Real Numbers

CHAPTER **1**

SOLUTIONS

- 1. (c) : Let HCF be *x*, then LCM = 14xWe have, $14x + x = 600 \implies x = 40$ Let other number be *y*. As we know that, product of two numbers = product of their LCM & HCF
- \Rightarrow $y \times 80 = 40 \times 14 \times 40 \Rightarrow y = 280$

2. (a) : The prime factorization of 280 and 674 are $280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$ and $674 = 2 \times 337$

∴ HCF (280, 674) = 2.

EXAM

3. (c) : If the sum of 3 prime numbers is even, then one of the numbers must be 2.

Let the second number be *x*.

x + (x + 36) + 2 = 100

 $\Rightarrow 2x = 62 \Rightarrow x = 31$

So, the numbers are 2, 31 and 67. Hence, largest number is 67.

4. (b) : Using factor tree method, we have



 \therefore x = 21; y = 84

5. (d) : 12 - 7 = 5, 15 - 10 = 5 and 16 - 11 = 5 Hence, the desired number is 5 less than the LCM of 12, 15, 16.

LCM of 12, 15 and 16 is 240.

Hence, the least number = 240 - 5 = 235

6. (c) : Since LCM
$$(a, b) = \frac{a \times b}{\text{HCF}(a, b)} = \frac{1800}{12} = 150$$

7. Given that, $a = x^5y^3 = x \times x \times x \times x \times x \times y \times y \times y$ and $b = x^3y^4 = x \times x \times x \times y \times y \times y \times y$

 \therefore HCF of *a* and *b* = HCF (x^5y^3 , x^3y^4)

 $= x \times x \times x \times y \times y \times y = x^3 y^3$

8. Clearly, 2 is neither a factor of *p* nor that of *q*

 \therefore *p* and *q* are both odd.

So, (p + q) must be an even number, which is divisible by 2. Hence, the least prime factor of (p + q) is 2.

9. Since, p is prime, then p and p + 1 has no common factor other than 1.

:. HCF of *p* and (p + 1) is 1 and LCM of *p* and (p + 1) is p(p + 1).

- **10.** The factors common to given numbers are 2^2 , 5 and 7^2
- :. HCF = $2^2 \times 5 \times 7^2 = 980$.

11 HCF =
$$\frac{\text{Product of two numbers}}{11}$$

12. (i) Here $80 = 2^4 \times 5$, $85 = 17 \times 5$

and $90 = 2 \times 3^2 \times 5$

L.C.M of 80, 85 and $90 = 2^4 \times 3 \times 3 \times 5 \times 17 = 12240$ Hence, the minimum distance each should walk when they at first time is 12240 cm.

(ii) Here $594 = 2 \times 3^3 \times 11$ and $189 = 3^3 \times 7$ HCF of 594 and $189 = 3^3 = 27$ Hence, the maximum number of columns in which they can march is 27.

(iii) Here 768 = $2^8 \times 3$ and $420 = 2^2 \times 3 \times 5 \times 7$

HCF of 768 and $420 = 2^2 \times 3 = 12$

So, the container which can measure fuel of either tanker exactly must be of 12 litres.

(iv) Here, Length = 825 cm, Breadth = 675 cm and Height = 450 cm

Also, $825 = 5 \times 5 \times 3 \times 11$, $675 = 5 \times 5 \times 3 \times 3 \times 3$ and $450 = 2 \times 3 \times 3 \times 5 \times 5$

 $HCF = 5 \times 5 \times 3 = 75$

Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

(v) LCM of 8 and 12 is 24.

- :. The least number of pack of pens = 24/8 = 3
- :. The least number of pack of note pads = 24/12 = 2

13. (i) (b) : Here $\sqrt{8} = 2\sqrt{2}$ = product of rational and irrational numbers = irrational number

(ii) (c) : Here,
$$\sqrt{9} = 3$$

So, $2 + 2\sqrt{9} = 2 + 6 = 8$, which is not irrational.

(iii) (b) : Here $\sqrt{15}$ and $\sqrt{10}$ are both irrational and difference of two irrational numbers is also irrational.

(iv) (c) : As $\sqrt{5}$ is irrational, so its reciprocal is also irrational.

(v) (d): We know that $\sqrt{6}$ is irrational.

So, $15 + 3\sqrt{6}$ is irrational.

Similarly, $\sqrt{24} - 9 = 2\sqrt{6} - 9$ is irrational.

And
$$5\sqrt{150} = 5 \times 5\sqrt{6} = 25\sqrt{6}$$
 is irrational.

14. (i) (b) : LCM of x and $y = p^3 q^3$ and HCF of x and

2

 $y = p^{2}q$ Also, LCM = $pq^{2} \times$ HCF. (ii) (d) : Number of marbles = 5m + 2 or 6n + 2. Thus, number of marbles, p = (multiple of 5×6) + 2 = 30k + 2 = 2(15k + 1)= which is an even number but not prime (iii) (d) : Here, required numbers = HCF (398 - 7, 436 - 11, 542 - 15) = HCF (391, 425, 527) = 17(iv) (b) : LCM of 126 and $600 = 2 \times 3 \times 21 \times 100 = 12600$ The least positive integer which on adding 1 is exactly

divisible by 126 and 600 = 12600 - 1 = 12599

(v) (a) : Here 85*C* – 340*A* = 109 and 425*A* + 85*B* = 146 On adding them, we get

 $85A + 85B + 85C = 255 \Rightarrow A + B + C = 3$, which is divisible by 3.

15. The greatest possible speed of the bird is the HCF of 45 and 336.

Now, $45 = 3 \times 3 \times 5$

 $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

 \therefore HCF (45, 336) = 3

 \therefore Required speed is 3 km/h.

 \therefore Prime factorisation of 429 = 3 × 11 × 13

(ii) 5 7325 5 1465 293 293 1

 $\therefore \quad \text{Prime factorsiation of } 7325 = 5 \times 5 \times 293$

17. We have, 3 × 5 × 13 × 46 + 23

= $3 \times 5 \times 13 \times 2 \times 23 + 23 = 23 (3 \times 5 \times 13 \times 2 + 1)$ = 23×391 , which is a product of two numbers. So, the given number is composite.

18. LCM of 252, 308 and 198 = 2772.

So, *A*, *B* and *C* will again meet at the starting point in 2772 seconds *i.e.*, 46 minutes 12 seconds.

OR

Let the required numbers be x, 2x and 3x. Then their HCF = x. So, x = 12

- :. The numbers are 12, 24 and 36.
- \therefore Required number = $\sqrt{36} = 6$

19. Divisors of 99 are 1, 3, 9, 11, 33 and 99

Divisors of 101 are 1 and 101

Divisors of 176 are 1, 2, 4, 8, 16, 11, 22, 44, 88 and 176 Divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182 Hence, 176 has the most number of divisors.

20. Prime factorisation of 144, 112, 418 are
$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

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 $112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$ $418 = 2 \times 11 \times 19$ LCM (144, 112, 418) = $2^4 \times 3^2 \times 7 \times 11 \times 19 = 210672$ *.*.. **21.** $99 = 1 \times 3 \times 3 \times 11; 101 = 1 \times 101;$ $176 = 1 \times 2 \times 2 \times 2 \times 2 \times 11;$ $182 = 1 \times 2 \times 7 \times 13$ So, divisors of 99 are 1, 3, 9, 11, 33 and 99; divisors of 101 are 1 and 101; divisors of 176 are 1, 2, 4, 8, 16, 11, 22, 44, 88 and 176; divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182 Hence, 176 has the most number of divisors. **22.** Let $x = 1.3\overline{7} = 1.37777...$ Multiplying both sides by 10, we get 10x = 13.7777......(i) Multiplying (i) by 10, we get 100x = 137.777......(ii) Subtracting (i) from (ii), we get 90x = 124

$$\Rightarrow \quad x = \frac{124}{90} = \frac{62}{45} = \frac{62}{3^2 \times 5} = \frac{p}{q}$$

Hence, q = 45 has prime factors 3 and 5 (or is not of the form $2^m \times 5^n$).

23. LCM of *n* and *p* is 21879

21879 = $3^2 \times 11 \times 13 \times 17$ Since *p* is a prime, LCM of *n*, *p* is *np* and HCF (*n*, *p*) = 1 If *p* = 13, *n* = 9 × 11 × 17 = 1683 \Rightarrow *n* + *p* ≠ 2000 If *p* = 17, *n* = 9 × 11 × 13 = 1287 \Rightarrow *n* + *p* ≠ 2000 If *p* = 11, *n* = 9 × 13 × 17 = 1989 \Rightarrow *n* + *p* = 2000 \therefore *p* = 11, *n* = 1989.

24. Using the factor tree method, we have



 \Rightarrow 404 = 2 × 2 × 101 and 96 = 2 × 2 × 2 × 2 × 2 × 3

 \therefore HCF of 404 and 96 = 2 × 2 = 4

LCM of 404 and 96 = 2 × 2 × 2 × 2 × 2 × 3 × 101 = 9696 Also, 404 × 96 = 38784

 $LCM \times HCF = 9696 \times 4 = 38784$

Thus, HCF × LCM = Product of two numbers.

25. The number of rooms will be minimum if each room accommodates maximum number of participants. Since in each room the same number of participants are to be seated and all of them must be of the same subject. Therefore the number of participants in each room must be the HCF of 60, 84 and 108.

We have, $60 = 2^2 \times 3 \times 5$, $84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$

Real Numbers

∴ HCF of 60, 84 and 108 is 2² × 3 = 12
Therefore, in each room 12 participants can be seated.
∴ Number of rooms required

 $= \frac{\text{Total number of participants}}{12}$

$$=\frac{60+84+108}{12}=\frac{252}{12}=21$$

26. If possible, let there be a positive integer *n* for which

 $\sqrt{n+1} + \sqrt{n-1}$ is rational and equal to $\frac{a}{b}$ (say), where *a*, *b* are positive integers. Then,

$$\begin{aligned} \frac{a}{b} &= \sqrt{n+1} + \sqrt{n-1} & \dots(i) \\ \Rightarrow & \frac{b}{a} &= \frac{1}{\sqrt{n+1} + \sqrt{n-1}} \\ &= \frac{\sqrt{n+1} - \sqrt{n-1}}{\{\sqrt{n+1} + \sqrt{n-1}\}\{\sqrt{n+1} - \sqrt{n-1}\}} \\ &= \frac{\sqrt{n+1} - \sqrt{n-1}}{(n+1) - (n-1)} = \frac{\sqrt{n+1} - \sqrt{n-1}}{2} \end{aligned}$$

$$\Rightarrow \quad \frac{2b}{a} = \sqrt{n+1} - \sqrt{n-1} \qquad \qquad \dots (ii)$$

Adding (i) and (ii) and subtracting (ii) from (i), we get

$$2\sqrt{n+1} = \frac{a}{b} + \frac{2b}{a} \text{ and } 2\sqrt{n-1} = \frac{a}{b} - \frac{2b}{a}$$
$$\Rightarrow \sqrt{n+1} = \frac{a^2 + 2b^2}{2ab} \text{ and } \sqrt{n-1} = \frac{a^2 - 2b^2}{2ab}$$

 $\Rightarrow \sqrt{n+1}$ and $\sqrt{n-1}$ are rationals.

$$\left[\therefore a \text{ and } b \text{ are integers} \therefore \frac{a^2 + 2b^2}{2ab} \text{ and } \frac{a^2 - 2b^2}{2ab} \right]$$
are rationals.

 \Rightarrow (*n* + 1) and (*n* - 1) are perfect squares of positive integers.

This is not possible as any two perfect squares differ at least by 3.

Hence, there is no positive integer *n* for which $(\sqrt{n-1} + \sqrt{n+1})$ is rational.

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