

# Triangles

## CHAPTER 6



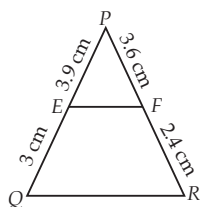
### SOLUTIONS

#### EXERCISE - 6.1

- All circles are similar.
  - All squares are similar.
  - All equilateral triangles are similar.
  - Two polygons of the same number of sides are similar, if
    - their corresponding angles are equal and
    - their corresponding sides are proportional.
- (a) Any two circles are similar figures.
  - (b) Any two squares are similar figures.
- (a) A circle and a triangle are non-similar figures.
  - (b) An isosceles triangle and a scalene triangle are non-similar figures.
- On observing the given figures, we find that their corresponding sides are proportional but their corresponding angles are not equal.  
 $\therefore$  The given figures are not similar.

#### EXERCISE - 6.2

- Since  $DE \parallel BC$  [Given]  
 $\therefore$  Using the basic proportionality theorem, we have  
 $\frac{AD}{DB} = \frac{AE}{EC}$   
 Since,  $AD = 1.5$  cm,  $DB = 3$  cm and  $AE = 1$  cm  
 $\therefore \frac{1.5 \text{ cm}}{3 \text{ cm}} = \frac{1 \text{ cm}}{EC}$   
 $\Rightarrow EC \times 1.5 = 1 \times 3$   
 $\Rightarrow EC = \frac{1 \times 3}{1.5} = \frac{1 \times 3 \times 10}{15} \Rightarrow EC = 2$  cm
  - In  $\triangle ABC$ ,  $DE \parallel BC$   
 $\therefore$  Using the basic proportionality theorem, we have  
 $\frac{AD}{DB} = \frac{AE}{EC}$   
 $\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow AD \times 5.4 = 1.8 \times 7.2$   
 $\Rightarrow AD = \frac{1.8 \times 7.2}{5.4} = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54} = \frac{24}{10} = 2.4$   
 $\therefore AD = 2.4$  cm
- We have,  $PE = 3.9$  cm,  $EQ = 3$  cm,  $PF = 3.6$  cm and  $FR = 2.4$  cm  
 $\therefore \frac{PE}{EQ} = \frac{3.9 \text{ cm}}{3 \text{ cm}} = \frac{1.3}{1}$   
 And  $\frac{PF}{FR} = \frac{3.6 \text{ cm}}{2.4 \text{ cm}} = \frac{1.5}{1}$   
 $\therefore \frac{1.3}{1} \neq \frac{1.5}{1} \therefore \frac{PE}{EQ} \neq \frac{PF}{FR}$



$\Rightarrow EF$  is not parallel to  $QR$ .

- We have,  $PE = 4$  cm,  $QE = 4.5$  cm,  
 $PF = 8$  cm and  $RF = 9$  cm

$$\therefore \frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$$

$$\text{And } \frac{PF}{FR} = \frac{8}{9}$$

$$\text{Since, } \frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow EF$  is parallel to  $QR$ .

- We have,  $PE = 0.18$  cm,  $PQ = 1.28$  cm,  $PF = 0.36$  cm and  $PR = 2.56$  cm

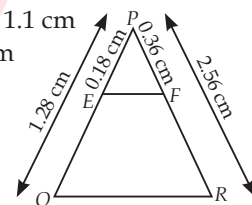
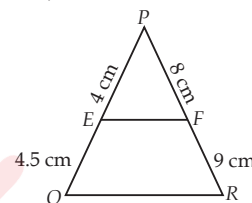
$$\therefore EQ = PQ - PE = 1.28 - 0.18 = 1.1 \text{ cm}$$

$$FR = PR - PF = 2.56 - 0.36 = 2.2 \text{ cm}$$

$$\therefore \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{18}{110} = \frac{9}{55}$$

$$\text{And } \frac{PF}{FR} = \frac{0.36}{2.2} = \frac{36}{220} = \frac{9}{55}$$

$$\text{Since, } \frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow EF \text{ is parallel to } QR.$$



- In  $\triangle ABC$ ,  $LM \parallel CB$  [Given]

$\therefore$  Using the basic proportionality theorem, we have

$$\frac{AM}{MB} = \frac{AL}{LC} \Rightarrow \frac{MB}{AM} + 1 = \frac{LC}{AL} + 1$$

$$\Rightarrow \frac{MB + AM}{AM} = \frac{LC + AL}{AL} \Rightarrow \frac{AB}{AM} = \frac{AC}{AL}$$

$$\Rightarrow \frac{AM}{AB} = \frac{AL}{AC} \quad \dots(i)$$

Similarly, in  $\triangle ACD$ ,  $LN \parallel CD$

$\therefore$  Using the basic proportionality theorem, we have

$$\frac{AL}{AC} = \frac{AN}{AD} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{AM}{AB} = \frac{AL}{AC} = \frac{AN}{AD} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

- In  $\triangle ABC$ ,  $DE \parallel AC$  [Given]

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad [\text{By basic proportionality theorem}] \quad \dots(i)$$

In  $\triangle ABE$ ,  $DF \parallel AE$  [Given]

$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad [\text{By basic proportionality theorem}] \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{BF}{FE} = \frac{BE}{EC}$$

5. In  $\Delta PQO$ ,  $DE \parallel OQ$  [Given]

$\therefore$  Using the basic proportionality theorem, we have  

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \dots(i)$$

Similarly, in  $\Delta POR$ ,  $DF \parallel OR$  [Given]

$\therefore$  Using the basic proportionality theorem, we have  

$$\frac{PD}{DO} = \frac{PF}{FR} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PD}{DO} = \frac{PF}{FR} \Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in  $\Delta PQR$ ,  $E$  and  $F$  are two distinct points on  $PQ$  and  $PR$  respectively and  $\frac{PE}{EQ} = \frac{PF}{FR}$  i.e.,  $E$  and  $F$  divide

the two sides  $PQ$  and  $PR$  in the same ratio.

$\therefore$  By converse of basic proportionality theorem,  $EF \parallel QR$ .

6. In  $\Delta PQR$ ,  $O$  is a point and  $OP$ ,  $OQ$  and  $OR$  are joined. We have points  $A$ ,  $B$ , and  $C$  on  $OP$ ,  $OQ$  and  $OR$  respectively such that  $AB \parallel PQ$  and  $AC \parallel PR$ .

Now, in  $\Delta OPQ$ ,  $AB \parallel PQ$  [Given]

$\therefore$  Using the basic proportionality theorem, we have  

$$\frac{OA}{AP} = \frac{OB}{BQ} \quad \dots(i)$$

Again, in  $\Delta OPR$ ,  $AC \parallel PR$  [Given]

$\therefore$  Using the basic proportionality theorem, we have  

$$\frac{OA}{AP} = \frac{OC}{CR} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OA}{AP} = \frac{OC}{CR} \Rightarrow \frac{OB}{BQ} = \frac{OC}{CR}$$

Now, in  $\Delta OQR$ ,  $B$  is a point on  $OQ$ ,  $C$  is a point on  $OR$

and  $\frac{OB}{BQ} = \frac{OC}{CR}$

i.e.,  $B$  and  $C$  divide the sides  $OQ$  and  $OR$  in the same ratio

$\therefore BC \parallel QR$

[By converse of basic proportionality theorem]

7. Given,  $\Delta ABC$ , in which  $D$  is the mid-point of  $AB$  and  $E$  is a point on  $AC$  such that  $DE \parallel BC$ .

$\therefore$  Using basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \dots(i)$$

But  $D$  is the mid-point of  $AB$

$\therefore AD = DB$

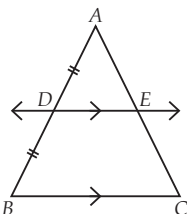
$$\Rightarrow \frac{AD}{DB} = 1 \quad \dots(ii)$$

From (i) and (ii), we get

$$1 = \frac{AE}{EC} \Rightarrow EC = AE$$

$\Rightarrow E$  is the mid-point of  $AC$ . Hence, it is proved that a line through the mid-point of one side of a triangle parallel to another side bisects the third side.

8. We have  $\Delta ABC$ , in which  $D$  and  $E$  are the mid-points of sides  $AB$  and  $AC$  respectively.



$\therefore AD = DB$  and  $AE = EC$

$$\Rightarrow \frac{AD}{DB} = 1 \text{ and } \frac{AE}{EC} = 1$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow DE \parallel BC$$

[By converse of basic proportionality theorem]

9. We have, a trapezium  $ABCD$  such that  $AB \parallel DC$ . The diagonals  $AC$  and  $BD$  intersect each other at  $O$ .

Let us draw  $OE$  parallel to either  $AB$  or  $DC$ .

In  $\Delta ADC$ ,  $OE \parallel DC$  [By construction]

$\therefore$  Using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{AO}{CO} \quad \dots(i)$$

In  $\Delta ABD$ ,  $OE \parallel AB$  [By construction]

$\therefore$  Using basic proportionality theorem, we get

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

**Note :** Remember this as a result.

10. It is given that  $\frac{AO}{BO} = \frac{CO}{DO} \Rightarrow \frac{AO}{CO} = \frac{BO}{DO} \quad \dots(i)$

Through  $O$ , draw  $OE \parallel BA$

In  $\Delta ADB$ ,  $OE \parallel AB$  [By construction]

$\therefore$  Using basic proportionality theorem, we get

$$\frac{DE}{EA} = \frac{DO}{BO} \Rightarrow \frac{EA}{DE} = \frac{BO}{DO} \quad \dots(ii)$$

From (i) and (ii), we have

$$\frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$$

i.e., the points  $O$  and  $E$  on the sides  $AC$  and  $AD$  (of  $\Delta ADC$ ) respectively are in the same ratio.

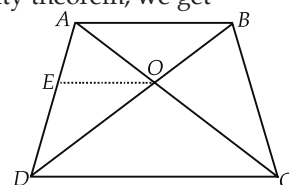
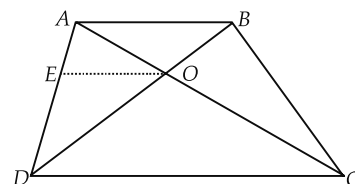
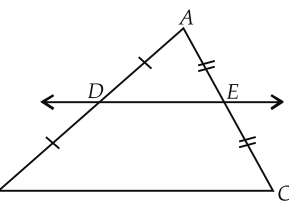
$\therefore$  Using basic proportionality theorem, we get

$$OE \parallel DC$$

Also,  $OE \parallel AB$

$$\Rightarrow AB \parallel DC$$

$$\Rightarrow ABCD \text{ is a trapezium.}$$



### EXERCISE - 6.3

1. (i) In  $\Delta ABC$  and  $\Delta PQR$ ,

$$\angle A = \angle P = 60^\circ$$

$$\angle B = \angle Q = 80^\circ$$

$$\angle C = \angle R = 40^\circ$$

∴ The corresponding angles are equal.

∴  $\triangle ABC \sim \triangle PQR$  [By AAA similarity criterion]

(ii) In  $\triangle ABC$  and  $\triangle QRP$ ,

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2},$$

$$\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2} \Rightarrow \frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

∴  $\triangle ABC \sim \triangle QRP$  [By SSS similarity criterion]

(iii) In  $\triangle PML$  and  $\triangle DEF$ ,

$$\frac{PM}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{ML}{EF} = \frac{2.7}{5} = \frac{27}{50}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \frac{PM}{DE} \neq \frac{ML}{EF} \neq \frac{LP}{DF} \Rightarrow \text{Triangles are not similar.}$$

(iv) In  $\triangle MNL$  and  $\triangle QPR$ ,

$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}, \frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2} \text{ and}$$

$$\angle NML = \angle PQR = 70^\circ$$

∴  $\triangle MNL \sim \triangle QPR$  [By SAS similarity criterion]

(v) In  $\triangle ABC$  and  $\triangle FDE$ ,

$$\angle A = \angle F = 80^\circ$$

Here,  $\frac{AB}{AC}$  and  $\frac{FD}{DE}$  are unknown.

∴ The triangles cannot be said similar.

(vi) In  $\triangle DEF$  and  $\triangle PQR$ ,  $\angle D = \angle P = 70^\circ$

$$[\because \angle P = 180^\circ - (80^\circ + 30^\circ) = 180^\circ - 110^\circ = 70^\circ]$$

$$\angle E = \angle Q = 80^\circ$$

$$\angle F = \angle R = 30^\circ [\because \angle F = 180^\circ - (80^\circ + 70^\circ) = 30^\circ]$$

∴  $\triangle DEF \sim \triangle PQR$  [By AAA similarity criterion]

2. We have,  $\angle BOC = 125^\circ$  and  $\angle CDO = 70^\circ$

Since,  $\angle DOC + \angle BOC = 180^\circ$

[Linear pair]

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ \quad \dots(i)$$

Using the angle sum property in  $\triangle ODC$ , we get

$$\angle DOC + \angle ODC + \angle DCO = 180^\circ$$

$$\Rightarrow 55^\circ + 70^\circ + \angle DCO = 180^\circ$$

$$\Rightarrow \angle DCO = 180^\circ - 55^\circ - 70^\circ = 55^\circ$$

Also,  $\angle OAB = \angle DCO = 55^\circ \quad \dots(ii)$

[Corresponding angles of similar triangles]

Thus, from (i) and (ii)

$$\angle DOC = 55^\circ, \angle DCO = 55^\circ \text{ and } \angle OAB = 55^\circ.$$

3. We have a trapezium  $ABCD$  in which  $AB \parallel DC$ . The diagonals  $AC$  and  $BD$  intersect at  $O$ .

In  $\triangle OAB$  and  $\triangle OCD$ ,

∴  $AB \parallel DC$  and  $AC$  and  $BD$  are transversals.

$$\therefore \angle OBA = \angle ODC$$

[Alternate angles]

$$\text{and } \angle OAB = \angle OCD$$

[Alternate angles]

$$\therefore \triangle OAB \sim \triangle OCD$$

[By AA similarity criterion]

$$\text{So, } \frac{OB}{OD} = \frac{OA}{OC} \quad [\text{Ratios of corresponding sides of the similar triangles}]$$

4. In  $\triangle PQR$ ,  $\angle 1 = \angle 2$

[Given]

$$\therefore PR = QP \quad \dots(i)$$

[Sides opposite to equal angles are equal]

$$\text{Also, } \frac{QR}{QS} = \frac{QT}{PR} \quad [\text{Given}] \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{QR}{QS} = \frac{QT}{QP} \Rightarrow \frac{QS}{QR} = \frac{QP}{QT} \quad [\text{By taking reciprocals}] \quad \dots(iii)$$

Now, in  $\triangle PQS$  and  $\triangle TQR$ ,

$$\frac{QS}{QR} = \frac{QP}{QT}$$

[From (iii)]

and  $\angle SQP = \angle RQT = \angle 1$

∴  $\triangle PQS \sim \triangle TQR$  [By SAS similarity criterion]

5. In  $\triangle PQR$ ,

$T$  is a point on  $QR$  and  $S$  is a point on  $PR$  such that  $\angle RTS = \angle P$ .

Now, in  $\triangle RPQ$  and  $\triangle RTS$ ,

$$\angle RPQ = \angle RTS$$

$$\angle PRQ = \angle TRS$$

[Given]

[Common]

$$\therefore \triangle RPQ \sim \triangle RTS$$

[By AA similarity criterion]

6. We have,  $\triangle ABE \cong \triangle ACD$

∴ Their corresponding parts are equal,

i.e.,  $AB = AC$ ,  $AE = AD$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{AE}{AD} = 1 \therefore \frac{AB}{AC} = \frac{AE}{AD} \Rightarrow \frac{AB}{AE} = \frac{AC}{AD}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad [\because AE = AD] \quad \dots(i)$$

$$\text{Now in } \triangle ADE \text{ and } \triangle ABC, \frac{AB}{AD} = \frac{AC}{AE} \quad [\text{From (i)}]$$

and  $\angle DAE = \angle BAC$

[common]

$$\therefore \triangle ADE \sim \triangle ABC$$

[By SAS similarity criterion]

7. We have a  $\triangle ABC$  in which altitude  $AD$  and  $CE$  intersect each other at  $P$ .

$$\Rightarrow \angle D = \angle E = 90^\circ \quad \dots(1)$$

(i) In  $\triangle AEP$  and  $\triangle CDP$ ,

$$\angle AEP = \angle CDP$$

[From (1)]

$$\angle EPA = \angle DPC$$

[Vertically opposite angles]

$$\therefore \triangle AEP \sim \triangle CDP$$

[By AA similarity criterion]

(ii) In  $\triangle ABD$  and  $\triangle CBE$ ,

$$\angle ADB = \angle CEB$$

[From (1)]

Also,  $\angle ABD = \angle CBE$

[Common]

$$\therefore \triangle ABD \sim \triangle CBE$$

[By AA similarity criterion]

(iii) In  $\triangle AEP$  and  $\triangle ADB$ ,

$$\angle AEP = \angle ADB$$

[From (1)]

Also,  $\angle EAP = \angle DAB$

[Common]

$$\therefore \triangle AEP \sim \triangle ADB$$

[By AA similarity criterion]

(iv) In  $\triangle PDC$  and  $\triangle BEC$ ,

$$\angle PDC = \angle BEC$$

[From (1)]

Also,  $\angle DCP = \angle ECB$

[Common]

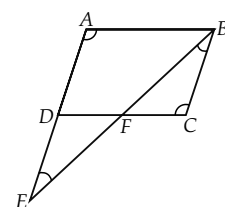
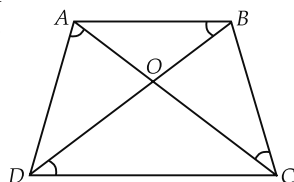
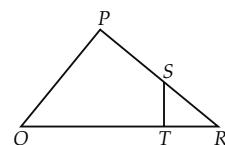
$$\therefore \triangle PDC \sim \triangle BEC$$

[By AA similarity criterion]

8. We have a parallelogram  $ABCD$  in which  $AD$  is produced to  $E$  and  $BE$  is joined such that  $BE$  intersects  $CD$  at  $F$ .

Now, in  $\triangle ABE$  and  $\triangle CFB$ ,

$$\angle BAE = \angle FCB \quad [\text{Opposite angles of a parallelogram}]$$



$$\angle AEB = \angle CBF$$

[Alternate angles, as  $AE \parallel BC$  and  $BE$  is a transversal.]

$$\therefore \triangle ABE \sim \triangle CBF \quad [\text{By AA similarity criterion}]$$

9. We have  $\triangle ABC$ , right angled at  $B$  and  $\triangle AMP$ , right angled at  $M$ .

$$\therefore \angle B = \angle M = 90^\circ \quad \dots(1)$$

(i) In  $\triangle ABC$  and  $\triangle AMP$ ,

$$\angle ABC = \angle AMP \quad [\text{From (1)}]$$

and  $\angle BAC = \angle MAP$  [Common]

$$\therefore \triangle ABC \sim \triangle AMP \quad [\text{By AA similarity criterion}]$$

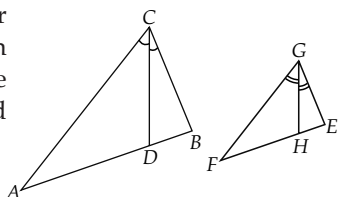
(ii)  $\therefore \triangle ABC \sim \triangle AMP$  [As proved above]

$\therefore$  Their corresponding sides are proportional.

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$$

10. We have, two similar  $\triangle ABC$  and  $\triangle FEG$  such that  $CD$  and  $GH$  are the bisectors of  $\angle ACB$  and  $\angle FGE$  respectively.

(i) In  $\triangle ACD$  and  $\triangle FGH$ ,  
 $\angle A = \angle F$



$$[\because \triangle ABC \sim \triangle FEG] \quad \dots(1)$$

Since  $\triangle ABC \sim \triangle FEG$

$$\therefore \angle C = \angle G \Rightarrow \frac{1}{2} \angle C = \frac{1}{2} \angle G$$

$$\Rightarrow \angle ACD = \angle FGH \quad \dots(2)$$

From (1) and (2), we have

$$\triangle ACD \sim \triangle FGH \quad [\text{By AA similarity criterion}]$$

$\therefore$  Their corresponding sides are proportional.

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

(ii) In  $\triangle DCB$  and  $\triangle HGE$ ,

$$\angle B = \angle E \quad [\because \triangle ABC \sim \triangle FEG] \quad \dots(1)$$

Again,  $\triangle ABC \sim \triangle FEG \Rightarrow \angle ACB = \angle FGE$

$$\therefore \frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

$$\Rightarrow \angle DCB = \angle HGE \quad \dots(2)$$

From (1) and (2), we have

$$\triangle DCB \sim \triangle HGE \quad [\text{By AA similarity criterion}]$$

(iii) In  $\triangle DCA$  and  $\triangle HGF$ ,

$$\therefore \triangle ABC \sim \triangle FEG \Rightarrow \angle CAB = \angle GFE$$

$$\Rightarrow \angle CAD = \angle GFH \Rightarrow \angle DAC = \angle HFG \quad \dots(1)$$

Also,  $\triangle ABC \sim \triangle FEG \Rightarrow \angle ACB = \angle FGE$

$$\therefore \frac{1}{2} \angle ACB = \frac{1}{2} \angle FGE$$

$$\Rightarrow \angle DCA = \angle HGF \quad \dots(2)$$

From (1) and (2), we have

$$\triangle DCA \sim \triangle HGF \quad [\text{By AA similarity criterion}]$$

11. We have an isosceles  $\triangle ABC$  in which  $AB = AC$ .

In  $\triangle ABD$  and  $\triangle ECF$ ,

$$\angle ACB = \angle ABC$$

$$\Rightarrow \angle ECF = \angle ABD \quad \dots(i)$$

Again,  $AD \perp BC$  and  $EF \perp AC$

$$\Rightarrow \angle ADB = \angle EFC = 90^\circ \quad \dots(ii)$$

From (i) and (ii), we have

$$\triangle ABD \sim \triangle ECF \quad [\text{By AA similarity criterion}]$$

12. We have  $\triangle ABC$  and  $\triangle PQR$  in which  $AD$  and  $PM$  are medians corresponding to sides  $BC$  and  $QR$  respectively

$$\text{such that } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{(1/2)BC}{(1/2)QR} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\therefore \triangle ABD \sim \triangle PQM \quad [\text{By SSS similarity criterion}]$$

$\therefore$  Their corresponding angles are equal.

$$\Rightarrow \angle ABD = \angle PQM \Rightarrow \angle ABC = \angle PQR$$

$$\text{Now, in } \triangle ABC \text{ and } \triangle PQR, \frac{AB}{PQ} = \frac{BC}{QR} \quad [\text{Given}]$$

Also,  $\angle ABC = \angle PQR$

$$\therefore \triangle ABC \sim \triangle PQR \quad [\text{By SAS similarity criterion}]$$

13. We have a  $\triangle ABC$  and a point  $D$  on its side  $BC$  such that  $\angle ADC = \angle BAC$ .

In  $\triangle BAC$  and  $\triangle ADC$ ,

$$\angle BAC = \angle ADC$$

[Given]

and  $\angle BCA = \angle ACD$

[Common]

$$\therefore \triangle BAC \sim \triangle ADC \quad [\text{By AA similarity criterion}]$$

$\therefore$  Their corresponding sides are proportional.

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA} \Rightarrow CA \times CA = CB \times CD$$

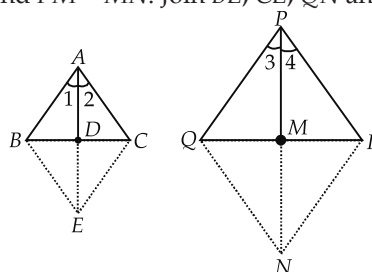
$$\Rightarrow CA^2 = CB \times CD$$

14. **Given :**  $\triangle ABC$  and  $\triangle PQR$  in which  $AD$  and  $PM$  are medians.

$$\text{Also, } \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \dots(i)$$

**To prove :**  $\triangle ABC \sim \triangle PQR$

**Construction :** Produce  $AD$  to  $E$  and  $PM$  to  $N$  such that  $AD = DE$  and  $PM = MN$ . Join  $BE$ ,  $CE$ ,  $QN$  and  $RN$ .



**Proof :** Quadrilaterals  $ABEC$  and  $PQNR$  are parallelograms, since their diagonals bisect each other at point  $D$  and  $M$  respectively.

$$\Rightarrow BE = AC \text{ and } QN = PR$$

$$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ} \quad [\text{From (i)}]$$

$$\text{i.e., } \frac{AB}{PQ} = \frac{BE}{QN} \quad \dots(ii)$$

$$\text{From (i), } \frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AE}{PN} \quad \dots(iii)$$

From (ii) and (iii), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow \triangle ABE \sim \triangle PQN$  [By SSS similarity criterion]

$\Rightarrow \angle 1 = \angle 3$  ... (iv)

Similarly, we can prove

$\triangle ACE \sim \triangle PRN \Rightarrow \angle 2 = \angle 4$  ... (v)

From (iv) and (v),  $\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle A = \angle P$  ... (vi)

Now, in  $\triangle ABC$  and  $\triangle PQR$ , we have

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{[From (i)]}$$

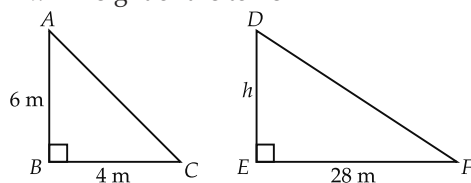
and  $\angle A = \angle P$  [From (vi)]

$\therefore \triangle ABC \sim \triangle PQR$  [By SAS similarity criterion]

**15.** Let  $AB = 6$  m be the pole and  $BC = 4$  m be its shadow (in right  $\triangle ABC$ ), whereas  $DE$  and  $EF$  denote the tower and its shadow respectively.

$EF =$  Length of the shadow of the tower  $= 28$  m

Let  $DE = h =$  Height of the tower



In  $\triangle ABC$  and  $\triangle DEF$ , we have  $\angle B = \angle E = 90^\circ$

$\angle A = \angle D$

[ $\because$  Angular elevation of the sun at the same time is equal]

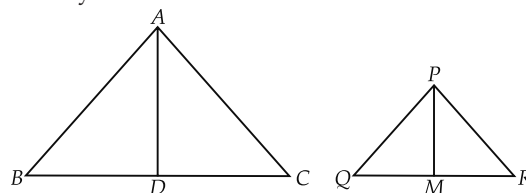
$\therefore \triangle ABC \sim \triangle DEF$  [By AA similarity criterion]

$\therefore$  Their sides are proportional i.e.,  $\frac{AB}{DE} = \frac{BC}{EF}$

$$\Rightarrow \frac{6}{h} = \frac{4}{28} \Rightarrow h = \frac{6 \times 28}{4} = 42 \text{ m}$$

Thus, the required height of the tower is 42 m.

**16.** We have  $\triangle ABC \sim \triangle PQR$  such that  $AD$  and  $PM$  are the medians corresponding to the sides  $BC$  and  $QR$  respectively.



$\therefore \triangle ABC \sim \triangle PQR$

$\Rightarrow$  The corresponding sides of similar triangles are proportional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad \text{... (i)}$$

$\therefore$  Corresponding angles are also equal in two similar triangles.

$\therefore \angle A = \angle P, \angle B = \angle Q$  and  $\angle C = \angle R$  ... (ii)

Since,  $AD$  and  $PM$  are medians.

$\therefore BC = 2BD$  and  $QR = 2QM$

$$\therefore \text{From (i), } \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM} \quad \text{... (iii)}$$

And  $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM$  ... (iv)

From (iii) and (iv), we have

$\triangle ABD \sim \triangle PQM$  [By SAS similarity criterion]

$\therefore$  Their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$



