CHAPTER 4

Quadratic Equations

NCERT FOCUS

SOLUTIONS

EXERCISE - 4.1

1. (i) We have, $(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0 \Rightarrow x^2 + 7 = 0$

Since, $x^2 + 7$ is a quadratic polynomial.

$\therefore (x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) We have, $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 2x - 2x + 6 = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Since, $x^2 - 4x + 6$ is a quadratic polynomial.

$\therefore x^2 - 2x = (-2)(3 - x)$ is a quadratic equation.

(iii) We have, $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0 \Rightarrow -3x + 1 = 0$

Since, $-3x + 1$ is a linear polynomial.

$\therefore (x-2)(x+1) = (x-1)(x+3)$ is not a quadratic equation.

(iv) We have, $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0 \Rightarrow x^2 - 10x - 3 = 0$

Since, $x^2 - 10x - 3$ is a quadratic polynomial.

$\therefore (x - 3)(2x + 1) = x(x + 5)$ is a quadratic equation.

(v) We have, $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0 \Rightarrow x^2 - 11x + 8 = 0$

Since, $x^2 - 11x + 8$ is a quadratic polynomial.

$\therefore (2x - 1)(x - 3) = (x + 5)(x - 1)$ is a quadratic equation.

(vi) We have, $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0 \Rightarrow 7x - 3 = 0$

Since, $7x - 3$ is a linear polynomial.

$\therefore x^2 + 3x + 1 = (x - 2)^2$ is not a quadratic equation.

(vii) We have, $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 = 2x^3 - 2x$

$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow x^3 + 6x^2 + 12x + 8 - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

Since, $-x^3 + 6x^2 + 14x + 8$ is a cubic polynomial.

$\therefore (x + 2)^3 = 2x(x^2 - 1)$ is not a quadratic equation.

(viii) We have, $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since, $2x^2 - 13x + 9$ is a quadratic polynomial.

$\therefore x^3 - 4x^2 - x + 1 = (x - 2)^3$ is a quadratic equation.

2. (i) Let the breadth = x metres

$\because$ Length = 2(Breadth) + 1

$\therefore$ Length = $(2x + 1)$metres

Since, length × breadth = Area

$\therefore (2x + 1) \times x = 528 \Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Thus, the required quadratic equation is $2x^2 + x - 528 = 0$.

(ii) Let the two consecutive positive integers be x and $(x + 1)$.

$\because$ Product of two consecutive positive integers = 306

$\therefore x(x + 1) = 306 \Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$

Thus, the required quadratic equation is $x^2 + x - 306 = 0$.

(iii) Let the present age of Rohan be x years.

$\therefore$ His mother's age = $(x + 26)$ years

After 3 years, Rohan's age = $(x + 3)$ years

After 3 years, his mother's age = $[(x + 26) + 3]$ years = $(x + 29)$ years

According to the condition, $(x + 3) \times (x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0 \Rightarrow x^2 + 32x - 273 = 0$

Thus, the required quadratic equation is $x^2 + 32x - 273 = 0$.

(iv) Let the speed of the train = u km/hr

Distance covered = 480 km

Time taken = $\dfrac{\text{Distance}}{\text{Speed}} = \dfrac{480}{u}$ hours

In other case, speed = $(u - 8)$ km/hour

$\therefore$ Time taken = $\dfrac{\text{Distance}}{\text{Speed}} = \dfrac{480}{(u-8)}$ hours

According to the condition, $\dfrac{480}{u-8} - \dfrac{480}{u} = 3$

$\Rightarrow 480u - 480(u - 8) = 3u(u - 8)$

$\Rightarrow 480u - 480u + 3840 = 3u^2 - 24u$

$\Rightarrow 3u^2 - 24u - 3840 = 0 \Rightarrow u^2 - 8u - 1280 = 0$

Thus, the required quadratic equation is $u^2 - 8u - 1280 = 0$.

EXERCISE - 4.2

1. (i) We have, $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0 \Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x - 5 = 0$ or $x + 2 = 0 \Rightarrow x = 5$ or $x = -2$

Thus, the required roots are 5 and –2.

(ii) We have, $2x^2 + x - 6 = 0 \Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0 \Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0 \Rightarrow x = -2$ or $x = 3/2$

Thus, the required roots are –2 and 3/2.

(iii) We have, $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow \quad (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow \quad x + \sqrt{2} = 0$ or $\sqrt{2}x + 5 = 0$

$\Rightarrow \quad x = -\sqrt{2}$ or $x = \dfrac{-5}{\sqrt{2}} = \dfrac{-5\sqrt{2}}{2}$

Thus, the required roots are $-\sqrt{2}$ and $\dfrac{-5\sqrt{2}}{2}$.

(iv) We have, $2x^2 - x + \dfrac{1}{8} = 0$

$\Rightarrow \quad 16x^2 - 8x + 1 = 0 \Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0 \Rightarrow (4x - 1)(4x - 1) = 0$

$\Rightarrow \quad 4x - 1 = 0 \Rightarrow x = 1/4$

Thus, the required roots are 1/4 and 1/4.

(v) We have, $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0 \Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow \quad (10x - 1) = 0 \Rightarrow x = 1/10$

Thus, the required roots are $\dfrac{1}{10}$ and $\dfrac{1}{10}$.

2. (i) Let John had x marbles and Jivanti had $(45 - x)$ marbles.

According to question,

$(x - 5) \times (45 - x - 5) = 124$

$\Rightarrow \quad (x - 5) \times (40 - x) = 124 \Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0 \Rightarrow x(x - 9) - 36(x - 9) = 0$

$\Rightarrow \quad (x - 9)(x - 36) = 0$

$\Rightarrow \quad x - 9 = 0$ or $x - 36 = 0 \Rightarrow x = 9$ or $x = 36$

$\therefore$ If John had 9 marbles, then Jivanti had 45 – 9 = 36 marbles.

If John had 36 marbles, then Jivanti had 45 – 36 = 9 marbles.

(ii) Let the number of toys produced in a day be x.

Then, cost of 1 toy $= \dfrac{750}{x}$

According to question, $\dfrac{750}{x} = 55 - x$

$\Rightarrow \quad 750 = 55x - x^2 \Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow \quad x^2 - 30x - 25x + 750 = 0$

$\Rightarrow \quad x(x - 30) - 25(x - 30) = 0 \Rightarrow (x - 30)(x - 25) = 0$

$\Rightarrow \quad x - 30 = 0$ or $x - 25 = 0 \Rightarrow x = 30$ or $x = 25$

Hence, number of toys produced on that day is either 30 or 25.

3. Let one of the numbers be x.

$\therefore$ Other number $= 27 - x$

According to the condition,

$x(27 - x) = 182 \Rightarrow 27x - x^2 = 182$

$\Rightarrow \quad x^2 - 27x + 182 = 0 \Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow \quad x(x - 13) - 14(x - 13) = 0 \Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow \quad x - 13 = 0$ or $x - 14 = 0 \Rightarrow x = 13$ or $x = 14$

Thus, the required numbers are 13 and 14.

4. Let the two consecutive positive integers be x and $(x + 1)$.

Since, the sum of the squares of the numbers is 365.

$\therefore \quad x^2 + (x + 1)^2 = 365 \Rightarrow x^2 + x^2 + 2x + 1 = 365$

$\Rightarrow \quad 2x^2 + 2x - 364 = 0 \Rightarrow x^2 + x - 182 = 0$

$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0$

$\Rightarrow \quad x(x + 14) - 13(x + 14) = 0 \Rightarrow (x + 14)(x - 13) = 0$

$\Rightarrow \quad x + 14 = 0$ or $x - 13 = 0 \Rightarrow x = -14$ or $x = 13$

Since, x has to be a positive integer, so $x = -14$ is rejected.

$\therefore \quad x = 13 \Rightarrow x + 1 = 13 + 1 = 14$

Thus, the required consecutive positive integers are 13 and 14.

5. Let the base of the given right triangle be x cm.

$\therefore$ Its altitude $= (x - 7)$ cm

$\because$ Hypotenuse $= \sqrt{(\text{Base})^2 + (\text{Altitude})^2}$

[By Pythagoras theorem]

$\therefore \quad 13 = \sqrt{x^2 + (x - 7)^2}$

On squaring both sides, we get, $169 = x^2 + (x - 7)^2$

$\Rightarrow \quad 169 = x^2 + x^2 - 14x + 49 \Rightarrow 2x^2 - 14x + 49 - 169 = 0$

$\Rightarrow \quad 2x^2 - 14x - 120 = 0 \Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0 \Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow \quad (x - 12)(x + 5) = 0$

$\Rightarrow \quad x - 12 = 0$ or $x + 5 = 0 \Rightarrow x = 12$ or $x = -5$

But the sides of a triangle can never be negative, so, $x = -5$ is rejected.

$\therefore \quad x = 12$

$\therefore$ Length of base = 12 cm

$\Rightarrow$ Length of altitude = (12 – 7) cm = 5 cm

Thus, the required base is 12 cm and altitude is 5 cm.

6. Let the number of articles produced in a day $= x$

$\therefore$ Cost of production of each article = ₹ $(2x + 3)$

Total cost = ₹ 90

$\therefore \quad x \times (2x + 3) = 90 \Rightarrow 2x^2 + 3x = 90$

$\Rightarrow \quad 2x^2 + 3x - 90 = 0 \Rightarrow 2x^2 - 12x + 15x - 90 = 0$

$\Rightarrow \quad 2x(x - 6) + 15(x - 6) = 0 \Rightarrow (x - 6)(2x + 15) = 0$

$\Rightarrow \quad x - 6 = 0$ or $2x + 15 = 0 \Rightarrow x = 6$ or $x = \dfrac{-15}{2}$

But the number of articles produced can never be negative,

so, $x = \dfrac{-15}{2}$ is rejected.

$\therefore \quad x = 6$

$\therefore$ Cost of production of each article = ₹ (2 × 6 + 3) = ₹ 15

Thus, the required number of articles produced is 6 and the cost of each article is ₹ 15.

EXERCISE - 4.3

1. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -3$ and $c = 5$.

$\therefore \quad D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

$\therefore$ The given quadratic equation has no real roots.

(ii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 3$, $b = -4\sqrt{3}$ and $c = 4$.

$\therefore \quad D = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4) = (16 \times 3) - 48$
$= 48 - 48 = 0$

$\therefore$ The given quadratic equation has two real roots which are equal. Hence, the roots are

$\frac{-b}{2a}$ and $\frac{-b}{2a}$ *i.e.*, $\frac{-(-4\sqrt{3})}{2\times 3}$ and $\frac{-(-4\sqrt{3})}{2\times 3}$ *i.e.*, $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = -6$ and $c = 3$.

$\therefore \quad D = b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$

Since, $b^2 - 4ac$ is positive.

$\therefore$ The given quadratic equation has two real and distinct roots, which are given by $x = \frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow \quad x = \frac{-(-6) \pm \sqrt{12}}{2\times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Thus, the roots are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

2. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$.

$\therefore \quad D = b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

$\because$ For a quadratic equation to have equal roots, $D = 0$

$\Rightarrow \quad b^2 - 4ac = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k = \pm\sqrt{24} \Rightarrow k = \pm 2\sqrt{6}$

Thus, the required values of k are $2\sqrt{6}$ and $-2\sqrt{6}$.

(ii) $kx(x - 2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$

Comparing $kx^2 - 2kx + 6 = 0$ with $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$.

$\therefore \quad D = b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

Since, the roots are equal.

$\therefore \quad D = b^2 - 4ac = 0 \Rightarrow 4k^2 - 24k = 0$

$\Rightarrow \quad 4k(k - 6) = 0 \Rightarrow 4k = 0$ or $k - 6 = 0 \Rightarrow k = 0$ or $k = 6$

But k cannot be 0, otherwise, the given equation is not quadratic. Thus, the required value of k is 6.

3. Let the breadth be x m. $\therefore$ Length $= 2x$ m

Now, Area = Length × Breadth $= 2x \times x = 2x^2$ m^2

According to the given condition, $2x^2 = 800$

$\Rightarrow \quad x^2 = \frac{800}{2} = 400 \Rightarrow x^2 - 400 = 0$

Here, $a = 1$, $b = 0$ and $c = -400$

$\therefore \quad D = b^2 - 4ac = 0 - 4(1)(-400) = 1600 > 0$

So, the roots are real and distinct.

$\therefore \quad x = \frac{0 \pm \sqrt{1600}}{2(1)} = \pm\frac{40}{2} = \pm 20$

Therefore, $x = 20$ or $x = -20$

But $x = -20$ is not possible. [$\because$ Breadth cannot be negative]

$\therefore \quad x = 20 \Rightarrow 2x = 2 \times 20 = 40$

Thus, it is possible to design a rectangular mango grove with length = 40 m and breadth = 20 m.

4. Let the age of one friend = x years

$\therefore$ Age of other friend = $(20 - x)$ years

Four years ago,

Age of one friend = $(x - 4)$ years

Age of other friend = $(20 - x - 4)$ years = $(16 - x)$ years

According to the condition, $(x - 4) \times (16 - x) = 48$

$\Rightarrow \quad 16x - 64 - x^2 + 4x = 48 \Rightarrow -x^2 + 20x - 64 - 48 = 0$

$\Rightarrow \quad -x^2 + 20x - 112 = 0 \Rightarrow x^2 - 20x + 112 = 0 \quad \ldots(1)$

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$.

$\therefore \quad D = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$

$\therefore$ The quadratic equation (1) has no real roots.

Thus, the given situation is not possible.

5. Let the breadth of the rectangle = x m.

Since, the perimeter of the rectangle = 80 m

$\therefore \quad$ 2(Length + Breadth) = 80 $\Rightarrow$ 2(Length + x) = 80

$\Rightarrow \quad$ Length + x = 80/2 = 40 $\Rightarrow$ Length = $(40 - x)$ m

$\therefore$ Area of the rectangle $= (40 - x) \times x = (40x - x^2)$ m^2

According to the given condition,

Area of the rectangle = 400 m^2

$\Rightarrow \quad 40x - x^2 = 400 \Rightarrow x^2 - 40x + 400 = 0 \quad \ldots(1)$

Comparing equation (1) with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$.

$\therefore \quad D = b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

$\therefore$ Equation (1) has two equal and real roots. Hence, the roots are $\frac{-b}{2a}$ and $\frac{-b}{2a}$ *i.e.*, $\frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

$\therefore$ Breadth = x m = 20 m, Length = $40 - x = 40 - 20 = 20$ m

Thus, it is possible to design a rectangular park of given perimeter and area.

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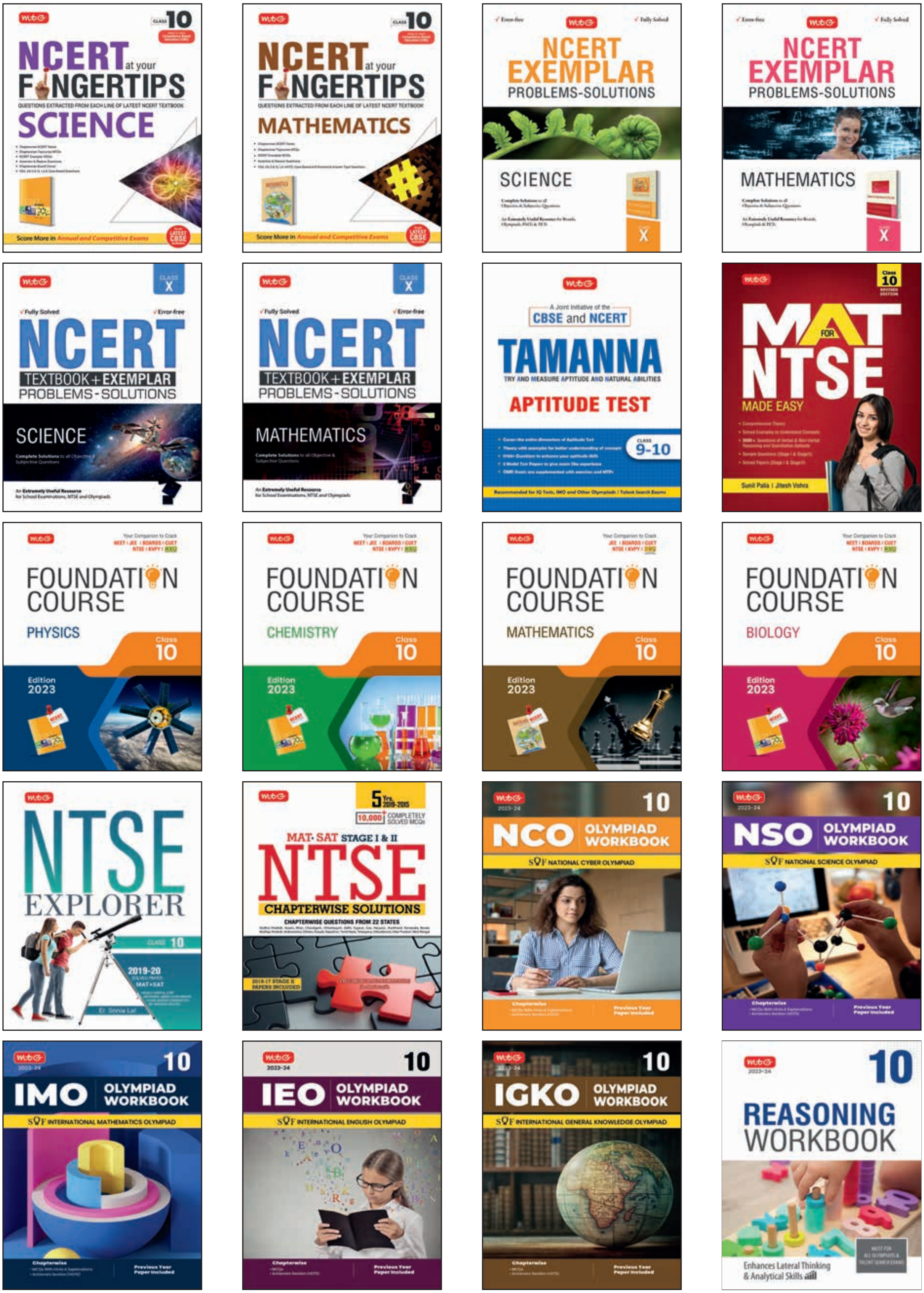

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