$ cm (Given)]

$= 2.8 + 2.2 + 2.4 = 7.4$ cm

14. We have, a ΔABC in which AD is a median and E is the mid-point of AD.

Let us draw $DP \parallel BF$.



Now in ΔADP, E is the mid-point of AD and $EF \parallel DP$.

$\therefore$ F is mid-point of AP.

$\Rightarrow AF = FP$ [By converse of mid-point theorem]

Now in ΔFBC, D is mid-point of BC and $DP \parallel BF$.

$\therefore$ P is mid-point of FC.

$\Rightarrow FP = PC$ [By converse of mid-point theorem]

Hence, $AF = FP = PC$

$\therefore AF = \frac{1}{3}AC$

15. Since, D and E are mid-points of BC and AC respectively. Therefore, by mid-point theorem, we have

$DE = \frac{1}{2}AB$ ...(i)

Now, E and F are the mid-points of AC and AB respectively.

$\therefore EF = \frac{1}{2}BC$ ...(ii)

And F and D are the mid-points of AB are BC respectively.

$\therefore FD = \frac{1}{2}AC$ ...(iii)

Now, ΔABC is an equilateral triangle

$\therefore AB = BC = CA$

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

$\Rightarrow DE = EF = FD$ [Using (i), (ii) and (iii)]

Hence, ΔDEF is an equilateral triangle.