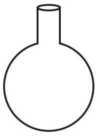
Surface Areas and Volumes

EXAM DRILL

CHAPTER 12

SOLUTIONS

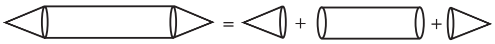
1. (a) : The shape of Surahi is as  , which is combination of sphere and cylinder.

2. (b) : Let the radius of the cone be r cm.

Since, the height and diameter of the base of the largest right circular cone = Edge of the cube.

$\therefore$ $2r = 8$ cm $\Rightarrow r = 4$ cm

3. (c) : The given figure is

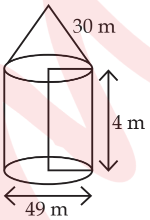


= Cone + Cylinder + Cone

= Two cones and a cylinder

4. (b) : Because the sphere is enclosed in the cylinder, therefore the diameter of sphere is equal to the diameter of cylinder which is $2r$ cm.

5. (a) : Area of canvas required = Curved surface area of cylinder + Curved surface area of cone



$= 2\pi rh + \pi rl$

$= \pi r[2h + l] = \frac{22}{7} \times \frac{98}{2}[2 \times 4 + 30]$

$= 5852 \text{ m}^2$

6. When, we join two solid hemispheres along their bases of radius r, we get a solid sphere. Also, curved surface area of a hemisphere is $2\pi r^2$.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2$ $= 4\pi r^2$.

7. Because solid ball is exactly fitted inside the cubical box of side a. So, a is the diameter of the solid ball.

$\therefore$ Radius of the ball $= \frac{a}{2}$ [$\because$ diameter = 2 × radius]

So, volume of the ball $= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{1}{6}\pi a^3$

8. (i) Lateral surface area of *Hermika* which is cubical in shape $= 4a^2 = 4 \times (8)^2 = 256 \text{ m}^2$

(ii) Diameter of cylindrical base = 42 m

$\therefore$ Radius of cylindrical base (r) = 21 m

Height of cylindrical base (h) = 12 m

$\therefore$ Number of bricks used $= \dfrac{\frac{22}{7} \times 21 \times 21 \times 12}{0.01}$

$= 1663200$

(iii) Given, diameter of *Anda* which is hemispherical in shape = 42 m

$\Rightarrow$ Radius of *Anda* (r) = 21 m

$\therefore$ Volume of *Anda* $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 44 \times 21 \times 21 = 19404 \text{ m}^3$

(iv) Given, radius of *Pradakshina Path* (r) = 25 m

$\therefore$ Perimeter of *path* $= 2\pi r$

$= \left(2 \times \frac{22}{7} \times 25\right)$ m

$\therefore$ Distance covered by priest $= 14 \times 2 \times \frac{22}{7} \times 25$

$= 2200$ m

(v) $\because$ Radius of *Anda* (r) = 21 m

$\therefore$ Curved surface area of *Anda* $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 21 \times 21 = 2772 \text{ m}^2$

9. We have, radius of each coin = 3.5 cm

$= \frac{35}{10} \text{ cm} = \frac{7}{2} \text{ cm}$

Thickness of each coin = 0.5 cm $= \frac{1}{2}$ cm

So, height of cylinder made by Meera$(h_1) = 12 \times \frac{1}{2} = 6$ cm

and height of cylinder made by Dhara (h_2)

$= 8 \times \frac{1}{2} = 4$ cm

(i) (b) : Curved surface area of cylinder made by Meera

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 6 = 132 \text{ cm}^2$

(ii) (b) : Required ratio

$= \dfrac{\text{Curved surface area of cylinder made by Meera}}{\text{Curved surface area of cylinder made by Dhara}}$

$= \frac{2\pi rh_1}{2\pi rh_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ *i.e.*, 3 : 2

(iii) (a) : Volume of cylinder made by Dhara $= \pi r^2 h_2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4 = 154 \text{ cm}^3$

(iv) (c) : Required ratio

$$= \frac{\text{Volume of cylinder made by Meera}}{\text{Volume of cylinder made by Dhara}}$$

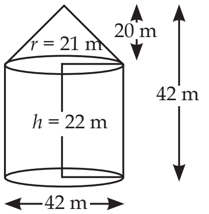
$$= \frac{\pi r^2 h_1}{\pi r^2 h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2} \text{ i.e., } 3:2$$

(v) (a) : When two coins are shifted from Meera's cylinder to Dhara's cylinder, then length of both cylinders become equal.

So, volume of both cylinders become equal.

10. (i) (c) : Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh = \pi r(l + 2h)$



$= \frac{22}{7} \times 21\ (29 + 44) = 4818\ \text{m}^2$

$$\left[\because\ l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29\ \text{m}\right]$$

(ii) (b): Area of floor = πr^2

$= \frac{22}{7} \times 21 \times 21 = 1386\ \text{m}^2$

Number of persons that can be accommodated in the tent $= \frac{1386}{126} = 11$

(iii) (d): Since, cost of 100 m^2 of canvas = ₹ 425

$\therefore$ Cost of 1 m^2 of canvas = ₹ 4.25

Thus, cost of 4818 m^2 of canvas = ₹ 20476.50

(iv) (c) : Volume of tent = Volume of cone + Volume of cylinder $= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h = \pi r^2\left(\frac{1}{3}h_1 + h\right)$

$$= \frac{22}{7} \times (21)^2 \left[\frac{20}{3} + 22\right] = \frac{9702}{7} \times \frac{86}{3} = 39732\ \text{m}^3$$

(v) (a): Required number of persons

$$= \frac{\text{Volume of tent}}{\text{Space required by one person}} = \frac{39732}{1892} = 21$$

11. (i) Curved surface area of two identical cylindrical parts = $2 \times 2\pi rh = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times 5$

$= 78.57\ \text{cm}^2$

(ii) Volume of big cylindrical part = $\pi r^2 h$

$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12 = 190.93\ \text{cm}^3$

(iii) Volume of two hemispherical ends $= 2 \times \frac{2}{3}\pi r^3$

$= \frac{2 \times 2}{3} \times \frac{22}{7} \times \left(\frac{2.5}{2}\right)^3 = 8.18\ \text{cm}^3$

(iv) Curved surface area of two hemispherical ends

$= 2 \times 2\pi r^2 = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times \frac{2.5}{2} = 19.64\ \text{cm}^2$

(v) Difference of volume of bigger cylinder to two small hemispherical ends = 190.93 – 8.18 = 182.75 cm^3

12. Radius of the sphere $(r) = \frac{18}{2}\ \text{cm} = 9\ \text{cm}$

Radius of the cylinder $(R) = \frac{36}{2}\ \text{cm} = 18\ \text{cm}$

Let us assume that the water level in the cylinder rises by h cm.

After the sphere is completely submerged,

Volume of the sphere = Volume of water raised in the cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h \Rightarrow \frac{4}{3}\pi(9)^3 = \pi(18)^2 \times h$$

$$\Rightarrow h = \frac{4 \times 9 \times 9 \times 9}{3 \times 18 \times 18} \Rightarrow h = 3$$

Thus, the water level in the cylinder rises by 3 cm.

13. Let l, r and h be the slant height, radius and height of the conical top respectively.

$\therefore\ l^2 = r^2 + h^2 = 3^2 + 4^2 = 25 = 5^2 \Rightarrow l = 5$ m

Curved surface area of cylindrical part = $2\pi rh$

$= 2\pi(3)(10) = 60\pi\ \text{m}^2$

Curved surface area of conical part = $\pi rl = \pi(3)(5)$

$= 15\pi\ \text{m}^2$

$\therefore$ Curved surface area of pillar

= Curved surface area of cylindrical part + Curved surface area of conical part = $60\pi + 15\pi = 75\pi\ \text{m}^2$

14. Volume of one cube = 64 cm^3

$\Rightarrow (\text{Edge})^3 = 64\ \text{cm}^3 \Rightarrow$ Edge = 4 cm

Length of the cuboid (l) = 5 × Edge = 5 × 4 = 20 cm

breadth (b) = 4 cm and height (h) = 4 cm

$\therefore$ Surface area of cuboid = $2(lb + bh + hl)$

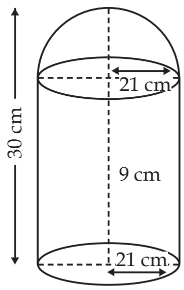
= 2[20 × 4 + 4 × 4 + 4 × 20] = 2 × 176 = 352 cm^2

Volume of the cuboid = $l \times b \times h$

= 20 × 4 × 4 = 320 cm^3

15. Radius of the hemisphere and cylinder, r

$= \frac{42}{2} = 21$ cm



Height of the cylinder, h

= 30 – 21 = 9 cm

$\therefore$ Internal surface area of the vessel = Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 21(9 + 21) = 3960\ \text{cm}^2$

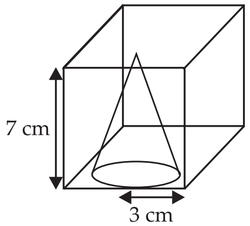
16. We know that, capacity of cylindrical vessel = $\pi r^2 h$ cm^3

and capacity of hemisphere $= \dfrac{2}{3}\pi r^3$ cm

From the figure, capacity of the cylindrical vessel

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2[3h - 2r]$$

17. Given that, side of a solid cube (a)= 7 cm
Height of conical cavity (h) = 7 cm
Radius of conical cavity (r) = 3 cm



Now, volume of cube
$= a^3 = (7)^3 = 343$ cm^3
Volume of conical cavity

$$= \frac{1}{3}\pi \times r^2 \times h \;\; = \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 7$$

$= 66$ cm^3

$\therefore$ Volume of remaining solid
= Volume of cube – Volume of conical cavity
$= 343 - 66 = 277$ cm^3

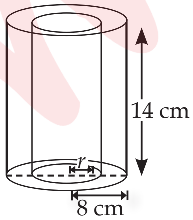
18. Here, radius of cylindrical portion (r) = Radius of conical portion (r) $= \dfrac{105}{2}$ m

Height of cylindrical portion (h) = 3 m
Slant height of conical portion (l) = 53 m
Total canvas used in making the tent
= Curved surface area of cylindrical portion + Curved surface area of conical portion

$$= 2\pi rh + \pi rl = 2\times\frac{22}{7}\times\frac{105}{2}\times 3 + \frac{22}{7}\times\frac{105}{2}\times 53$$

$= 990 + 8745 = 9735$ m^2

19. Total curved surface area of hollow cylinder



$= 2\pi RH + 2\pi rH = 1320$

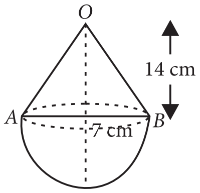
$\Rightarrow \quad 2 \times \dfrac{22}{7} \times 14(8 + r) = 1320$

$\Rightarrow \quad 88(8 + r) = 1320$

$\Rightarrow \quad 8 + r = \dfrac{1320}{88} \Rightarrow 8 + r = 15 \Rightarrow r = 7$

$\therefore$ Internal diameter $= 2r = 14$ cm

20. Radius of cone (r) = 7 cm = Radius of hemisphere
Height of cone (h) = 14 cm



$\therefore$ Volume of solid
= Volume of cone + volume of hemisphere

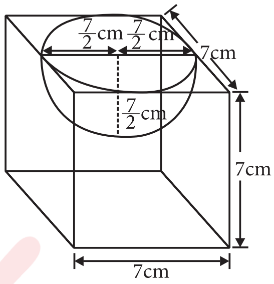
$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}[h + 2r]$$

$$= \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times[14 + (2\times 7)]$$

$$= \frac{22}{3}\times 7\times 28 = \frac{4312}{3} = 1437.33 \text{ cm}^3$$

21. Diameter of hemisphere = Edge of cube = 7 cm

Radius of hemisphere



$(r) = \dfrac{7}{2}$ cm

Required surface area = surface area of cube – area of top of hemisphere + curved surface area of hemisphere
$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$$= 6(7)^2 + \pi\left(\frac{7}{2}\right)^2$$

$$= 6\times 49 + \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$= 294 + 38.5 = 332.5$ cm^2

22. Given, ice-cream cone is the combination of a hemisphere and a cone.

Also, radius of hemisphere, r = 5 cm

$\therefore$ Volume of hemisphere $= \dfrac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\frac{22}{7}\times(5)^3 = \frac{5500}{21} = 261.90 \text{ cm}^3$$

Now, radius of the cone = 5 cm
and height of the cone, h
= height of ice-cream cone – radius of hemisphere
$= 10 - 5 = 5$ cm

$\therefore$ Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times(5)^2\times 5 = \frac{2750}{21} = 130.95 \text{ cm}^3$$

Now, total volume of ice-cream cone =
volume of hemisphere + volume of cone
$= 261.90 + 130.95 = 392.85$ cm^3

Since, $\dfrac{1}{6}$ part is left unfilled with ice-cream.

$\therefore$ Required volume of ice-cream

$= 392.85 - 392.85 \times \dfrac{1}{6} = 392.85 - 65.475 = 327.4$ cm^3