Triangles

SOLUTIONS

1. **(a) :** We have $\Delta ABC \sim \Delta DEF$

$\therefore \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

Given, $\angle A = 47°$, $\angle E = 83°$

$\therefore \quad \angle B = 83°$

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 47° - 83° = 50°$

2. **(a) :** We have, $\Delta ABC \sim \Delta XYZ$

$$\Rightarrow \quad \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \Rightarrow \frac{4}{x} = \frac{6}{7.2} = \frac{5}{6} \Rightarrow x = \frac{4 \times 7.2}{6}$$

$\Rightarrow \quad x = 4.8$ cm

3. **(d) :** In triangle CAB, if DE divides CA and CB in the same ratio, then $DE \parallel AB$.

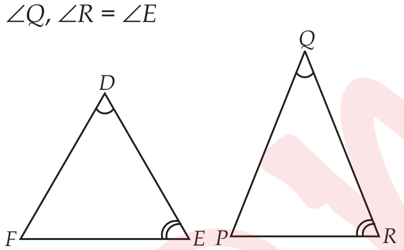
$$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB} \Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$\Rightarrow \quad 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow \quad 6x = 12 \Rightarrow x = 2$

4. **(b) :** Given, in ΔDEF and ΔPQR,

$\angle D = \angle Q$, $\angle R = \angle E$



$\therefore \quad \Delta DEF \sim \Delta QRP$ [By AA similarity criterion]

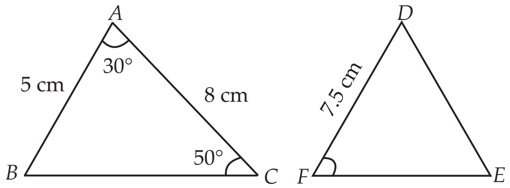
$\Rightarrow \quad \angle F = \angle P$

[Corresponding angles of similar triangles]

$$\therefore \quad \frac{DE}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$$

5. **(b) :** Given, $\Delta ABC \sim \Delta DFE$, then

$\angle A = \angle D = 30°$, $\angle C = \angle E = 50°$



$\therefore \quad \angle B = \angle F = 180° - (30° + 50°) = 100°$

Also, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm

$$\therefore \quad \frac{AB}{DF} = \frac{AC}{DE} \Rightarrow \frac{5}{7.5} = \frac{8}{DE}$$

$$\therefore \quad DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

Hence, $DE = 12$ cm, $\angle F = 100°$

6. **(b) :** In ΔABC, $DE \parallel BC$ [Given]

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By B.P.T.]}$$

$$\Rightarrow \quad \frac{x}{x+1} = \frac{x+3}{x+5}$$

$\Rightarrow \quad x(x + 5) = (x + 3)(x + 1)$

$\Rightarrow \quad x^2 + 5x = x^2 + 3x + x + 3 \Rightarrow x = 3$

7. **(a)**

8. In triangle ABC, $DE \parallel BC$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By B.P.T.]}$$

$$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$$

9. We have, $\Delta ABE \cong \Delta ACD$

$\therefore \quad AB = AC$ and $AD = AE$ [By C.P.C.T.] ...(i)

Now, in ΔADE and ΔABC,

$\angle A = \angle A$ [Common]

$$\frac{AB}{AD} = \frac{AC}{AE} \quad \text{[Using (i)]}$$

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

10. Let ΔABC and ΔDEF be two similar triangles such that $AB = 9$ cm.

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$

[∵ Ratio of corresponding sides of similar triangles is equal to the ratio of their perimeters]

$$\Rightarrow \quad \frac{9}{DE} = \frac{36}{48} \Rightarrow DE = 12 \text{ cm}$$

11. Basic proportionality theorem : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

12. SAS similarity criterion : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

13. Two quadrilaterals are similar, if their corresponding angles are equal and corresponding sides must also be proportional.

14. In ΔABD and ΔASR, $RS \parallel DB$

$\therefore \quad \angle ABD = \angle ASR$ [Corresponding angles]

$\angle A = \angle A$ [Common]

$\therefore \quad \Delta ABD \sim \Delta ASR$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{AB}{AS} = \frac{AD}{AR} = \frac{BD}{RS} \Rightarrow \frac{3+3}{3} = \frac{x}{y} \Rightarrow x = 2y$$

15. (i) (c) : Since, $\angle B = \angle D = 90°$, $\angle AMB = \angle CMD$

($\because$ Angle of incident = Angle of reflection)

$\therefore$ By AA similarity criterion, $\Delta ABM \sim \Delta CDM$

(ii) (a)

(iii) (c) : $\because \Delta ABM \sim \Delta CDM$

$$\therefore \frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{1.8} = \frac{2.5}{1.5}$$

$$\Rightarrow AB = \frac{2.5 \times 1.8}{1.5} = 3 \text{ m}$$

(iv) (b) : Since, $\Delta ABM \sim \Delta CDM$

$\therefore \angle A = \angle C = 30°$

[$\because$ Corresponding angles of similar triangles are also equal]

(v) (b) : Since, $\Delta ABM \sim \Delta CDM$

$$\therefore \frac{AB}{CD} = \frac{BM}{MD} \Rightarrow \frac{AB}{6} = \frac{24}{8} \Rightarrow AB = 18 \text{ cm}$$

16. (i) In ΔPAB and ΔPQR,

$\angle P = \angle P$ (Common)

$\angle A = \angle Q$ (Corresponding angles)

By AA similarity criterion, $\Delta PAB \sim \Delta PQR$

$$\therefore \frac{AB}{QR} = \frac{PA}{PQ} \Rightarrow \frac{AB}{12} = \frac{6}{24} \Rightarrow AB = 3 \text{ m}$$

(ii) Similarly, ΔPCD and ΔPQR are similar.

$$\therefore \frac{PC}{PQ} = \frac{CD}{QR} \Rightarrow \frac{14}{24} = \frac{CD}{12} \Rightarrow CD = 7 \text{ m}$$

(iii) Area of whole empty land

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 15 = 90 \text{ m}^2$$

(iv) In ΔPCD, $AB \parallel CD$

$$\therefore \frac{PA}{AC} = \frac{PB}{BD} \text{ (By Thales thereom)}$$

$$\Rightarrow \frac{6}{8} = \frac{PB}{BD} \Rightarrow \frac{PB}{BD} = \frac{3}{4}$$

(v) We know that, if two triangles are similar, then their corresponding angles are equal.

$\therefore \angle D = \angle R$, $\angle E = \angle P$ and $\angle F = \angle Q$

17. (i) (b) : If ΔAED and ΔBEC, are similar by SAS similarity rule, then their corresponding proportional sides are $\dfrac{BE}{AE} = \dfrac{CE}{DE}$

(ii) (a) : Since, ΔADE and ΔBCE are similar.

$$\therefore \frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \frac{AD}{BC}$$

$$\Rightarrow \frac{2}{3} = \frac{AD}{5} \Rightarrow AD = \frac{5 \times 2}{3} = \frac{10}{3} \text{ cm}$$

(iii) (b) : $\dfrac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \dfrac{ED}{CE}$

$$\Rightarrow \frac{2}{3} = \frac{ED}{4} \Rightarrow ED = \frac{4 \times 2}{3} = \frac{8}{3} \text{ cm}$$

(iv) (a) : $CD = CE + ED = 4 + \dfrac{8}{3} = \dfrac{12+8}{3} = \dfrac{20}{3}$ cm

(v) (d) : $\dfrac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta BCE} = \dfrac{AE}{BE} \Rightarrow \dfrac{2}{3} BE = AE$

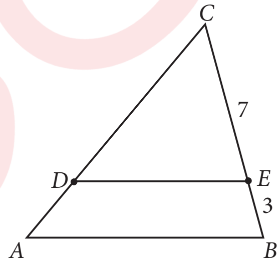
$$\Rightarrow AE = \frac{2}{3}\sqrt{BC^2 - CE^2}$$

Also, in ΔAED, $AE = \sqrt{AD^2 - DE^2}$

18. (i) Let ΔABC is the triangle formed by both hotels and mountain top. ΔCDE is the triangle formed by both huts and mountain top.

Clearly, $DE \parallel AB$ and so

$\Delta ABC \sim \Delta DEC$ [By AA-similarity criterion]



Now, required ratio = Ratio of their corresponding sides

$= \dfrac{BC}{EC} = \dfrac{10}{7}$ *i.e.*, 10 : 7.

(ii) Since, $DE \parallel AB$, therefore

$$\frac{CD}{AD} = \frac{CE}{EB} \Rightarrow \frac{10}{AD} = \frac{7}{3} \Rightarrow AD = \frac{10 \times 3}{7} = 4.29 \text{ miles}$$

(iii) Since, $\Delta ABC \sim \Delta DEC$

$\therefore \dfrac{BC}{EC} = \dfrac{AB}{DE}$ [$\because$ Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{10}{7} = \frac{AB}{8} \Rightarrow AB = \frac{80}{7} = 11.43 \text{ miles}$$

(iv) Given, $DC = 5 + BC$.

Clearly, $BC = 10 - 5 = 5$ miles

Now, $CE = \dfrac{7}{10} \times BC = \dfrac{7}{10} \times 5 = 3.5$ miles

(v) When corresponding angles of two triangles are equal, then they are known as equiangular triangle.

19. In ΔACF, $BP \parallel CF$

$\therefore \dfrac{AB}{BC} = \dfrac{AP}{PF}$ [By B.P.T.]

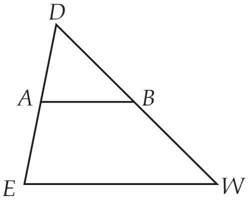
$\Rightarrow \dfrac{2}{8-2} = \dfrac{AP}{PF} \Rightarrow \dfrac{AP}{PF} = \dfrac{1}{3}$ ...(i)

In ΔAEF, $DP \parallel EF$

$\therefore \quad \frac{AD}{DE} = \frac{AP}{PF}$ [By B.P.T.]

$\Rightarrow \quad \frac{AD}{DE} = \frac{1}{3}$ [Using (i)]

20. In ΔDEW, $AB \parallel EW$,



$\therefore \quad \frac{DA}{AE} = \frac{DB}{BW}$ [By B.P.T.]

$\Rightarrow \quad \frac{DA}{DE - AD} = \frac{DB}{DW - DB}$

$\Rightarrow \quad \frac{4}{12 - 4} = \frac{DB}{24 - DB}$

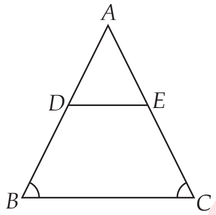
[DA = 4 cm, DE = 12 cm, DW = 24 cm]

$\Rightarrow \quad \frac{4}{8} = \frac{DB}{24 - DB} \Rightarrow \frac{1}{2} = \frac{DB}{24 - DB}$

$\Rightarrow \quad 24 - DB = 2DB \Rightarrow 24 = 3DB$

$\Rightarrow \quad DB = 24/3 = 8$ cm

21. In ΔABC, we have



$\angle B = \angle C \quad \Rightarrow \quad AC = AB$

$\Rightarrow \quad AE + EC = AD + DB$

$\Rightarrow \quad AE + CE = AD + CE$

[$\because BD = CE$]

$\Rightarrow \quad AE = AD$

Thus, we have

$AD = AE$ and $BD = CE$

$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE}$

$\Rightarrow \quad DE \parallel BC$ [By the converse of B.P.T.]

22. Here, the corresponding two sides and the perimeters of two triangles are proportional, then third side of both triangles will also in proportion.

Yes, the two triangles are similar.

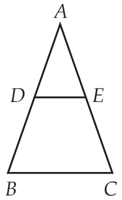
23. In ΔADE and ΔABC, $DE \parallel BC$

$\Rightarrow \quad \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$

[Corresponding angles]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$\Rightarrow \quad \frac{AB}{AD} = \frac{BC}{DE}$ ...(i)



Given, $\frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{DB}{AD} = \frac{3}{2}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{3}{2} + 1 \Rightarrow \frac{DB + AD}{AD} = \frac{3+2}{2}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{5}{2}$ ...(ii)

From (i) and (ii), we get $\frac{BC}{DE} = \frac{5}{2}$

24. Since, $AB \parallel DC$

$\therefore \quad \angle OAB = \angle OCQ$ [Alternate angles]

and $\angle APO = \angle OQC$ [Alternate angles]

Now, in ΔOAP and ΔOCQ,

$\angle OAP = \angle OCQ$ [Proved above]

$\angle APO = \angle OQC$ [Proved above]

$\angle AOP = \angle QOC$ [Vertically opposite angles]

$\therefore \quad \Delta OAP \sim \Delta OCQ$ [By AAA similarity criterion]

$\Rightarrow \quad \frac{OA}{OC} = \frac{OP}{OQ} = \frac{AP}{CQ}$

$\Rightarrow \quad OA \cdot CQ = OC \cdot AP$

25. In ΔABC, $\angle A + \angle B + \angle C = 180°$

[By angle sum property]

$\Rightarrow \quad \angle A = 180° - 30° - 20° = 130°$

Also, $\frac{DE}{AC} = \frac{7}{63} = \frac{1}{9}$ and $\frac{EF}{AB} = \frac{5}{45} = \frac{1}{9}$

Now, in ΔABC and ΔEFD,

$\angle A = \angle E = 130°$

$\frac{DE}{AC} = \frac{EF}{AB}$

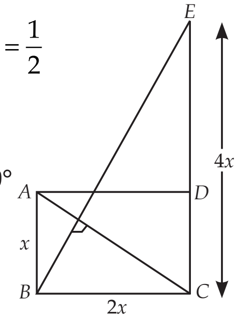
$\therefore \quad \Delta ABC \sim \Delta EFD$ [By SAS similarity criterion]

$\Rightarrow \quad \angle A = \angle E, \angle B = \angle F, \angle C = \angle D$

$\therefore \quad \angle D = 20°$ and $\angle F = 30°$.

26. Let $AB = x$

$\Rightarrow \quad BC = 2x$ and $CE = 4x$



Now, in ΔABC and ΔBCE, $\frac{AB}{BC} = \frac{x}{2x} = \frac{1}{2}$

$\frac{BC}{CE} = \frac{2x}{4x} = \frac{1}{2}$

$\therefore \quad \frac{AB}{BC} = \frac{BC}{CE} = \frac{1}{2}$ and $\angle B = \angle C = 90°$

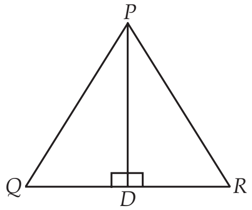
$\therefore \quad \Delta ABC \sim \Delta BCE$

[By SAS similarity criterion]

$\therefore \quad \frac{AB}{BC} = \frac{BC}{CE} = \frac{AC}{BE}$

$\Rightarrow \quad \frac{AC}{BE} = \frac{1}{2}$ or $AC : BE = 1 : 2$

27. False



In ΔPQD and ΔRPD,

$PD = PD$ [common side]

$\angle PDQ = \angle PDR$ [each 90°]

Here, no other sides or angles are equal, so we can say that ΔPQD is not similar to ΔRPD.

But if $\angle P = 90°$, then $\angle DPQ = \angle PRD$

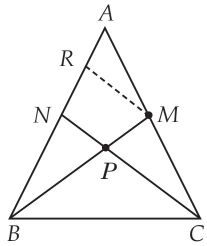
[Each equal to 90° – $\angle Q$ and by *ASA* similarity criterion,

$\Delta PQD \sim \Delta RPD$]

28. Given, $AM : MC = 3 : 4$, $BP : PM = 3 : 2$ and $BN = 12$ cm

Draw MR parallel to CN which meets AB at the point R.

In ΔBMR, $PN \parallel MR$



$\therefore \quad \frac{BN}{NR} = \frac{BP}{PM}$ [By B.P.T.]

$\Rightarrow \quad \frac{12}{NR} = \frac{3}{2} \Rightarrow NR = \frac{12 \times 2}{3} = 8$ cm

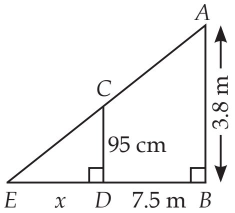
In ΔANC, $RM \parallel NC$

$\therefore \quad \frac{AR}{RN} = \frac{AM}{MC}$ [By B.P.T.]

$\Rightarrow \quad \frac{AR}{8} = \frac{3}{4} \Rightarrow AR = \frac{3 \times 8}{4} = 6$ cm

$\therefore \quad AN = AR + RN = 6 + 8 = 14$ cm

OR



Let AB be the lamp post and CD be the boy after walking 5 seconds. Let $DE = x$ m be the length of his shadow such that $BD = 1.5 \times 5 = 7.5$ m.

In ΔABE and ΔCDE,

$\angle B = \angle D$ [Each equals 90°]

$\angle E = \angle E$ [Common]

$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$\Rightarrow \quad \frac{BE}{DE} = \frac{AB}{CD} \Rightarrow \frac{7.5 + x}{x} = \frac{3.8}{0.95}$

[$\because AB = 3.8$ m, $CD = 95$ cm $= 0.95$ m and $BE = BD + DE = (7.5 + x)$m]

$\Rightarrow \quad 7.125 + 0.95x = 3.8x \Rightarrow 7.125 = 3.8x - 0.95x$

$\Rightarrow \quad 7.125 = 2.85x \Rightarrow x = 7.125 \div 2.85 \Rightarrow x = 2.5$

Hence, the length of his shadow after 5 seconds is 2.5 m.

29. Since, $\Delta NSQ \cong \Delta MTR$

$\therefore \quad \angle SQN = \angle TRM$

$\Rightarrow \quad \angle Q = \angle R$

In ΔPQR,

$\angle P + \angle Q + \angle R = 180°$ [By angle sum property]

$\Rightarrow \quad \angle Q + \angle Q = 180° - \angle P$

$\Rightarrow \angle Q = \frac{1}{2}(180° - \angle P) \Rightarrow \angle Q = \angle R = 90° - \frac{1}{2}\angle P$ ...(i)

Again, in ΔPST, $\angle 1 = \angle 2$ [Given]

and $\angle P + \angle 1 + \angle 2 = 180°$ [By angle sum property]

$\Rightarrow \quad \angle 1 + \angle 1 = 180° - \angle P$

$\Rightarrow \quad \angle 1 = \frac{1}{2}(180° - \angle P)$

$\Rightarrow \quad \angle 1 = \angle 2 = 90° - \frac{1}{2}\angle P$ ...(ii)

Now, in ΔPTS and ΔPRQ

$\angle 1 = \angle Q$ [From (i) and (ii)]

$\angle P = \angle P$ [Common]

$\therefore \quad \Delta PTS \sim \Delta PRQ$ [By AA similarity criterion]

30. Given, in ΔABC, $AB = 4$ cm, and in ΔDEF, $DE = 6$ cm. $EF = 9$ cm and $FD = 12$ cm

Also, $\Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Rightarrow \quad \frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$

On taking first two terms, we get

$\frac{4}{6} = \frac{BC}{9} \Rightarrow BC = \frac{4 \times 9}{6} = 6$ cm

On taking first and last terms, we get

$\frac{4}{6} = \frac{AC}{12}$

$\Rightarrow \quad AC = \frac{4 \times 12}{6} = 8$ cm

Now, perimeter of $\Delta ABC = AB + BC + AC$
$= 4 + 6 + 8 = 18$ cm

31. Since, $DEFG$ is a square.

$\therefore \quad \angle BDG = 90° = \angle FEC$

Also, $DG = GF = FE = DE$ ...(i)

In ΔBAC and ΔBDG,

$\angle ABC = \angle DBG$ (Common)

$\angle BAC = \angle BDG$ (Each 90°)

$\therefore \quad \Delta BAC \sim \Delta BDG$ (By AA similarity criterion)

$\Rightarrow \quad \frac{AB}{BD} = \frac{AC}{DG} \Rightarrow \frac{AB}{AC} = \frac{BD}{DG}$ ...(ii)

In ΔBAC and ΔFEC,

$\angle ACB = \angle ECF$ (Common)

$\angle BAC = \angle FEC$ (Each 90°)

$\therefore \quad \Delta BAC \sim \Delta FEC$ (By AA similarity criterion)

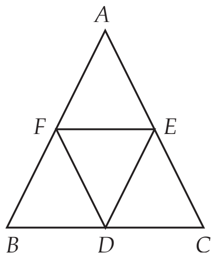
$\Rightarrow \quad \frac{AB}{FE} = \frac{AC}{EC} \Rightarrow \frac{AB}{AC} = \frac{FE}{EC}$ ...(iii)

From (ii) and (iii), we get

$\frac{BD}{DG} = \frac{FE}{EC} \Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$ [Using (i)]

$\Rightarrow \quad DE^2 = BD \times EC$

32. Given, ΔABC in which D, E and F are the mid-points of sides BC, CA and AB respectively.



Since, F and E are mid-points of AB and AC respectively.

$\therefore \quad FE \parallel BC$

$\Rightarrow \quad \angle AFE = \angle B$ [Corresponding angles]

Thus, in ΔAFE and ΔABC, we have

$\angle AFE = \angle B$ [Proved above]

and $\angle A = \angle A$ [Common]

$\therefore \quad \Delta AFE \sim \Delta ABC$ [By AA similarity criterion]

Similarly, we have

$\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall prove that $\Delta DEF \sim \Delta ABC$.

Clearly, $ED \parallel AF$ and $DF \parallel EA$.

$\therefore \quad AFDE$ is a parallelogram.

$\Rightarrow \quad \angle EDF = \angle A$

[$\because$ Opposite angles of a parallelogram are equal]

Similarly, $BDEF$ is a parallelogram.

$\therefore \quad \angle DEF = \angle B$

Thus, in ΔDEF and ΔABC, we have

$\angle EDF = \angle A$ and $\angle DEF = \angle B$

$\therefore \quad \Delta DEF \sim \Delta ABC$ [By AA similarity criterion]

Thus, each one of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

33. In ΔDFG and ΔDAB,

$AB \parallel FE \Rightarrow \angle 1 = \angle 2$ [Corresponding angles]

$\angle FDG = \angle ADB$ [Common]

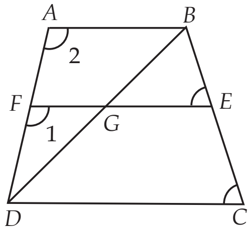
$\therefore \quad \Delta DFG \sim \Delta DAB$ [By AA similarity criterion]

$\Rightarrow \quad \frac{DF}{DA} = \frac{FG}{AB}$ ...(i)

In trapezium $ABCD$, we have

$EF \parallel AB \parallel DC$

$\therefore \quad \frac{AF}{DF} = \frac{BE}{EC} \Rightarrow \frac{AF}{DF} = \frac{3}{4}$ $\left[\because \frac{BE}{EC} = \frac{3}{4}\text{(given)}\right]$



$\Rightarrow \quad \dfrac{AF}{DF}+1=\dfrac{3}{4}+1$

$\Rightarrow \quad \dfrac{AF+DF}{DF}=\dfrac{3+4}{4}$

$\Rightarrow \quad \dfrac{AD}{DF}=\dfrac{7}{4} \Rightarrow \dfrac{DF}{AD}=\dfrac{4}{7}$ ...(ii)

From (i) and (ii), we get

$\dfrac{FG}{AB}=\dfrac{4}{7} \Rightarrow FG=\dfrac{4}{7}AB$ ...(iii)

In ΔBEG and ΔBCD,

$EF \parallel CD \Rightarrow \angle BEG = \angle BCD$ [Corresponding angles]

$\angle B = \angle B$ [Common]

$\therefore \quad \Delta BEG \sim \Delta BCD$ [By AA similarity criterion]

$\Rightarrow \quad \dfrac{BE}{BC}=\dfrac{EG}{CD} \Rightarrow \dfrac{3}{7}=\dfrac{EG}{CD}$

$\left[\because \dfrac{BE}{EC}=\dfrac{3}{4} \Rightarrow \dfrac{EC}{BE}=\dfrac{4}{3} \Rightarrow \dfrac{EC}{BE}+1=\dfrac{4}{3}+1 \Rightarrow \dfrac{BC}{BE}=\dfrac{7}{3}\right]$

$\Rightarrow \quad EG=\dfrac{3}{7}CD \Rightarrow EG=\dfrac{3}{7}\times 2AB$ [$\because CD = 2AB$ (Given)]

$\Rightarrow \quad EG=\dfrac{6}{7}AB$ ...(iv)

Adding (iii) and (iv), we get

$FG+EG=\dfrac{4}{7}AB+\dfrac{6}{7}AB \Rightarrow FE=\dfrac{10}{7}AB$

$\Rightarrow \quad 7\,FE = 10\,AB$

34. **(i)** In ΔABC, we have $DE \parallel BC$

$\Rightarrow \quad \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ [Corresponding angles]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity criterion]

$\Rightarrow \quad \dfrac{AD}{AB}=\dfrac{DE}{BC}$ ...(i)

Now, $\dfrac{AD}{DB}=\dfrac{5}{4} \Rightarrow \dfrac{DB}{AD}=\dfrac{4}{5} \Rightarrow \dfrac{DB}{AD}+1=\dfrac{4}{5}+1$

$\Rightarrow \quad \dfrac{DB+AD}{AD}=\dfrac{4+5}{5} \Rightarrow \dfrac{AB}{AD}=\dfrac{9}{5}$

$\Rightarrow \quad \dfrac{AD}{AB}=\dfrac{5}{9}$

$\Rightarrow \quad \dfrac{DE}{BC}=\dfrac{5}{9}$ [Using (i)]

(ii) In ΔDEF and ΔCBF, we have

$\angle DFE = \angle CFB$ [Vertically opposite angles]

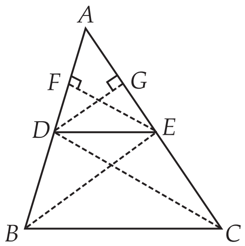
$\angle DEF = \angle FBC$ [Alternate angles]

$\therefore \quad \Delta DEF \sim \Delta CBF$ [By AA similarity criterion]

OR

Given : A triangle ABC in which $DE \parallel BC$ and DE intersects AB at D and AC at E.

To Prove : $\dfrac{AD}{DB}=\dfrac{AE}{EC}$



Construction : Join BE, CD and draw $EF \perp AB$, $DG \perp AC$.

Proof : In ΔEAD and ΔEDB, EF is perpendicular to AB, therefore, EF is the height for both triangles EAD and EDB.

Now, area of $\Delta EAD = \dfrac{1}{2}\times(\text{Base}\times\text{height}) = \dfrac{1}{2}\times AD\times EF$

Again, area of $\Delta EDB = \dfrac{1}{2}\times(\text{Base}\times\text{height}) = \dfrac{1}{2}\times DB\times EF$

$\therefore \quad \dfrac{\text{ar}(\Delta EAD)}{\text{ar}(\Delta EDB)}=\dfrac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times DB\times EF}=\dfrac{AD}{DB}$ ...(i)

Similarly, $\dfrac{\text{ar}(\Delta EAD)}{\text{ar}(\Delta ECD)}=\dfrac{\frac{1}{2}\times AE\times DG}{\frac{1}{2}\times EC\times DG}=\dfrac{AE}{EC}$ ...(ii)

Since, triangles EDB and ECD are on the same base DE and between the same parallel lines DE and BC.

$\therefore \quad \text{ar}(\Delta EDB) = \text{ar}(\Delta ECD)$ ...(iii)

From (i), (ii) and (iii), we get $\dfrac{AD}{DB}=\dfrac{AE}{EC}$

Now, we are given that $AB \parallel DE$ and $BC \parallel EF$.

In ΔOED, $AB \parallel DE$

$\therefore \quad \dfrac{OA}{AD}=\dfrac{OB}{BE}$ ..(i)

In ΔOEF, $BC \parallel EF$

$\therefore \quad \dfrac{OB}{BE}=\dfrac{OC}{CF}$ ...(ii)

From (i) and (ii), we get $\dfrac{OA}{AD}=\dfrac{OC}{CF}$

$\therefore \quad AC \parallel DF$ (By converse of above proved result)

35. In order to prove that the points B, C, E and D are concyclic, it is sufficient to show that

$\angle ABC + \angle CED = 180°$ and $\angle ACB + \angle BDE = 180°$.

In ΔABC, we have $AB = AC$ and $AD = AE$

$\Rightarrow \quad AB - AD = AC - AE \Rightarrow DB = EC$

Thus, we have $AD = AE$ and $DB = EC$

$\Rightarrow \quad \dfrac{AD}{DB}=\dfrac{AE}{EC}$

$\Rightarrow \quad DE \parallel BC$ (By converse of Thales theorem)

$\Rightarrow \quad \angle ABC = \angle ADE$ (Corresponding angles)

$\Rightarrow \quad \angle ABC + \angle BDE = \angle ADE + \angle BDE$ (Adding $\angle BDE$ on both sides)

$\Rightarrow \quad \angle ABC + \angle BDE = 180°$ (Linear pair)

$\Rightarrow \quad \angle ACB + \angle BDE = 180°$ ($\because AB = AC \therefore \angle ABC = \angle ACB$)

Now, $\angle ACB = \angle AED$ (Corresponding angles as $DE \parallel BC$)

$\Rightarrow \quad \angle ACB + \angle CED = \angle AED + \angle CED$ (Adding $\angle CED$ on both sides)

$\Rightarrow \quad \angle ACB + \angle CED = 180°$ (Linear pair)

$\Rightarrow \quad \angle ABC + \angle CED = 180°$ ($\because \angle ABC = \angle ACB$)

Thus, $BDEC$ is a quadrilateral such that

$\angle ACB + \angle BDE = 180°$ and $\angle ABC + \angle CED = 180°$

Therefore, $BDEC$ is a cyclic quadrilateral. Hence, B, C, E and D are concyclic points.