Polynomials

CHAPTER 2

NCERT FOCUS

SOLUTIONS

EXERCISE - 2.1

1. (i) The given graph is parallel to x-axis, it does not intersect the x-axis.

$\therefore$ It has no zero.

(ii) The given graph intersects the x-axis at one point only.

$\therefore$ It has one zero.

(iii) The given graph intersects the x-axis at three points.

$\therefore$ It has three zeroes.

(iv) The given graph intersects the x-axis at two points.

$\therefore$ It has two zeroes.

(v) The given graph intersects the x-axis at four points.

$\therefore$ It has four zeroes.

(vi) The given graph meets the x-axis at three points.

$\therefore$ It has three zeroes.

EXERCISE - 2.2

1. (i) We have, $p(x) = x^2 - 2x - 8$

$= x^2 + 2x - 4x - 8 = x(x + 2) - 4(x + 2) = (x - 4)(x + 2)$

For $p(x) = 0$, we must have $(x - 4)(x + 2) = 0$

Either $x - 4 = 0 \Rightarrow x = 4$ or $x + 2 = 0 \Rightarrow x = -2$

$\therefore$ The zeroes of $x^2 - 2x - 8$ are 4 and – 2

Now, sum of the zeroes $= 4 + (-2) = 2 = \dfrac{-(-2)}{1}$

$= \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 4 \times (-2) = \dfrac{-8}{1} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $x^2 - 2x - 8$ is verified.

(ii) We have, $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

For $p(s) = 0$, we have, $(2s - 1) = 0 \Rightarrow s = \dfrac{1}{2}$

$\therefore$ The zeroes of $4s^2 - 4s + 1$ are $\dfrac{1}{2}$ and $\dfrac{1}{2}$

Sum of the zeroes $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = \dfrac{-(-4)}{4} = \dfrac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

and product of zeroes $= \left(\dfrac{1}{2}\right) \times \left(\dfrac{1}{2}\right) = \dfrac{1}{4} = \dfrac{\text{Constant term}}{\text{Coefficient of } s^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $4s^2 - 4s + 1$ is verified.

(iii) We have, $p(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$

$= (3x + 1)(2x - 3)$

For $p(x) = 0$, we have,

Either $(3x + 1) = 0 \Rightarrow x = -\dfrac{1}{3}$

or $(2x - 3) = 0 \Rightarrow x = \dfrac{3}{2}$

Thus, the zeroes of $6x^2 - 3 - 7x$ are $-\dfrac{1}{3}$ and $\dfrac{3}{2}$.

Now, sum of the zeroes $= -\dfrac{1}{3} + \dfrac{3}{2} = \dfrac{7}{6} = \dfrac{-(-7)}{6}$

$= \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes $= \left(-\dfrac{1}{3}\right) \times \dfrac{3}{2} = \dfrac{-3}{6}$

$= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $6x^2 - 3 - 7x$ is verified.

(iv) We have, $p(u) = 4u^2 + 8u = 4u(u + 2)$

For $p(u) = 0$, we have

Either $4u = 0 \Rightarrow u = 0$

or $u + 2 = 0 \Rightarrow u = -2$

$\therefore$ The zeroes of $4u^2 + 8u$ are 0 and – 2.

Now, $4u^2 + 8u$ can be written as $4u^2 + 8u + 0$.

Sum of the zeroes $= 0 + (-2) = -2 = \dfrac{-(8)}{4}$

$= \dfrac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

and the product of zeroes $= 0 \times (-2) = 0 = \dfrac{0}{4}$

$= \dfrac{\text{Constant term}}{\text{Coefficient of } u^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $4u^2 + 8u$ is verified.

(v) We have, $p(t) = t^2 - 15 = (t^2) - (\sqrt{15})^2$

$= (t + \sqrt{15})(t - \sqrt{15})$ $[\because a^2 - b^2 = (a + b)(a - b)]$

For $p(t) = 0$, we have

Either $(t + \sqrt{15}) = 0 \Rightarrow t = -\sqrt{15}$

or $t - \sqrt{15} = 0 \Rightarrow t = \sqrt{15}$

$\therefore$ The zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$

Now, we can write $t^2 - 15$ as $t^2 + 0t - 15$.

$\therefore$ Sum of the zeroes $= -\sqrt{15} + \sqrt{15} = 0 = \dfrac{-(0)}{1}$

$= \dfrac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes = $(-\sqrt{15})\times(\sqrt{15})=\frac{-(15)}{1}$

$=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$

Thus, the relationship between zeroes and the coefficients in the polynomial $t^2 - 15$ is verified.

(vi) We have, $p(x) = 3x^2 - x - 4$

$= 3x^2 + 3x - 4x - 4 = 3x(x + 1) - 4(x + 1) = (x + 1)(3x - 4)$

For $p(x) = 0$, we have

Either $(x + 1) = 0 \Rightarrow x = -1$

or $3x - 4 = 0 \Rightarrow x = 4/3$

$\therefore$ The zeroes of $3x^2 - x - 4$ are -1 and $4/3$

Now, sum of the zeroes = $(-1)+\frac{4}{3}=\frac{1}{3}=\frac{-(-1)}{3}$

$=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes = $(-1)\times\frac{4}{3}=\frac{(-4)}{3}$

$=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $3x^2 - x - 4$ is verified.

2. (i) Since, sum of the zeroes, $(\alpha+\beta)=\frac{1}{4}$

Product of the zeroes, $\alpha\beta = -1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-\left(\frac{1}{4}\right)x+(-1)=x^2-\frac{1}{4}x-1=\frac{1}{4}(4x^2-x-4)$

Since, $\frac{1}{4}(4x^2-x-4)$ and $(4x^2 - x - 4)$ have same zeroes, therefore $(4x^2 - x - 4)$ is the required quadratic polynomial.

(ii) Since, sum of the zeroes, $(\alpha+\beta)=\sqrt{2}$

Product of zeroes, $\alpha\beta=\frac{1}{3}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-(\sqrt{2})x+\left(\frac{1}{3}\right)=\frac{1}{3}(3x^2-3\sqrt{2}x+1)$

Since, $\frac{1}{3}(3x^2-3\sqrt{2}x+1)$ and $(3x^2-3\sqrt{2}x+1)$ have same zeroes, therefore

$(3x^2-3\sqrt{2}x+1)$ is the required quadratic polynomial.

(iii) Since, sum of zeroes, $(\alpha + \beta) = 0$

Product of zeroes, $\alpha\beta=\sqrt{5}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-(0)x+\sqrt{5}=x^2+\sqrt{5}$

(iv) Since, sum of zeroes, $(\alpha + \beta) = 1$

Product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (1)x + 1 = x^2 - x + 1$

(v) Since, sum of the zeroes, $(\alpha+\beta)=-\frac{1}{4}$

Product of zeroes, $\alpha\beta = 1/4$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-\left(-\frac{1}{4}\right)x+\frac{1}{4}=x^2+\frac{x}{4}+\frac{1}{4}=\frac{1}{4}(4x^2+x+1)$

Since, $\frac{1}{4}(4x^2+x+1)$ and $(4x^2+x+1)$ have same zeroes, therefore, the required quadratic polynomial is $(4x^2+x+1)$.

(vi) Since, sum of zeroes, $(\alpha + \beta) = 4$ and product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 4x + 1$.