

Arithmetic Progressions

CHAPTER 5



SOLUTIONS

EXERCISE - 5.1

1. (i) Let us consider, first term, a_1 = Fare for the first 1 km = ₹ 15 since, the taxi fare after the first 1 km is ₹ 8 for each additional km.

∴ Fare for 2 km = ₹ 15 + ₹ 8 = ₹ 23

Fare for 3 km = ₹ 23 + ₹ 8 = ₹ 31 = ₹ 15 + 2 × ₹ 8

Fare for 4 km = ₹ 31 + ₹ 8 = ₹ 39 = ₹ 15 + 3 × ₹ 8

Fare for 5 km = ₹ 39 + ₹ 8 = ₹ 47 = ₹ 15 + 4 × ₹ 8

We see that fare for each km forms an A.P., with common difference 8.

(ii) Let the amount of air in the cylinder = x

∴ Air removed in 1st stroke = $x/4$

⇒ Air left after 1st stroke = $x - \frac{x}{4} = \frac{3x}{4}$

Air left after 2nd stroke

$$= \frac{3x}{4} - \frac{1}{4} \left(\frac{3x}{4} \right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$$

Air left after 3rd stroke

$$= \frac{9x}{16} - \frac{1}{4} \left(\frac{9x}{16} \right) = \frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64}$$

Air left after 4th stroke

$$= \frac{27x}{64} - \frac{1}{4} \left(\frac{27x}{64} \right) = \frac{27x}{64} - \frac{27x}{256} = \frac{81x}{256}$$

Thus, the terms are $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \frac{81x}{256}, x$

Here, $\frac{3x}{4} - x = \frac{-x}{4}, \frac{9x}{16} - \frac{3x}{4} = \frac{-3x}{16}$

Since, $\left(\frac{-x}{4} \right) \neq \left(\frac{-3x}{16} \right)$.

The above terms are not in A.P.

(iii) Here, the cost of digging for first 1 metre = ₹ 150

The cost of digging for first 2 metres

$$= ₹ 150 + ₹ 50 = ₹ 200$$

The cost of digging for first 3 metres

$$= ₹ 200 + ₹ 50 = ₹ 250 = ₹ 150 + 2 \times (₹ 50)$$

The cost of digging for first 4 metres

$$= ₹ 250 + ₹ 50 = ₹ 300 = ₹ 150 + 3 \times (₹ 50)$$

We see that the cost of digging a well for each subsequent metre form an A.P., with common difference = 50.

(iv) ∴ The amount at the end of 1st year

$$= 10000 \left(1 + \frac{8}{100} \right)^1$$

The amount at the end of 2nd year = $10000 \left(1 + \frac{8}{100} \right)^2$

The amount at the end of 3rd year = $10000 \left(1 + \frac{8}{100} \right)^3$

The amount at the end of 4th year = $10000 \left(1 + \frac{8}{100} \right)^4$

∴ The terms are $[10000], \left[10000 \left(1 + \frac{8}{100} \right) \right],$

$$\left[10000 \left(1 + \frac{8}{100} \right)^2 \right], \left[10000 \left(1 + \frac{8}{100} \right)^3 \right], \dots$$

Obviously, $\left[10000 \left(1 + \frac{8}{100} \right) \right] - [10000]$

$$\neq \left[10000 \left(1 + \frac{8}{100} \right)^2 \right] - \left[10000 \left(1 + \frac{8}{100} \right) \right]$$

∴ The above terms are not in A.P.

2. (i) Here, $a = 10$ and $d = 10$

We have, first term, $a = a_1 = 10$

Second term, $a_2 = 10 + 10 = 20$

Third term, $a_3 = 20 + 10 = 30$ and

Fourth term, $a_4 = 30 + 10 = 40$

Thus, the first four terms are 10, 20, 30 and 40.

(ii) Here, $a = -2$ and $d = 0$, we have

Since, $d = 0$, so each term of given A.P. will be same as the first term of the A.P.

Thus, the first four terms of the A.P. are -2, -2, -2 and -2.

(iii) Here, $a = 4$ and $d = -3$,

We have, first term, $a = a_1 = 4$

Second term, $a_2 = 4 + (-3) = 1$

Third term, $a_3 = 1 + (-3) = -2$ and

Fourth term, $a_4 = -2 + (-3) = -5$

Thus, the first four terms are 4, 1, -2 and -5.

(iv) Here, $a = -1$ and $d = 1/2$

We have, first term, $a = a_1 = -1$,

Second term, $a_2 = -1 + \frac{1}{2} = -\frac{1}{2}$,

Third term, $a_3 = -\frac{1}{2} + \frac{1}{2} = 0$ and

Fourth term, $a_4 = 0 + \frac{1}{2} = \frac{1}{2}$

∴ Thus, the first four terms are $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$.

(v) Here, $a = -1.25$ and $d = -0.25$

We have, first term, $a = a_1 = -1.25$

Second term, $a_2 = -1.25 + (-0.25) = -1.50$,

Third term, $a_3 = -1.50 + (-0.25) = -1.75$ and

Fourth term, $a_4 = -1.75 + (-0.25) = -2.0$

Thus, the first four terms are -1.25, -1.50, -1.75 and -2.0.

3. (i) We have ; 3, 1, -1, -3, ...

$\therefore a_1 = 3 \therefore$ First term = 3

Also, $a_2 = 1, a_3 = -1, a_4 = -3$

$\therefore a_2 - a_1 = 1 - 3 = -2$

$a_4 - a_3 = -3 - (-1) = -3 + 1 = -2$

$\Rightarrow$ Common difference, $d = -2$

(ii) We have ; -5, -1, 3, 7, ...

$\therefore a_1 = -5 \therefore$ First term = -5

Also, $a_2 = -1, a_3 = 3, a_4 = 7$

$\therefore a_2 - a_1 = -1 - (-5) = -1 + 5 = 4$

and $a_4 - a_3 = 7 - 3 = 4 \Rightarrow$ Common difference, $d = 4$

(iii) We have ; $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

$\therefore a_1 = \frac{1}{3} \therefore$ First term = $\frac{1}{3}$

Also, $a_2 = \frac{5}{3}, a_3 = \frac{9}{3}, a_4 = \frac{13}{3}$

$\therefore a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$ and $a_4 - a_3 = \frac{13}{3} - \frac{9}{3} = \frac{4}{3}$

$\Rightarrow$ Common difference, $d = 4/3$

(iv) We have ; 0.6, 1.7, 2.8, 3.9,

$\therefore a_1 = 0.6$

$\therefore$ First term = 0.6

Also, $a_2 = 1.7, a_3 = 2.8, a_4 = 3.9$

$\therefore a_2 - a_1 = 1.7 - 0.6 = 1.1$

and $a_4 - a_3 = 3.9 - 2.8 = 1.1 \Rightarrow$ Common difference, $d = 1.1$

4. (i) We have ; 2, 4, 8, 16,

Here, $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$\therefore a_2 - a_1 = 4 - 2 = 2$ and $a_4 - a_3 = 16 - 8 = 8$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(ii) We have ; $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

Here, $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}$

$\therefore a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}, a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$ and

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \frac{1}{2}$

$\Rightarrow$ Common difference, $d = 1/2$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = \frac{7}{2} + \frac{1}{2} = 4, a_6 = 4 + \frac{1}{2} = \frac{9}{2}$ and $a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) We have ; -1.2, -3.2, -5.2, -7.2,

Here, $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$\therefore a_2 - a_1 = -3.2 + 1.2 = -2,$

$a_3 - a_2 = -5.2 + 3.2 = -2$ and $a_4 - a_3 = -7.2 + 5.2 = -2$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -2$

$\Rightarrow$ Common difference, $d = -2$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = -7.2 + (-2) = -9.2,$

$a_6 = -9.2 + (-2) = -11.2$ and $a_7 = -11.2 + (-2) = -13.2$

(iv) We have ; -10, -6, -2, 2,

Here, $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$

$\therefore a_2 - a_1 = -6 + 10 = 4,$

$a_3 - a_2 = -2 + 6 = 4$ and $a_4 - a_3 = 2 + 2 = 4$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 4$

$\Rightarrow$ Common difference, $d = 4$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = 2 + 4 = 6,$

$a_6 = 6 + 4 = 10$

and $a_7 = 10 + 4 = 14$

(v) We have ; $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

Here, $a_1 = 3, a_2 = 3 + \sqrt{2}, a_3 = 3 + 2\sqrt{2}, a_4 = 3 + 3\sqrt{2}$

$\therefore a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2},$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$ and

$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \sqrt{2}$

$\Rightarrow$ Common difference, $d = \sqrt{2}$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2},$

$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$ and

$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) We have ; 0.2, 0.22, 0.222, 0.2222,

Here, $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$\therefore a_2 - a_1 = 0.22 - 0.2 = 0.02$ and

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(vii) We have ; 0, -4, -8, -12,

Here, $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$\therefore a_2 - a_1 = -4 - 0 = -4,$

$a_3 - a_2 = -8 + 4 = -4$

and $a_4 - a_3 = -12 + 8 = -4$

$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -4$

$\Rightarrow$ Common difference, $d = -4$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = a_4 + (-4) = -12 + (-4) = -16$

$a_6 = a_5 + (-4) = -16 + (-4) = -20$

and $a_7 = a_6 + (-4) = -20 + (-4) = -24$

(viii) We have ; $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

Here, $a_1 = a_2 = a_3 = a_4 = -\frac{1}{2}$

$\therefore a_2 - a_1 = 0, a_3 - a_2 = 0, a_4 - a_3 = 0$

$\Rightarrow$ Common difference, $d = 0$

$\therefore$ The given numbers form an A.P.

Now, $a_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$ and $a_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

(ix) We have ; 1, 3, 9, 27, ...

Here, $\left. \begin{matrix} a_1 = 1 \\ a_2 = 3 \end{matrix} \right\} \Rightarrow a_2 - a_1 = 3 - 1 = 2$

$$\text{Also, } \left. \begin{matrix} a_3 = 9 \\ a_4 = 27 \end{matrix} \right\} \Rightarrow a_4 - a_3 = 27 - 9 = 18$$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(x) We have ; $a, 2a, 3a, 4a, \dots$

Here, $a_1 = a, a_2 = 2a, a_3 = 3a, a_4 = 4a$

$$\therefore a_2 - a_1 = 2a - a = a, a_3 - a_2 = 3a - 2a = a$$

$$\text{and } a_4 - a_3 = 4a - 3a = a$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a$$

$\Rightarrow$ Common difference, $d = a$

$\therefore$ The given numbers form an A.P.

$$\text{Now, } a_5 = 4a + a = 5a,$$

$$a_6 = 5a + a = 6a \text{ and } a_7 = 6a + a = 7a$$

(xi) We have ; $a, a^2, a^3, a^4, \dots$

$$\text{Here, } \left. \begin{matrix} a_1 = a \\ a_2 = a^2 \end{matrix} \right\} \Rightarrow a_2 - a_1 = a^2 - a = a(a - 1)$$

$$\text{Also, } \left. \begin{matrix} a_3 = a^3 \\ a_4 = a^4 \end{matrix} \right\} \Rightarrow a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(xii) We have ; $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

$$a_1 = \sqrt{2}, a_2 = \sqrt{8}, a_3 = \sqrt{18}, a_4 = \sqrt{32}$$

$$\therefore a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2},$$

$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2},$$

$$\text{and } a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \sqrt{2}$$

$\Rightarrow$ Common difference, $d = \sqrt{2}$

$\therefore$ The given numbers form an A.P.

$$\text{Now, } a_5 = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50},$$

$$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72} \text{ and}$$

$$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$$

(xiii) We have ; $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

$$\text{Here, } \left. \begin{matrix} a_1 = \sqrt{3} \\ a_2 = \sqrt{6} \end{matrix} \right\} \Rightarrow a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$$

$$\text{Also, } \left. \begin{matrix} a_3 = \sqrt{9} \\ a_4 = \sqrt{12} \end{matrix} \right\} \Rightarrow a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3 \\ = \sqrt{3}(2 - \sqrt{3})$$

$$\therefore a_2 - a_1 \neq a_4 - a_3$$

$\therefore$ The given numbers do not form an A.P.

(xiv) We have ; $1^2, 3^2, 5^2, 7^2, \dots$

$$\text{Here, } \left. \begin{matrix} a_1 = 1^2 = 1 \\ a_2 = 3^2 = 9 \end{matrix} \right\} \Rightarrow a_2 - a_1 = 9 - 1 = 8$$

$$\text{Also, } \left. \begin{matrix} a_3 = 5^2 = 25 \\ a_4 = 7^2 = 49 \end{matrix} \right\} \Rightarrow a_4 - a_3 = 49 - 25 = 24$$

Since, $a_2 - a_1 \neq a_4 - a_3$

$\therefore$ The given numbers do not form an A.P.

(xv) We have ; $1^2, 5^2, 7^2, 73, \dots$

Here, $a_1 = 1^2, a_2 = 5^2, a_3 = 7^2, a_4 = 73$

$$\therefore a_2 - a_1 = 25 - 1 = 24, a_3 - a_2 = 49 - 25 = 24 \text{ and } a_4 - a_3 \\ = 73 - 7^2 = 73 - 49 = 24$$

$$\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 24$$

$\Rightarrow$ Common difference, $d = 24$

$\therefore$ The given numbers form an A.P.

$$\text{Now, } a_5 = 73 + 24 = 97,$$

$$a_6 = 97 + 24 = 121 \text{ and } a_7 = 121 + 24 = 145$$

EXERCISE - 5.2

1. (i) $a_n = a + (n - 1)d$

$$\Rightarrow a_8 = 7 + (8 - 1)3 = 7 + 7 \times 3 = 7 + 21$$

$$\therefore a_8 = 28$$

(ii) $a_n = a + (n - 1)d$

$$\Rightarrow a_{10} = -18 + (10 - 1)d \Rightarrow 0 = -18 + 9d$$

$$\Rightarrow 9d = 18 \Rightarrow d = 18/9 = 2$$

$$\therefore d = 2$$

(iii) $a_n = a + (n - 1)d$

$$\Rightarrow a_{18} = a + (18 - 1) \times (-3) \Rightarrow -5 = a + 17 \times (-3)$$

$$\Rightarrow -5 = a - 51 \Rightarrow a = -5 + 51 = 46$$

$$\therefore a = 46$$

(iv) $a_n = a + (n - 1)d$

$$\Rightarrow 3.6 = -18.9 + (n - 1) \times 2.5$$

$$\Rightarrow (n - 1) \times 2.5 = 3.6 + 18.9$$

$$\Rightarrow (n - 1) \times 2.5 = 22.5 \Rightarrow n - 1 = \frac{22.5}{2.5} = 9$$

$$\Rightarrow n = 9 + 1 = 10$$

$$\therefore n = 10$$

(v) $a_n = a + (n - 1)d \Rightarrow a_{105} = 3.5 + (105 - 1) \times 0$

$$\Rightarrow a_{105} = 3.5 + 104 \times 0 \Rightarrow a_{105} = 3.5 + 0 = 3.5$$

$$\therefore a_{105} = 3.5$$

2. (i) (c) : Here, $a = 10, n = 30$ and $d = 7 - 10 = -3$

$$\therefore a_n = a + (n - 1)d$$

$$\therefore a_{30} = 10 + (30 - 1) \times (-3)$$

$$= 10 + 29 \times (-3) = 10 - 87 = -77$$

(ii) (b) : Here, $a = -3, n = 11$ and

$$d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$$

$$\therefore a_n = a + (n - 1)d$$

$$\therefore a_{11} = -3 + (11 - 1) \times 5/2 = -3 + 25 = 22$$

3. (i) Here, $a = 2, a_3 = 26$

Let common difference = d

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow a_3 = 2 + (3 - 1)d \Rightarrow 26 = 2 + 2d$$

$$\Rightarrow 2d = 26 - 2 = 24 \Rightarrow d = 24/2 = 12$$

$$\therefore \text{The missing term} = a + d = 2 + 12 = 14$$

(ii) Let the first term = a

and common difference = d

$$\text{Here, } a_2 = 13 \text{ and } a_4 = 3$$

$$a_2 = a + d = 13, a_4 = a + 3d = 3$$

$$\therefore a_4 - a_2 = (a + 3d) - (a + d) = 3 - 13$$

$$\Rightarrow 2d = -10 \Rightarrow d = -10/2 = -5$$

$$\text{Now, } a + d = 13 \Rightarrow a + (-5) = 13$$

$$\Rightarrow a = 13 + 5 = 18$$

Thus, missing terms are a and $a + 2d$
i.e., 18 and $18 + (-10) = 8$

$$\text{(iii) Here, } a = 5 \text{ and } a_4 = 9 \frac{1}{2} = \frac{19}{2}$$

$$\text{since, } a_4 = a + 3d$$

$$\Rightarrow \frac{19}{2} = 5 + 3d \Rightarrow 3d = \frac{19}{2} - 5 = \frac{9}{2}$$

$$\Rightarrow d = \frac{9}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$$

$$\therefore \text{The missing terms are : } a_2 = a + d = 5 + \frac{3}{2} = 6 \frac{1}{2}$$

$$\text{and } a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

$$\text{(iv) Here, } a = -4, a_6 = 6$$

$$\therefore a_n = a + (n - 1)d$$

$$\therefore a_6 = -4 + (6 - 1)d$$

$$\Rightarrow 6 = -4 + 5d \Rightarrow 5d = 10 \Rightarrow d = 2$$

$$\therefore a_2 = a + d = -4 + 2 = -2,$$

$$a_3 = a + 2d = -4 + 2(2) = 0,$$

$$a_4 = a + 3d = -4 + 3(2) = 2$$

$$\text{and } a_5 = a + 4d = -4 + 4(2) = 4$$

$$\therefore \text{The missing terms are } -2, 0, 2 \text{ and } 4$$

$$\text{(v) Here, } a_2 = 38 \text{ and } a_6 = -22$$

$$\therefore a_2 = a + d = 38, a_6 = a + 5d = -22$$

$$\Rightarrow a_6 - a_2 = a + 5d - (a + d) = -22 - 38$$

$$\Rightarrow 4d = -60 \Rightarrow d = -60/4 = -15$$

$$\therefore a + d = 38 \Rightarrow a + (-15) = 38$$

$$\Rightarrow a = 38 + 15 = 53$$

$$\text{Now, } a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23,$$

$$a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$$

$$\text{and } a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$$

Thus, missing terms are 53, 23, 8 and -7

$$4. \text{ Let the } n^{\text{th}} \text{ term} = 78$$

$$\text{Here, } a = 3 \Rightarrow a_1 = 3 \text{ and } a_2 = 8$$

$$\therefore d = a_2 - a_1 = 8 - 3 = 5$$

$$\text{And, } a_n = a + (n - 1)d$$

$$\Rightarrow 78 = 3 + (n - 1) \times 5 \Rightarrow 78 - 3 = (n - 1) \times 5$$

$$\Rightarrow 75 = (n - 1) \times 5 \Rightarrow (n - 1) = 15 \Rightarrow n = 16$$

Thus, 78 is the 16th term of the given A.P.

$$5. \text{ (i) Here, } a = 7, d = 13 - 7 = 6$$

Let total number of terms be n .

$$\therefore a_n = 205. \text{ Now, } a_n = a + (n - 1) \times d$$

$$\Rightarrow 7 + (n - 1) \times 6 = 205$$

$$\Rightarrow (n - 1) \times 6 = 205 - 7 = 198$$

$$\therefore n = 33 + 1 = 34.$$

Thus, the required number of terms is 34.

$$\text{(ii) Here, } a = 18, d = 15 \frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{-5}{2}$$

Let the n^{th} term = -47

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow -47 = 18 + (n - 1) \times \left(-\frac{5}{2}\right)$$

$$\Rightarrow -47 - 18 = (n - 1) \times \left(-\frac{5}{2}\right) \Rightarrow -65 = (n - 1) \times \left(-\frac{5}{2}\right)$$

$$\Rightarrow n - 1 = -65 \times \left(-\frac{2}{5}\right) \Rightarrow n - 1 = 26$$

$$\Rightarrow n = 26 + 1 = 27$$

Thus, the required number of terms is 27.

$$6. \text{ For the given A.P.,}$$

$$\text{we have } a = 11, d = 8 - 11 = -3$$

Let -150 be the n^{th} term of the given A.P.

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow -150 = 11 + (n - 1) \times (-3) \Rightarrow -150 - 11 = (n - 1) \times (-3)$$

$$\Rightarrow -161 = (n - 1) \times (-3) \Rightarrow n - 1 = \frac{-161}{-3} = \frac{161}{3}$$

$$\Rightarrow n = \frac{161}{3} + 1 = \frac{164}{3}, \text{ which is a fraction}$$

But, n must be a positive integer.

Thus, -150 is not a term of the given A.P.

$$7. \text{ Here, } a_{11} = 38 \text{ and } a_{16} = 73$$

If the first term = a and the common difference = d .

$$\text{Then, } a + (11 - 1)d = 38 \Rightarrow a + 10d = 38 \quad \dots(i)$$

$$\text{and } a + (16 - 1)d = 73 \Rightarrow a + 15d = 73 \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$(a + 15d) - (a + 10d) = 73 - 38$$

$$\Rightarrow 5d = 35 \Rightarrow d = 35/5 = 7$$

$$\text{From (i), } a + 10(7) = 38$$

$$\Rightarrow a + 70 = 38 \Rightarrow a = 38 - 70 = -32$$

$$\therefore a_{31} = -32 + (31 - 1) \times 7$$

$$= -32 + 30 \times 7 = -32 + 210 = 178$$

Thus, the 31st term is 178.

$$8. \text{ Here, } n = 50, a_3 = 12, a_n = 106 \Rightarrow a_{50} = 106$$

If the first term = a and the common difference = d

$$\therefore a_3 = a + 2d = 12 \quad \dots(i)$$

$$a_{50} = a + 49d = 106 \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$\Rightarrow a_{50} - a_3 = a + 49d - (a + 2d) = 106 - 12$$

$$\Rightarrow 47d = 94 \Rightarrow d = 94/47 = 2$$

$$\text{From (i), we have } a + 2d = 12$$

$$\Rightarrow a + 2(2) = 12 \Rightarrow a = 12 - 4 = 8$$

$$\text{Now, } a_{29} = a + (29 - 1)d = 8 + (28) \times 2 = 8 + 56 = 64$$

Thus, the 29th term is 64.

$$9. \text{ Here, } a_3 = 4 \text{ and } a_9 = -8$$

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow a_3 = a + 2d = 4 \quad \dots(i)$$

$$a_9 = a + 8d = -8 \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$(a + 8d) - (a + 2d) = -8 - 4$$

$$\Rightarrow 6d = -12 \Rightarrow d = -12/6 = -2$$

$$\text{Now, From (i), we have } a + 2d = 4$$

$$\Rightarrow a + 2(-2) = 4 \Rightarrow a = 4 + 4 = 8$$

Let the n^{th} term of the A.P. be 0.

$$\therefore a_n = a + (n - 1)d = 0$$

$$\Rightarrow 8 + (n - 1) \times (-2) = 0 \Rightarrow (n - 1) \times (-2) = -8$$

$$\Rightarrow n - 1 = -8/-2 = 4 \Rightarrow n = 4 + 1 = 5$$

Thus, the 5th term of given A.P. is 0.

10. Let a be the first term and d the common difference of the given A.P.

Now, using $a_n = a + (n - 1)d$, we have

$$a_{17} = a + 16d, a_{10} = a + 9d$$

According to the question, $a_{10} + 7 = a_{17}$

$\Rightarrow (a + 9d) + 7 = a + 16d$
 $\Rightarrow a + 9d - a - 16d = -7 \Rightarrow -7d = -7 \Rightarrow d = 1$
 Thus, the common difference is 1.

11. Here, $a = 3$, $d = 15 - 3 = 12$

Using $a_n = a + (n - 1)d$, we get

$$a_{54} = a + 53d = 3 + 53 \times 12 = 3 + 636 = 639$$

Let a_n be 132 more than its 54th term.

$$\therefore a_n = a_{54} + 132 \Rightarrow a_n = 639 + 132 = 771$$

$$\text{Now, } a_n = 771 \Rightarrow a + (n - 1)d = 771$$

$$\Rightarrow 3 + (n - 1) \times 12 = 771$$

$$\Rightarrow (n - 1) \times 12 = 771 - 3 = 768$$

$$\Rightarrow (n - 1) = 768/12 = 64 \Rightarrow n = 64 + 1 = 65$$

Thus, 132 more than 54th term is the 65th term.

12. Let for the 1st A.P., the first term = a

$$\Rightarrow a_{100} = a + 99d$$

And for the 2nd A.P., the first term = a'

$$\Rightarrow a'_{100} = a' + 99d$$

According to the condition, we have $a_{100} - a'_{100} = 100$

$$\Rightarrow a + 99d - (a' + 99d) = 100$$

$$\Rightarrow a - a' = 100$$

$$\text{Let, } a_{1000} - a'_{1000} = x$$

$$\therefore a + 999d - (a' + 999d) = x$$

$$\Rightarrow a - a' = x \Rightarrow x = 100$$

$\therefore$ The difference between their 1000th terms is 100.

13. The first three digit number divisible by 7 is 105.

The last such three digit number is 994.

$\therefore$ The A.P. is 105, 112, 119,, 994

Here, $a = 105$ and $d = 7$

Let n be the required number of terms.

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow 994 = 105 + (n - 1) \times 7$$

$$\Rightarrow (n - 1) \times 7 = 994 - 105 = 889$$

$$\Rightarrow (n - 1) = 889/7 = 127$$

$$\Rightarrow n = 127 + 1 = 128$$

Thus, there are 128 three-digits numbers which are divisible by 7.

14. The multiple of 4 that lie between 10 and 250 are : 12, 16,, 248, which is an A.P.

Here, $a = 12$ and $d = 4$

Let the number of terms = n

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$$a_n = 12 + (n - 1) \times 4$$

$$\Rightarrow 248 = 12 + (n - 1) \times 4$$

$$\Rightarrow (n - 1) \times 4 = 248 - 12 = 236$$

$$\Rightarrow n - 1 = 236/4 = 59 \Rightarrow n = 59 + 1 = 60$$

Thus, the required number of terms = 60.

15. For the 1st A.P.

$$a = 63 \text{ and } d = 65 - 63 = 2$$

$$\therefore a_n = a + (n - 1)d = 63 + (n - 1) \times 2$$

For the 2nd A.P.

$$\therefore a = 3 \text{ and } d = 10 - 3 = 7$$

$$\therefore a_n = a + (n - 1)d = 3 + (n - 1) \times 7$$

Now, according to the question

$$3 + (n - 1) \times 7 = 63 + (n - 1) \times 2$$

$$\Rightarrow (n - 1) \times 7 - (n - 1) \times 2 = 63 - 3$$

$$\Rightarrow 7n - 7 - 2n + 2 = 60$$

$$\Rightarrow 5n - 5 = 60 \Rightarrow 5n = 60 + 5 = 65 \Rightarrow n = 65/5 = 13$$

Thus, the 13th terms of the two given A.P.'s are equal.

16. Let the first term = a and the common difference = d

$\therefore$ Using, $a_n = a + (n - 1)d$, we have

$$a_3 = a + 2d \Rightarrow a + 2d = 16 \quad \dots(i)$$

$$\text{And } a_7 = a + 6d, a_5 = a + 4d$$

According to the question, $a_7 - a_5 = 12$

$$\Rightarrow (a + 6d) - (a + 4d) = 12$$

$$\Rightarrow a + 6d - a - 4d = 12$$

$$\Rightarrow 2d = 12 \Rightarrow d = 6 \quad \dots(ii)$$

Now, from (i) and (ii), we have $a + 2(6) = 16$

$$\Rightarrow a + 12 = 16 \Rightarrow a = 16 - 12 = 4$$

$\therefore$ The required A.P. is 4, $[4 + 6]$, $[4 + 2(6)]$,

$[4 + 3(6)]$, or 4, 10, 16, 22,

17. We have, the last term, $l = 253$

Here, $d = 8 - 3 = 5$

Since the n^{th} term from the last term is given by, $l - (n - 1)d$,

$\therefore$ We have 20th term from the end

$$= l - (20 - 1) \times 5 = 253 - 19 \times 5 = 253 - 95 = 158$$

18. Let the first term = a and the common difference = d

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$$a_4 + a_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow 2a + 10d = 24 \Rightarrow a + 5d = 12 \quad \dots(i)$$

$$\text{And } a_6 + a_{10} = 44$$

$$\Rightarrow (a + 5d) + (a + 9d) = 44$$

$$\Rightarrow 2a + 14d = 44 \Rightarrow a + 7d = 22 \quad \dots(ii)$$

Now, subtracting (i) from (ii), we get

$$(a + 7d) - (a + 5d) = 22 - 12$$

$$\Rightarrow 2d = 10 \Rightarrow d = 5 \quad \dots(iii)$$

$$\therefore \text{From (i), } a + 5 \times 5 = 12$$

$$\Rightarrow a = 12 - 25 = -13$$

Now, the first three terms of the A.P. are given by a ,

$$(a + d), (a + 2d)$$

or -13, $(-13 + 5)$, $[-13 + 2(5)]$ or -13, -8, -3.

19. Here, $a = ₹ 5000$ and $d = ₹ 200$

Let in the n^{th} year he gets ₹ 7000.

$\therefore$ Using $a_n = a + (n - 1)d$, we get

$$7000 = 5000 + (n - 1) \times 200$$

$$\Rightarrow (n - 1) \times 200 = 7000 - 5000 = 2000$$

$$\Rightarrow n - 1 = 2000/200 = 10 \Rightarrow n = 10 + 1 = 11$$

Thus, the income becomes ₹ 7000 in 11 years i.e., in year 2006.

20. Here, $a = ₹ 5$ and $d = ₹ 1.75$

$\therefore$ In the n^{th} week her savings become ₹ 20.75.

$$\therefore a_n = ₹ 20.75$$

$\therefore$ Using $a_n = a + (n - 1)d$, we have

$$20.75 = 5 + (n - 1) \times (1.75)$$

$$\Rightarrow (n - 1) \times 1.75 = 20.75 - 5 \Rightarrow (n - 1) \times 1.75 = 15.75$$

$$\Rightarrow n - 1 = \frac{15.75}{1.75} = 9 \Rightarrow n = 9 + 1 = 10$$

Thus, the required number of years = 10.

EXERCISE - 5.3

1. (i) Given A.P. is 2, 7, 12, ..., to 10 terms.

Here, $a = 2$, $d = 7 - 2 = 5$, $n = 10$

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$$

$$= 5[4 + 9 \times 5] = 5[49] = 245$$

Thus, the sum of first 10 terms is 245.

- (ii) Given A.P. is -37, -33, -29, ..., to 12 terms.

Here $a = -37$, $d = -33 - (-37) = 4$, $n = 12$

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{12} = \frac{12}{2}[2(-37) + (12-1) \times 4]$$

$$= 6[-74 + 11 \times 4] = 6[-74 + 44] = 6 \times [-30] = -180$$

Thus, the sum of first 12 terms is -180.

- (iii) Given A.P. is 0.6, 1.7, 2.8, ..., to 100 terms.

Here, $a = 0.6$, $d = 1.7 - 0.6 = 1.1$, $n = 100$

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{100} = \frac{100}{2}[2(0.6) + (100-1) \times 1.1]$$

$$= 50[1.2 + 99 \times 1.1] = 50[1.2 + 108.9]$$

$$= 50[110.1] = 5505$$

Thus, the sum of first 100 terms is 5505.

- (iv) Given A.P. is
- $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$
- , to 11 terms.

Here, $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$, $n = 11$

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{11} = \frac{11}{2} \left[\left(2 \times \frac{1}{15} \right) + (11-1) \times \frac{1}{60} \right]$$

$$= \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right] = \frac{11}{2} \left[\frac{4+5}{30} \right] = \frac{11}{2} \times \frac{9}{30} = \frac{99}{60} = \frac{33}{20}$$

Thus, the sum of first 11 terms is $33/20$.

2. (i) The given numbers are : 7,
- $10\frac{1}{2}$
- , 14, ..., 84

Here, $a = 7$, $d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$, $l = 84$ Let n be the number of terms then, $a_n = a + (n-1)d$

$$\Rightarrow 84 = 7 + (n-1) \times \frac{7}{2} \Rightarrow (n-1) \times \frac{7}{2} = 84 - 7 = 77$$

$$\Rightarrow n-1 = 77 \times \frac{2}{7} = 22 \Rightarrow n = 22 + 1 = 23$$

$$\text{Now, } S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{23} = \frac{23}{2}(7+84) = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$$

Thus, the required sum is $1046\frac{1}{2}$.

- (ii) The given numbers are : 34, 32, 30, ..., 10

Here, $a = 34$, $d = 32 - 34 = -2$, $l = 10$ Let the number of terms be n .

$$\text{then, } a_n = a + (n-1)d$$

$$\Rightarrow 10 = 34 + (n-1) \times (-2) \Rightarrow (n-1) \times (-2) = -24$$

$$\Rightarrow n-1 = \frac{-24}{-2} = 12 \Rightarrow n = 13$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_{13} = \frac{13}{2}[68 + 12 \times (-2)] = \frac{13}{2}[68 - 24]$$

$$= \frac{13}{2}[44] = 13 \times 22 = 286$$

Thus, the required sum is 286.

- (iii) The given numbers are : -5, -8, -11, ..., -230

Here, $a = -5$, $d = -8 - (-5) = -3$, $l = -230$ Let n be the number of terms.

$$\text{then, } a_n = a + (n-1)d$$

$$\Rightarrow -230 = -5 + (n-1) \times (-3)$$

$$\Rightarrow (n-1) \times (-3) = -230 + 5 = -225$$

$$\Rightarrow n-1 = \frac{-225}{-3} = 75 \Rightarrow n = 75 + 1 = 76$$

$$\text{Now, } S_n = \frac{n}{2}[a+l]$$

$$\text{So, } S_{76} = \frac{76}{2}[(-5) + (-230)] = 38 \times (-235) = -8930.$$

 $\therefore$ The required sum is -8930.

3. (i) Here,
- $a = 5$
- ,
- $d = 3$
- and
- $a_n = 50 = l$

$$\therefore a_n = a + (n-1)d \Rightarrow 50 = 5 + (n-1) \times 3$$

$$\Rightarrow 50 - 5 = (n-1) \times 3 \Rightarrow (n-1) \times 3 = 45$$

$$\Rightarrow (n-1) = \frac{45}{3} = 15 \Rightarrow n = 15 + 1 = 16$$

$$\text{Now, } S_n = \frac{n}{2}(a+l) \Rightarrow S_{16} = \frac{16}{2}(5+50) = 8(55) = 440$$

Thus, $n = 16$ and $S_n = 440$

- (ii) Here,
- $a = 7$
- and
- $a_{13} = 35 = l$

$$\therefore a_{13} = a + (13-1)d \Rightarrow 35 = 7 + (13-1)d$$

$$\Rightarrow 35 - 7 = 12d \Rightarrow 28 = 12d \Rightarrow d = \frac{28}{12} = \frac{7}{3}$$

$$\text{Now, } S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow S_{12} = \frac{12}{2}(4+37) = \frac{13}{2} \times 42 = 13 \times 21 = 273$$

Thus, $S_{13} = 273$ and $d = \frac{7}{3}$

- (iii) Here,
- $a_{12} = 37 = l$
- and
- $d = 3$

Let the first term of the A.P. be a .

$$\text{Now, } a_{12} = a + (12-1)d$$

$$\Rightarrow 37 = a + 11d \Rightarrow 37 = a + 11 \times 3$$

$$\Rightarrow 37 = a + 33 \Rightarrow a = 37 - 33 = 4$$

$$\text{Now, } S_n = \frac{n}{2}(a+l) \Rightarrow S_{12} = \frac{12}{2}(4+37) = 6 \times (41) = 246$$

Thus, $a = 4$ and $S_{12} = 246$.

- (iv) Here,
- $a_3 = 15$
- and
- $S_{10} = 125$

Let the first term of the A.P. be a and d be the common difference.

$$\therefore a_3 = a + 2d \Rightarrow a + 2d = 15$$

$$\text{Again, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$$\Rightarrow 125 = 5[2a + 9d] \Rightarrow 2a + 9d = \frac{125}{5} = 25$$

$$\Rightarrow 2a + 9d = 25$$

...(i)

Multiplying (i) by 2 and subtracting (ii) from it, we get

$$2a + 4d - 2a - 9d = 30 - 25$$

$$\Rightarrow -5d = 5 \Rightarrow d = -1.$$

$$\therefore \text{From (i), } a + 2(-1) = 15$$

$$\Rightarrow a = 17$$

$$\text{Now, } a_{10} = a + (10-1)d = 17 + 9 \times (-1) = 17 - 9 = 8$$

$$\text{Thus, } d = -1 \text{ and } a_{10} = 8$$

$$(v) \text{ Here, } d = 5 \text{ and } S_9 = 75$$

Let the first term of the A.P. is a .

$$\therefore S_9 = \frac{9}{2}[2a + (9-1) \times 5] \Rightarrow 75 = \frac{9}{2}[2a + 40]$$

$$\Rightarrow 75 \times \frac{2}{9} = 2a + 40 \Rightarrow \frac{50}{3} = 2a + 40$$

$$\Rightarrow 2a = \frac{50}{3} - 40 = \frac{-70}{3} \Rightarrow a = \frac{-70}{3} \times \frac{1}{2} = \frac{-35}{3}$$

$$\text{Now, } a_9 = a + (9-1)d$$

$$= \frac{-35}{3} + (8 \times 5) = \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$$

$$\text{Thus, } a = \frac{-35}{3} \text{ and } a_9 = \frac{85}{3}.$$

$$(vi) \text{ Here, } a = 2, d = 8 \text{ and } S_n = 90$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore 90 = \frac{n}{2}[2 \times 2 + (n-1) \times 8]$$

$$\Rightarrow 90 \times 2 = 4n + n(n-1) \times 8 \Rightarrow 180 = 4n + 8n^2 - 8n$$

$$\Rightarrow 180 = 8n^2 - 4n \Rightarrow 45 = 2n^2 - n$$

$$\Rightarrow 2n^2 - n - 45 = 0 \Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$\Rightarrow 2n(n-5) + 9(n-5) = 0 \Rightarrow (2n+9)(n-5) = 0$$

$$\therefore \text{Either, } 2n+9=0 \Rightarrow n = -9/2$$

$$\text{Or } n-5=0 \Rightarrow n=5$$

$$\text{But } n = -\frac{9}{2} \text{ is not possible, so } n=5$$

$$\text{Now, } a_n = a + (n-1)d$$

$$\Rightarrow a_5 = 2 + (5-1) \times 8 = 2 + 32 = 34$$

$$\text{Thus, } n=5 \text{ and } a_5=34$$

$$(vii) \text{ Here, } a=8, a_n=62=l \text{ and } S_n=210$$

Let the common difference = d

$$\text{Now, } S_n = \frac{n}{2}(a+l) \Rightarrow 210 = \frac{n}{2}(8+62) = \frac{n}{2} \times 70 = 35n$$

$$\therefore n = \frac{210}{35} = 6$$

$$\text{Again, } a_n = a + (n-1)d$$

$$\Rightarrow 62 = 8 + (6-1) \times d \Rightarrow 62 - 8 = 5d$$

$$\Rightarrow 54 = 5d \Rightarrow d = \frac{54}{5}. \text{ Thus, } n=6 \text{ and } d = \frac{54}{5}.$$

$$(viii) \text{ Here, } a_n=4, d=2 \text{ and } S_n=-14$$

Let the first term be ' a '.

$$\therefore a_n = 4 \therefore a + (n-1)2 = 4 \Rightarrow a = 4 - 2n + 2$$

$$\Rightarrow a = 6 - 2n$$

...(i)

$$\text{Also, } S_n = \frac{n}{2}(a+l) \Rightarrow -14 = \frac{n}{2}(a+4)$$

$$\Rightarrow n(a+4) = -28$$

...(ii)

Substituting the value of a from (i) into (ii), we get

$$n[6 - 2n + 4] = -28$$

$$\Rightarrow n[10 - 2n] = -28 \Rightarrow 2n[5 - n] = -28$$

$$\Rightarrow n(5 - n) = -14 \Rightarrow 5n - n^2 + 14 = 0$$

$$\Rightarrow n^2 - 5n - 14 = 0 \Rightarrow (n-7)(n+2) = 0$$

$$\therefore \text{Either, } n-7=0 \Rightarrow n=7$$

$$\text{Or } n+2=0 \Rightarrow n=-2$$

But n cannot be negative, so $n=7$

$$\text{Now, from (i), we have } a = 6 - 2 \times 7 \Rightarrow a = -8$$

$$\text{Thus, } a = -8 \text{ and } n = 7$$

$$(ix) \text{ Here, } a=3, n=8 \text{ and } S_n=192$$

Let d be the common difference.

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d] \therefore 192 = \frac{8}{2}[2(3) + (8-1)d]$$

$$\Rightarrow 192 = 4[6 + 7d] \Rightarrow 192 = 24 + 28d$$

$$\Rightarrow 28d = 192 - 24 = 168 \Rightarrow d = 6$$

$$\text{Thus, } d = 6.$$

$$(x) \text{ Here, } l = 28 \text{ and } S_9 = 144$$

Let the first term be ' a '.

$$\text{Thus } S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow S_9 = \frac{9}{2}(a+28) \Rightarrow 144 = \frac{9}{2}(a+28)$$

$$\Rightarrow a+28 = 144 \times \frac{2}{9} = 16 \times 2 = 32 \Rightarrow a = 32 - 28 = 4$$

$$\text{Thus, } a = 4.$$

$$4. \text{ Here, } a=9, d=17-9=8 \text{ and } S_n=636$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = 636$$

$$\therefore \frac{n}{2}[(2 \times 9) + (n-1) \times 8] = 636$$

$$\Rightarrow n[18 + (n-1) \times 8] = 1272 \Rightarrow 18n + 8n^2 - 8n = 1272$$

$$\Rightarrow 8n^2 + 10n = 1272 \Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow 4n^2 - 48n + 53n - 636 = 0$$

$$\Rightarrow 4n(n-12) + 53(n-12) = 0$$

$$\Rightarrow (n-12)(4n+53) = 0 \Rightarrow n=12, -53/4$$

As n can't be negative.

$$\therefore \text{Required number of terms} = 12.$$

$$5. \text{ Here, } a=5, l=45=a_n, S_n=400$$

$$\therefore a_n = a + (n-1)d$$

$$\therefore 45 = 5 + (n-1)d$$

$$\Rightarrow (n-1)d = 45 - 5 \Rightarrow (n-1)d = 40$$

...(i)

$$\text{Also } S_n = \frac{n}{2}(a+l) \Rightarrow 400 = \frac{n}{2}(5+45) \Rightarrow 400 \times 2 = n \times 50$$

$$\Rightarrow n = \frac{400 \times 2}{50} = 16$$

$$\text{From (i), we get } (16-1)d = 40 \Rightarrow 15d = 40 \Rightarrow d = 8/3$$

$$6. \text{ We have, first term } a=17, \text{ last term, } l=350=a_n \text{ and common difference } d=9$$

Let the number of terms be n .

$$\therefore a_n = a + (n-1)d$$

$$\therefore 350 = 17 + (n-1) \times 9 \Rightarrow (n-1) \times 9 = 350 - 17 = 333$$

$$\Rightarrow n-1 = 333/9 = 37 \Rightarrow n = 37 + 1 = 38$$

$$\text{Since, } S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{38} = \frac{38}{2}(17+350) = 19(367) = 6973$$

Thus, $n = 38$ and $S_n = 6973$.

7. Here, $n = 22$, $a_{22} = 149 = l$, $d = 7$

Let the first term of the A.P. be a .

$$\therefore a_n = a + (n-1)d$$

$$\therefore a_{22} = a + (22-1) \times 7 \Rightarrow a + 21 \times 7 = 149$$

$$\Rightarrow a + 147 = 149 \Rightarrow a = 149 - 147 = 2$$

$$\text{Now, } S_{22} = \frac{n}{2}[a+l] \Rightarrow S_{22} = \frac{22}{2}[2+149] = 11[151] = 1661$$

Thus, $S_{22} = 1661$.

8. Here, $n = 51$, $a_2 = 14$ and $a_3 = 18$

Let the first term of the A.P. be a and the common difference is d .

$$\text{We have } a_2 = a + d \Rightarrow a + d = 14 \quad \dots(i)$$

$$a_3 = a + 2d \Rightarrow a + 2d = 18 \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$$

From (i), we get

$$a + 4 = 14 \Rightarrow a = 14 - 4 = 10$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{51} = \frac{51}{2}[(2 \times 10) + (51-1) \times 4]$$

$$= \frac{51}{2}[20 + 200] = \frac{51}{2}[220] = 51 \times 110 = 5610$$

Thus, the sum of 51 terms is 5610.

9. Here, we have $S_7 = 49$ and $S_{17} = 289$

Let the first term of the A.P. be ' a ' and ' d ' be the common difference, then

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_7 = \frac{7}{2}[2a + (7-1)d] = 49$$

$$\Rightarrow 7(2a + 6d) = 2 \times 49 = 98$$

$$\Rightarrow 2a + 6d = \frac{98}{7} = 14 \Rightarrow 2[a + 3d] = 14$$

$$\Rightarrow a + 3d = \frac{14}{2} = 7 \Rightarrow a + 3d = 7 \quad \dots(i)$$

$$\text{Also, } S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$$

$$\Rightarrow \frac{17}{2}(2a + 16d) = 289$$

$$\Rightarrow a + 8d = \frac{289}{17} = 17 \Rightarrow a + 8d = 17 \quad \dots(ii)$$

Subtracting (i) from (ii), we have

$$a + 8d - a - 3d = 17 - 7$$

$$\Rightarrow 5d = 10 \Rightarrow d = 2$$

Now, from (i), we have

$$a + 3(2) = 7 \Rightarrow a = 7 - 6 = 1$$

$$\text{Now, } S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}[2n] = n \times n = n^2$$

Thus, the required sum of n terms $= n^2$.

10. (i) Here, $a_n = 3 + 4n$

Putting $n = 1, 2, 3, 4, \dots, n$, we get

$$a_1 = 3 + 4(1) = 7$$

$$a_2 = 3 + 4(2) = 11$$

$$a_3 = 3 + 4(3) = 15$$

$$a_4 = 3 + 4(4) = 19$$

$$\dots \dots \dots$$

$$a_n = 3 + 4n$$

$\therefore$ The A.P. in which $a = 7$ and $d = 11 - 7 = 4$ is 7, 11, 15, 19, ..., $(3 + 4n)$.

$$\text{Now, } S_{15} = \frac{15}{2}[(2 \times 7) + (15-1) \times 4]$$

$$= \frac{15}{2}[14 + (14 \times 4)] = \frac{15}{2}[14 + 56] = \frac{15}{2}[70]$$

$$= 15 \times 35 = 525$$

(ii) Here, $a_n = 9 - 5n$

Putting $n = 1, 2, 3, 4, \dots, n$, we get

$$a_1 = 9 - 5(1) = 4$$

$$a_2 = 9 - 5(2) = -1$$

$$a_3 = 9 - 5(3) = -6$$

$$a_4 = 9 - 5(4) = -11$$

$$\dots \dots \dots$$

$$a_n = 9 - 5n$$

$\therefore$ The A.P. is 4, -1, -6, -11, ..., $9 - 5n$ having first term as 4 and $d = -1 - 4 = -5$

$$\therefore S_{15} = \frac{15}{2}[(2 \times 4) + (15-1) \times (-5)]$$

$$= \frac{15}{2}[8 + 14 \times (-5)] = \frac{15}{2}[8 - 70] = \frac{15}{2} \times (-62)$$

$$= 15 \times (-31) = -465.$$

11. We have $S_n = 4n - n^2$

$$\therefore S_1 = 4(1) - (1)^2 = 4 - 1 = 3 \Rightarrow \text{First term} = 3$$

$$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$$

$$\Rightarrow \text{Sum of first two terms} = 4$$

$$\therefore \text{Second term } (S_2 - S_1) = 4 - 3 = 1$$

$$S_3 = 4(3) - (3)^2 = 12 - 9 = 3$$

$$\Rightarrow \text{Sum of first 3 terms} = 3$$

$$\therefore \text{Third term } (S_3 - S_2) = 3 - 4 = -1$$

$$S_9 = 4(9) - (9)^2 = 36 - 81 = -45$$

$$S_{10} = 4(10) - (10)^2 = 40 - 100 = -60$$

$$\therefore \text{Tenth term} = S_{10} - S_9 = [-60] - [-45] = -15$$

$$\text{Now, } S_n = 4(n) - (n)^2 = 4n - n^2$$

$$\text{Also, } S_{n-1} = 4(n-1) - (n-1)^2$$

$$= 4n - 4 - [n^2 - 2n + 1]$$

$$= 4n - 4 - n^2 + 2n - 1 = 6n - n^2 - 5$$

$$\therefore n^{\text{th}} \text{ term} = S_n - S_{n-1} = [4n - n^2] - [6n - n^2 - 5]$$

$$= 4n - n^2 - 6n + n^2 + 5 = 5 - 2n$$

Thus, $S_1 = 3$ and $a_1 = 3$

$$S_2 = 4 \text{ and } a_2 = 1$$

$$S_3 = 3 \text{ and } a_3 = -1$$

$$a_{10} = -15 \text{ and } a_n = 5 - 2n$$

12. $\therefore$ The first 40 positive integers divisible by 6 are 6, 12, 18,, (6×40)

And, these numbers are in A.P., such that $a = 6$

$$d = 12 - 6 = 6 \text{ and } a_{40} = 6 \times 40 = 240 = l$$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} \therefore S_{40} &= \frac{40}{2}[(2 \times 6) + (40-1) \times 6] \\ &= 20[12 + 39 \times 6] = 20[12 + 234] \\ &= 20 \times 246 = 4920 \end{aligned}$$

13. The first 15 multiples of 8 are 8, (8×2) , (8×3) , (8×4) ,, (8×15) or 8, 16, 24, 32,, 120.

These numbers are in A.P., where $a = 8$ and $l = 120$

$$\begin{aligned} \therefore S_{15} &= \frac{15}{2}[a + l] = \frac{15}{2}[8 + 120] \\ &= \frac{15}{2} \times 128 = 15 \times 64 = 960 \end{aligned}$$

Thus, the sum of first 15 multiples of 8 is 960.

14. Odd numbers between 0 and 50 are 1, 3, 5, 7,, 49.

These numbers are in A.P. such that $a = 1$ and $l = 49$

$$\text{Here, } d = 3 - 1 = 2 \therefore a_n = a + (n-1)d$$

$$\Rightarrow 49 = 1 + (n-1)2 \Rightarrow 49 - 1 = (n-1)2$$

$$\Rightarrow (n-1) = \frac{48}{2} = 24 \therefore n = 24 + 1 = 25$$

$$\text{Now, } S_{25} = \frac{25}{2}[1 + 49] = \frac{25}{2}[50] = 25 \times 25 = 625$$

Thus, the sum of odd numbers between 0 and 50 is 625.

15. Here, penalty for delay on

$$1^{\text{st}} \text{ day} = ₹ 200$$

$$2^{\text{nd}} \text{ day} = ₹ 250$$

$$3^{\text{rd}} \text{ day} = ₹ 300$$

.....

.....

Now, 200, 250, 300, are in A.P. such that

$$a = 200, d = 250 - 200 = 50$$

$$\begin{aligned} \therefore S_{30} \text{ is given by } S_{30} &= \frac{30}{2}[2(200) + (30-1) \times 50] \\ &\quad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d] \right] \\ &= 15[400 + 29 \times 50] = 15[400 + 1450] \\ &= 15 \times 1850 = 27750 \end{aligned}$$

Thus, penalty for the delay for 30 days is ₹ 27750.

16. Sum of all the prizes = ₹ 700

Let the first prize = a

$$\therefore 2^{\text{nd}} \text{ prize} = (a - 20)$$

$$3^{\text{rd}} \text{ prize} = (a - 40)$$

$$4^{\text{th}} \text{ prize} = (a - 60)$$

.....

Thus, we have, first term = a

Common difference = -20

Sum of 7 terms, $S_7 = 700$

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 700 = \frac{7}{2}[2(a) + (7-1) \times (-20)]$$

$$\Rightarrow 700 = \frac{7}{2}[2a + 6 \times (-20)] \Rightarrow 700 \times \frac{2}{7} = 2a - 120$$

$$\Rightarrow 200 = 2a - 120 \Rightarrow 2a = 320 \Rightarrow a = 320/2 = 160$$

Thus, the values of the seven prizes are ₹ 160, ₹(160 - 20), ₹(160 - 40), ₹(160 - 60), ₹(160 - 80), ₹(160 - 100) and ₹(160 - 120) = ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

17. Number of classes = 12

$\therefore$ Each class has 3 sections.

$$\therefore \text{Number of plants planted by class I} = 1 \times 3 = 3$$

$$\text{Number of plants planted by class II} = 2 \times 3 = 6$$

$$\text{Number of plants planted by class III} = 3 \times 3 = 9$$

$$\text{Number of plants planted by class IV} = 4 \times 3 = 12$$

$$\text{Number of plants planted by class XII} = 12 \times 3 = 36$$

Thus, the numbers 3, 6, 9, 12,, 36 are in A.P.

Here, $a = 3$ and $d = 6 - 3 = 3$

$$\therefore \text{Number of classes} = 12 \text{ i.e., } n = 12$$

$\therefore$ Sum the n terms of the above A.P., is given by

$$\begin{aligned} S_{12} &= \frac{12}{2}[2(3) + (12-1)3] \quad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d] \right] \\ &= 6[6 + 11 \times 3] = 6[6 + 33] = 6 \times 39 = 234 \end{aligned}$$

Thus, the total number of trees = 234.

18. Length of a semi-circle = Semi-circumference

$$= \frac{1}{2}(2\pi r) = \pi r$$

$$\therefore l_1 = \pi r_1 = 0.5 \pi \text{ cm} = 1 \times 0.5 \pi \text{ cm}$$

$$l_2 = \pi r_2 = 1.0 \pi \text{ cm} = 2 \times 0.5 \pi \text{ cm}$$

$$l_3 = \pi r_3 = 1.5 \pi \text{ cm} = 3 \times 0.5 \pi \text{ cm}$$

$$l_4 = \pi r_4 = 2.0 \pi \text{ cm} = 4 \times 0.5 \pi \text{ cm}$$

.....

$$l_{13} = \pi r_{13} \text{ cm} = 6.5 \pi \text{ cm} = 13 \times 0.5 \pi \text{ cm}$$

$$\begin{aligned} \text{Now, length of the spiral} &= l_1 + l_2 + l_3 + l_4 + \dots + l_{13} \\ &= 0.5\pi[1 + 2 + 3 + 4 + \dots + 13] \text{ cm} \quad \dots(i) \end{aligned}$$

$$\therefore 1, 2, 3, 4, \dots, 13 \text{ are in A.P. such that } a = 1 \text{ and } l = 13$$

$$\begin{aligned} \therefore S_{13} &= \frac{13}{2}[1 + 13] \quad \left[\text{Using } S_n = \frac{n}{2}(a + l) \right] \\ &= \frac{13}{2} \times 14 = 13 \times 7 = 91 \end{aligned}$$

$\therefore$ From (i), we have

$$\text{Total length of the spiral} = 0.5\pi[91] \text{ cm}$$

$$= \frac{5}{10} \times \frac{22}{7} \times 91 \text{ cm} = 11 \times 13 \text{ cm} = 143 \text{ cm}$$

19. The number of logs in

$$1^{\text{st}} \text{ row} = 20, 2^{\text{nd}} \text{ row} = 19 \text{ and } 3^{\text{rd}} \text{ row} = 18$$

Obviously, the numbers 20, 19, 18,, are in A.P., such that $a = 20$, $d = 19 - 20 = -1$

Let the number of rows be n .

$$\text{Since, } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 200 = \frac{n}{2}[2(20) + (n-1) \times (-1)] \Rightarrow 200 = \frac{n}{2}[40 - (n-1)]$$

$$\Rightarrow 200 = \frac{n}{2}[40 - (n-1)]$$

$$\Rightarrow 2 \times 200 = n \times 40 - n(n-1)$$

$$\Rightarrow 400 = 40n - n^2 + n \Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n(n-16) - 25(n-16) = 0$$

$$\Rightarrow (n-16)(n-25) = 0$$

$$\text{Either } n-16 = 0 \Rightarrow n = 16$$

$$\text{Or } n-25 = 0 \Rightarrow n = 25$$

$$a_n = 0 \Rightarrow a + (n-1)d = 0$$

$$\Rightarrow 20 + (n-1) \times (-1) = 0 \Rightarrow n-1 = 20$$

$$\Rightarrow n = 21 \text{ i.e., 21}^{\text{st}} \text{ term becomes 0}$$

$$\therefore n = 25 \text{ is not required.}$$

$$\therefore \text{Number of rows} = 16$$

$$\text{Now, } a_{16} = a + (16-1)d = 20 + 15 \times (-1) = 20 - 15 = 5$$

$$\therefore \text{Number of logs in the 16th (top) row is 5.}$$

20. Here, number of potatoes = 10

The up-down distance of the bucket :

$$\text{From the 1}^{\text{st}} \text{ potato} = [5\text{m}] \times 2 = 10 \text{ m}$$

$$\text{From the 2}^{\text{nd}} \text{ potato} = [(5+3)\text{m}] \times 2 = 16 \text{ m}$$

$$\text{From the 3}^{\text{rd}} \text{ potato} = [(5+3+3)\text{m}] \times 2 = 22 \text{ m}$$

$$\text{From the 4}^{\text{th}} \text{ potato} = [(5+3+3+3)\text{m}] \times 2 = 28 \text{ m}$$

$$\therefore 10, 16, 22, 28, \dots \text{ are in A.P. such that}$$

$$a = 10 \text{ and } d = 16 - 10 = 6$$

$$\therefore \text{Using } S_n = \frac{n}{2}[2a + (n-1)d], \text{ we have}$$

$$S_{10} = \frac{10}{2}[2(10) + (10-1) \times 6] = 5[20 + 54] = 5 \times 74 = 370$$

$$\text{Thus, the sum of above distance} = 370 \text{ m.}$$

$$\Rightarrow \text{The competitor has to run a total distance of 370 m.}$$

EXERCISE - 5.4

1. We have the A.P. having $a = 121$ and $d = 117 - 121 = -4$

$$\text{Now, } a_n = a + (n-1)d = 121 + (n-1) \times (-4) \\ = 121 - 4n + 4 = 125 - 4n$$

For the first negative term, we have $a_n < 0$

$$\Rightarrow (125 - 4n) < 0 \Rightarrow 125 < 4n$$

$$\Rightarrow \frac{125}{4} < n \Rightarrow 31\frac{1}{4} < n \text{ or } n > 31\frac{1}{4}$$

Thus, the first negative term is 32^{nd} term.

2. Here, $a_3 + a_7 = 6$ and $a_3 \times a_7 = 8$

Let the first term = a and the common difference = d

$$\therefore a_3 = a + 2d \text{ and } a_7 = a + 6d$$

$$\therefore a_3 + a_7 = 6$$

$$\therefore (a + 2d) + (a + 6d) = 6$$

$$\Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3 \quad \dots(i)$$

$$\text{Again, } a_3 \times a_7 = 8$$

$$\therefore (a + 2d) \times (a + 6d) = 8$$

$$\Rightarrow [(a + 4d) - 2d] \times [(a + 4d) + 2d] = 8$$

$$\Rightarrow (3 - 2d) \times (3 + 2d) = 8$$

$$\Rightarrow 3^2 - (2d)^2 = 8 \Rightarrow 9 - 4d^2 = 8$$

$$\Rightarrow -4d^2 = 8 - 9 = -1$$

$$\Rightarrow d^2 = \frac{-1}{-4} = \frac{1}{4} \Rightarrow d = \pm \frac{1}{2}$$

Case-I When $d = \frac{1}{2}$, from (i), we have

$$a + 2 = 3 \Rightarrow a = 3 - 2 = 1$$

Now, using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

The sum of first 16 terms,

$$S_{16} = \frac{16}{2} \left[2(1) + (16-1) \times \frac{1}{2} \right] = 8 \left[2 + \frac{15}{2} \right] = 16 + 60 = 76$$

Case-II When $d = -\frac{1}{2}$, from (i), we have

$$a + 4 \left(-\frac{1}{2} \right) = 3 \Rightarrow a - 2 = 3 \Rightarrow a = 5$$

So, the sum of first 16 terms,

$$S_{16} = \frac{16}{2} \left[2(5) + (16-1) \times \left(-\frac{1}{2} \right) \right] \\ = 8 \left[10 + \left(-\frac{15}{2} \right) \right] = 80 - 60 = 20$$

$$\text{3. Distance between bottom and top rungs} = 2\frac{1}{2} \text{ m} \\ = \frac{5}{2} \times 100 \text{ cm} = 250 \text{ cm}$$

Distance between two consecutive rungs = 25 cm

$$\therefore \text{Number of rungs, } n = 250/25 + 1 = 10 + 1 = 11$$

Length of the 1^{st} rung (bottom rung) = 45 cm

Length of the 11^{th} rung (top rung) = 25 cm

Let the length of each successive rung decrease by x cm.

$$\therefore \text{Total length of the rungs} = 45 \text{ cm} +$$

$$(45 - x) \text{ cm} + (45 - 2x) \text{ cm} + \dots + 25 \text{ cm}$$

Here, the number 45, $(45 - x)$, $(45 - 2x)$, ..., 25 are in an A.P. such that first term, $a = 45$ and last term, $l = 25$

Number of terms, $n = 11$

$$\therefore \text{Using, } S_n = \frac{n}{2}[a + l], \text{ we have } S_{11} = \frac{11}{2}[45 + 25]$$

$$\Rightarrow S_{11} = \frac{11}{2} \times 70 \Rightarrow S_{11} = 11 \times 35 = 385$$

$\therefore$ Total length of 11 rungs = 385 cm i.e., Length of wood required for the rungs is 385 cm.

4. We have the following consecutive numbers on the houses of a row; 1, 2, 3, 4, 5, ..., 49.

These numbers are in A.P., such that $a = 1$, $d = 2 - 1 = 1$, $n = 49$

Let one of the houses be numbered as x

$$\therefore \text{Number of houses preceding it} = x - 1$$

$$\text{Number of houses following it} = 49 - x$$

Now, the sum of the house-numbers preceding x is

$$S_{x-1} = \frac{x-1}{2}[2(1) + (x-1-1) \times 1]$$

$$= \frac{x-1}{2}[2 + x - 2] = \frac{x(x-1)}{2} = \frac{x^2}{2} - \frac{x}{2}$$

The houses beyond x are numbered as $(x+1)$, $(x+2)$, $(x+3)$,, 49

$\therefore$ For these house numbers (which are in an A.P.)

First term, $a = x+1$

Last term, $l = 49$

$\therefore$ Using $S_n = \frac{n}{2}[a+l]$, we have

$$S_{49-x} = \frac{49-x}{2}[(x+1) + 49]$$

$$= \frac{49-x}{2}[x+50] = \frac{49x}{2} - \frac{x^2}{2} + (49 \times 25) - 25x$$

$$= \left(\frac{49x}{2} - 25x\right) - \frac{x^2}{2} + (49 \times 25) = \frac{-x}{2} - \frac{x^2}{2} + (49 \times 25)$$

Now, [Sum of house numbers preceding x] = [Sum of house numbers following x]

i.e., $S_{x-1} = S_{49-x}$

$$\Rightarrow \frac{x^2}{2} - \frac{x}{2} = \frac{-x}{2} - \frac{x^2}{2} + (49 \times 25)$$

$$\Rightarrow \left(\frac{x^2}{2} + \frac{x^2}{2}\right) - \frac{x}{2} + \frac{x}{2} = (49 \times 25) \Rightarrow \frac{2x^2}{2} = (49 \times 25)$$

$$\Rightarrow x^2 = (49 \times 25) \Rightarrow x = \pm\sqrt{49 \times 25}$$

$$\Rightarrow x = \pm(7 \times 5) = \pm 35$$

But x cannot be taken as negative.

$$\therefore x = 35.$$

5. For 1st step : Length = 50 m, Breadth = $1/2$ m, Height = $1/4$ m

$\therefore$ Volume of concrete required to build the 1st step
= Volume of the cuboidal step
= Length $\times$ breadth $\times$ height

$$= 50 \times \frac{1}{2} \times \frac{1}{4} \text{ m}^3 = \frac{25}{4} \times 1 \text{ m}^3$$

For 2nd step : Length = 50 m, Breadth = $1/2$ m, Height = $\left(\frac{1}{4} + \frac{1}{4}\right)$ m = $2 \times \frac{1}{4}$ m

$\therefore$ Volume of concrete required to build the 2nd step

$$= 50 \times \frac{1}{2} \times \frac{1}{4} \times 2 \text{ m}^3 = \frac{25}{4} \times 2 \text{ m}^3$$

For 3rd step : Length = 50 m, Breadth = $1/2$ m, Height

$$= \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) \text{ m} = 3 \times \frac{1}{4} \text{ m}$$

$\therefore$ Volume of concrete required to build the 3rd step

$$= 50 \times \frac{1}{2} \times \frac{1}{4} \times 3 \text{ m}^3 = \frac{25}{4} \times 3 \text{ m}^3$$

Thus, the volumes (in m^3) of concrete required to build the various steps are :

$\left(\frac{25}{4} \times 1\right), \left(\frac{25}{4} \times 2\right), \left(\frac{25}{4} \times 3\right), \dots$ obviously, these numbers form an A.P. such that $a = 25/4$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

Here, total number of steps, $n = 15$

Total volume of concrete required to build 15 steps is given by the sum of their individual volumes.

On using $S_n = \frac{n}{2}[2a + (n-1)d]$, we have

$$S_{15} = \frac{15}{2} \left[2 \left(\frac{25}{4} \right) + (15-1) \times \frac{25}{4} \right]$$

$$= \frac{15}{2} \left[\frac{25}{2} + 14 \times \frac{25}{4} \right] = \frac{15}{2} \left[\frac{25}{2} + \frac{175}{2} \right]$$

$$= 15 \times 50 = 750 \text{ m}^3$$

Thus, the required volume of concrete is 750 m^3 .

