Surface Areas and Volumes

CHAPTER 12

NCERT FOCUS

SOLUTIONS

EXERCISE - 12.1

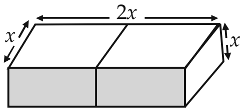
1. Let the edge of each cube = x

Given, volume of each cube

$= 64\text{ cm}^3$

$\therefore\quad x^3 = 64\text{ cm}^3$

$\Rightarrow\quad x = 4\text{ cm}$



Now, length of the resulting cuboid (l) = $2x$ cm = 8 cm

Breadth of the resulting cuboid (b) = x cm = 4 cm

Height of the resulting cuboid (h) = x cm = 4 cm

$\therefore$ Surface area of the cuboid = $2(lb + bh + hl)$

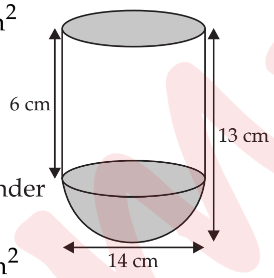
$= 2[(8 \times 4) + (4 \times 4) + (4 \times 8)]\text{ cm}^2$

$= 2[32 + 16 + 32]\text{ cm}^2 = 2[80]\text{ cm}^2 = 160\text{ cm}^2$

2. For hemispherical part, radius (r) $= \dfrac{14}{2} = 7$ cm

$\therefore$ Curved surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7\text{cm}^2 = 308\text{ cm}^2$$



Total height of vessel = 13 cm

$\therefore$ Height of cylinder

= (13 – 7) cm = 6 cm

and radius(r) = 7 cm

$\therefore$ Curved surface area of cylinder

$= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 6\text{cm}^2 = 264\text{cm}^2$$

$\therefore$ Inner surface area of vessel = $(308 + 264)\text{ cm}^2$

$= 572\text{ cm}^2$

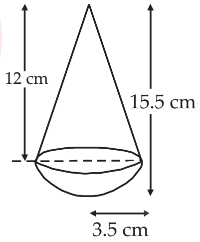
3. Given radius of cone (r)

= radius of hemisphere (r)

= 3.5 cm

Height of cone (h) = (15.5 – 3.5) cm

= 12 cm



Also, slant height (l) $= \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + (3.5)^2} = \sqrt{156.25}$

= 12.5 cm

Total surface area of the toy = curved surface area of conical part + curved surface area of hemispherical part

$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \dfrac{22}{7} \times \dfrac{35}{10}(12.5 + 2 \times 3.5)\text{ cm}^2$

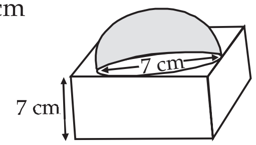
$= 11 \times (12.5 + 7)\text{ cm}^2 = 11 \times 19.5\text{ cm}^2 = 214.5\text{ cm}^2$

4. Let side of the block, (a) = 7 cm

$\therefore$ The greatest diameter of the hemisphere = 7 cm

Radius of hemisphere, (r)

= 7/2 cm



Surface area of the solid

= [Total surface area of the cubical block]

+ [Curved surface of the hemisphere]

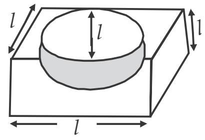
– [Base area of the hemisphere]

$= (6 \times a^2) + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2 = (6 \times 7^2) + \left(\dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}\right) = \left(294 + \dfrac{77}{2}\right) = 332.5\text{ cm}^2$

5. Given, side of the cube = diameter of the hemisphere = l

$\Rightarrow$ Radius of the hemisphere $= \dfrac{l}{2}$



$\therefore$ Surface area of hemisphere $= 2\pi r^2$

$= 2 \times \pi \times \dfrac{l}{2} \times \dfrac{l}{2} = \dfrac{\pi l^2}{2}$ sq. units

Base area of the hemisphere $= \pi\left(\dfrac{l}{2}\right)^2 = \dfrac{\pi l^2}{4}$ sq. units

Surface area of the cube = $6 \times l^2 = 6l^2$ sq. units

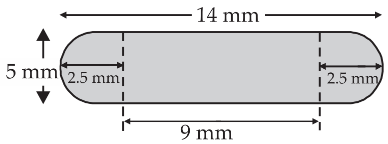
$\therefore$ Surface area of the remaining solid

$$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4} = \frac{24l^2 + 2\pi l^2 - \pi l^2}{4} = \frac{24l^2 + \pi l^2}{4}$$

$$= \frac{l^2}{4}(24 + \pi)\text{ sq.units.}$$

6. Radius of the hemispherical part (r) $= \dfrac{5}{2}$mm

= 2.5 mm



Curved surface area of one hemispherical part = $2\pi r^2$

$\therefore$ Surface area of both hemispherical parts

$$= 2(2\pi r^2) = 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{25}{10}\right)^2\text{ mm}^2$$

$$= 4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\text{mm}^2$$

Entire length of capsule = 14 mm

$\therefore$ Length of cylindrical part = 14 – 2 × 2.5 = 9 mm

$\therefore$ Area of cylindrical part = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.5 \times 9\text{ mm}^2 = 2 \times \frac{22}{7} \times \frac{25}{10} \times 9\text{ mm}^2$$

$\therefore$ Total surface area of capsule

$$= \left[2 \times \frac{22}{7} \times \frac{25}{10} \times 9\right] + \left[4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\right]\text{mm}^2$$

$$=\left(2\times\frac{22}{7}\times\frac{25}{10}\right)\left[9+\frac{50}{10}\right]\text{mm}^2=\frac{44\times25}{70}\times14\text{ mm}^2 = 220\text{ mm}^2$$

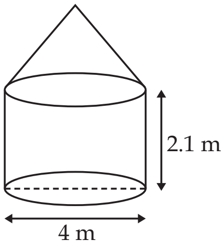
7. For cylindrical part :

Radius $(r) = \frac{4}{2}\text{m} = 2\text{m}$ and

height (h) = 2.1 m

$\therefore$ Curved surface area

$$= 2\pi rh = 2\times\frac{22}{7}\times2\times\frac{21}{10}\text{m}^2$$



For conical part :

Slant height (l) = 2.8 m and base radius (r) = 2 m

$\therefore$ Curved surface area $= \pi rl = \frac{22}{7}\times2\times\frac{28}{10}\text{m}^2$

$\therefore$ Total surface area

= [Curved surface area of the cylindrical part] + [Curved surface area of conical part]

$$=\left[2\times\frac{22}{7}\times2\times\frac{21}{10}\right]+\left[\frac{22}{7}\times2\times\frac{28}{10}\right]\text{m}^2$$

$$=2\times\frac{22}{7}\left[\frac{42}{10}+\frac{28}{10}\right]\text{m}^2 = 2\times\frac{22}{7}\times\frac{70}{10}\text{ m}^2 = 44\text{ m}^2$$

$\because$ Cost of 1 m^2 of canvas = ₹ 500

$\therefore$ Cost of 44 m^2 of canvas = ₹ (500 × 44) = ₹ 22000.

8. For cylindrical part :

Height (h) = 2.4 cm and diameter = 1.4 cm

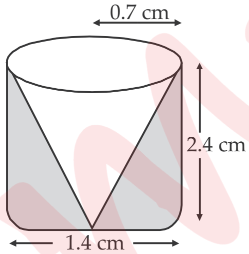
$\Rightarrow$ Radius (r) = 0.7 cm

$\therefore$ Total surface area of the cylindrical part

$= 2\pi rh + 2\pi r^2 = 2\pi r\,[h + r]$

$$= 2\times\frac{22}{7}\times\frac{7}{10}[2.4+0.7]$$

$$=\frac{44}{10}\times3.1=\frac{44\times31}{100}=\frac{1364}{100}\text{cm}^2$$



For conical part :

Base radius (r) = 0.7 cm and height (h) = 2.4 cm

$\therefore$ Slant height $(l) = \sqrt{r^2+h^2} = \sqrt{(0.7)^2+(2.4)^2}$

$= \sqrt{0.49+5.76} = \sqrt{6.25} = 2.5$ cm

$\therefore$ Curved surface area of the conical part

$$=\pi rl = \frac{22}{7}\times0.7\times2.5\text{ cm}^2 = \frac{550}{100}\text{cm}^2$$

Base area of the conical part

$$=\pi r^2 = \frac{22}{7}\times\left(\frac{7}{10}\right)^2\text{ cm}^2 = \frac{22\times7}{100}\text{cm}^2 = \frac{154}{100}\text{cm}^2$$

Total surface area of the remaining solid

= [(Total surface area of cylindrical part) + (Curved surface area of conical part) – (Base area of the conical part)]

$$=\left[\frac{1364}{100}+\frac{550}{100}-\frac{154}{100}\right]\text{cm}^2=\frac{1760}{100}\text{cm}^2 = 17.6\text{ cm}^2.$$

Hence, total surface area to the nearest cm^2 is 18 cm^2.

9. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

$\therefore$ Curved surface area = $2\pi rh$

$$= 2\times\frac{22}{7}\times\frac{35}{10}\times10\text{cm}^2 = 220\text{cm}^2$$

Curved surface area of a hemisphere = $2\pi r^2$

$\therefore$ Curved surface area of both hemispheres

$$= 2\times2\pi r^2 = 4\pi r^2 = 4\times\frac{22}{7}\times\frac{35}{10}\times\frac{35}{10}\text{cm}^2 = 154\text{ cm}^2$$

Total surface area of the remaining solid

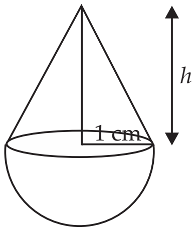
= (220 + 154) cm^2 = 374 cm^2

EXERCISE - 12.2

1. Here, r = 1 cm and h = 1 cm.

Volume of the conical part $= \frac{1}{3}\pi r^2 h$

and volume of the hemispherical part $= \frac{2}{3}\pi r^3$



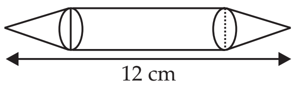
$\therefore$ Volume of the solid shape

$$=\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3=\frac{1}{3}\pi r^2[h+2r]$$

$$=\frac{1}{3}\pi(1)^2[1+2(1)]\text{ cm}^3=\frac{1}{3}\pi\times1\times3\text{ cm}^3=\pi\text{ cm}^3$$

2. Here, diameter = 3 cm

$\Rightarrow$ Radius $(r) = \frac{3}{2}$ cm



Total height = 12 cm

Height of a cone (h) = 2 cm

$\therefore$ Height of both cones = 2 × 2 = 4 cm

$\Rightarrow$ Height of the cylinder (h_1) = (12 – 4) cm = 8 cm

Now, volume of the cylindrical part = $\pi r^2 h$

Volume of both conical parts $= 2\left[\frac{1}{3}\pi r^2 h\right]$

$\therefore$ Volume of the whole model

$$=\pi r^2h_1+\frac{2}{3}\pi r^2h=\pi r^2\left[h_1+\frac{2}{3}h\right]$$

$$=\frac{22}{7}\times\left(\frac{3}{2}\right)^2\left[8+\frac{2}{3}(2)\right]=\frac{22}{7}\times\frac{9}{4}\times\left(\frac{24+4}{3}\right)$$

$$=\frac{22}{7}\times\frac{9}{4}\times\frac{28}{3}\text{cm}^3 = 66\text{ cm}^3.$$

3. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the gulab jamun = 5 cm.

Diameter = 2.8 cm $\Rightarrow$ Radius (r) = 1.4 cm

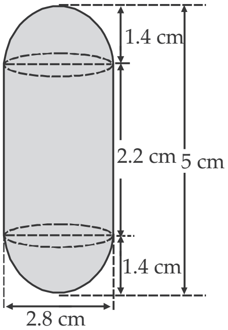
$\therefore$ Length (height) of the cylindrical part (h)

= 5 cm – (1.4 + 1.4) cm = 5 cm – 2.8 cm = 2.2 cm

Now, volume of the cylindrical part = $\pi r^2 h$

and volume of both the hemispherical ends

$= 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$



$\therefore$ Volume of a gulab jamun

$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left[h + \frac{4}{3}r\right]$

$= \frac{22}{7} \times (1.4)^2\left[2.2 + \frac{4}{3}(1.4)\right]$

$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left[\frac{22}{10} + \frac{56}{30}\right]$

$= \frac{22 \times 2 \times 14}{10 \times 10}\left[\frac{66+56}{30}\right] = \frac{44 \times 14}{100} \times \frac{122}{30}\text{cm}^3$

Volume of 45 gulab jamuns

$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30}\right] = \frac{15 \times 44 \times 14 \times 122}{1000}\text{cm}^3$

Since, the quantity of syrup in gulab jamuns

$= 30\%$ of [volume] $= 30\%$ of $\left[\frac{15 \times 44 \times 14 \times 122}{1000}\right]$

$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} = 338.184 \text{ cm}^3$

$= 338 \text{ cm}^2$ (approx.)

4. Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.

$\therefore$ Volume of the cuboid $= 15 \times 10 \times \frac{35}{10} = 525 \text{ cm}^3$

Since each depression is conical in shape with base radius (r) = 0.5 cm and depth (h) = 1.4 cm.

$\therefore$ Volume of each depression

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

Since there are 4 depressions.

$\therefore$ Total volume of 4 depressions $= 4 \times \frac{11}{30} = \frac{44}{30}\text{cm}^3$

Now, volume of the wood in entire stand

= [Volume of the wooden cuboid]

– [Volume of 4 depressions]

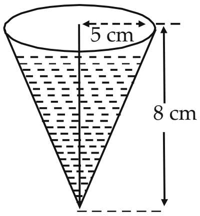
$= 525 - \frac{44}{30} = \frac{15750 - 44}{30} = \frac{15706}{30} = 523.53 \text{ cm}^3$

5. Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm

Volume of water in conical vessel

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 = \frac{4400}{21}\text{cm}^3$



Now, total volume of lead shots

$= \frac{1}{4}$ of [Volume of water in the cone]

$= \frac{1}{4} \times \frac{4400}{21} = \frac{1100}{21}\text{cm}^3$

Since, radius of spherical lead shot (r) = 0.5 cm

$\therefore$ Volume of 1 lead shot $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10}\text{cm}^3$

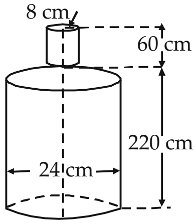
$\therefore$ Number of lead shots $= \frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}}$

$= = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]} = 100$

Thus, the required number of lead shots = 100

6. Height of the big cylinder (h)

= 220 cm



Base radius (r) $= \frac{24}{2}\text{cm} = 12 \text{ cm}$

$\therefore$ Volume of the big cylinder

$= \pi r^2 h = \pi(12)^2 \times 220 \text{ cm}^3$

Also, height of smaller cylinder (h_1)

= 60 cm

Base radius (r_1) = 8 cm

$\therefore$ Volume of the smaller cylinder $= \pi r_1^2 h_1 = \pi(8)^2 \times 60 \text{ cm}^3$

$\therefore$ Volume of iron pole = [Volume of big cylinder]

+ [Volume of the smaller cylinder]

$= (\pi \times 220 \times 12^2 + \pi \times 60 \times 8^2) \text{ cm}^3$

$= 3.14[220 \times 12 \times 12 + 60 \times 8 \times 8] \text{ cm}^3$

$= \frac{314}{100}[220 \times 144 + 60 \times 64]\text{ cm}^3$

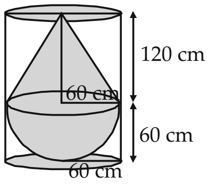
$= \frac{314}{100}[31680 + 3840]\text{ cm}^3 = \frac{314}{100} \times 35520 \text{ cm}^3$

Mass of pole $= \frac{8 \times 314 \times 35520}{100}\text{g} = \frac{89226240}{100}\text{g}$

$= \frac{8922624}{10000}\text{kg} = 892.2624 \text{ kg} = 892.26 \text{ kg}.$

7. Height of the conical part (h)

= 120 cm.



Base radius of the conical part (r)

= 60 cm.

$\therefore$ Volume of the conical part

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120 \text{ cm}^3$

Radius of the hemispherical part (r) = 60 cm

$\therefore$ Volume of the hemispherical part

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (60)^3 \text{cm}^3$

$\therefore$ Volume of the solid = [Volume of conical part]

+ [Volume of hemispherical part]

$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3\right]$

$= \frac{2}{3} \times \frac{22}{7} \times 60^2[60 + 60]$

$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 120 = \frac{6336000}{7}\text{ cm}^3$

Radius of cylinder (r_1) = 60 cm

and, Height of cylinder (h_1) = 180 cm

Volume of the cylinder = $\pi r_1^2 h_1$

$$= \frac{22}{7} \times 60^2 \times 180 = \frac{14256000}{7} \text{cm}^3$$

$\Rightarrow$ Volume of water in the cylinder $= \frac{14256000}{7} \text{cm}^3$

$\therefore$ Volume of the water left in the cylinder

$$= \left[\frac{14256000}{7} - \frac{6336000}{7}\right] = \frac{7920000}{7}$$

$= 1131428.571 \text{ cm}^3 = \frac{1131428.571}{1000000} \text{m}^3$

$= 1.131428571 \text{ m}^3 = 1.131 \text{ m}^3$ (approx).

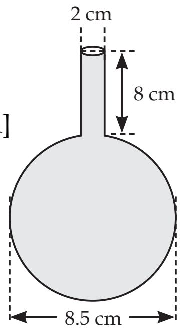
8. Volume of the cylindrical part = $\pi r^2 h$

$= 3.14 \times 1^2 \times 8 = \frac{314}{100} \times 8 \text{ cm}^3$

$\left[\because \text{Radius}(r) = \frac{2}{2} = 1 \text{ cm}, \text{height}(h) = 8 \text{ cm}\right]$

Radius of spherical part $(r_1) = \frac{8.5}{2}$ cm

Volume of the spherical part $= \frac{4}{3}\pi r_1^3$



$$= \frac{4}{3} \times \frac{314}{100} \times \frac{85}{20} \times \frac{85}{20} \times \frac{85}{20} \text{cm}^3$$

Total volume of the glass-vessel

$$= \left[\frac{314}{100} \times 8\right] + \left[\frac{4}{3} \times \frac{314}{100} \times \frac{85 \times 85 \times 85}{8000}\right]$$

$$= \frac{314}{100}\left[8 + \frac{4 \times 85 \times 85 \times 85}{24000}\right]$$

$$= \frac{314}{100}\left[8 + \frac{614125}{6000}\right]$$

$$= \frac{314}{100}\left[\frac{48000 + 614125}{6000}\right]$$

$$= \frac{314}{100}\left[\frac{662125}{6000}\right]$$

$= 346.51 \text{ cm}^3$ (approx.)

$\Rightarrow$ Volume of water in the vessel = 346.51 cm^3

Since, the child finds the volume as 345 cm^3

$\therefore$ The child's answer is not correct

The correct answer is 346.51 cm^3.