

Coordinate Geometry

**EXAM
DRILL**

SOLUTIONS

1. (b) : Distance between $A(0, 6)$ and $B(0, -2)$,

$$AB = \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{0+(-8)^2} = \sqrt{8^2} = 8 \text{ units}$$

2. (a) : Let $A(4, p)$ and $B(1, 0)$ be the given points.

Given, $AB = 5$ units

$$\Rightarrow \left(\sqrt{(1-4)^2 + (0-p)^2} \right)^2 = (5)^2$$

$$\Rightarrow 9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$$

3. (a) : Let $A(4, 7)$ be the given point. Suppose the perpendicular from A meet the y -axis at B .

So, the coordinates of B is $(0, 7)$.

$$\therefore AB = \sqrt{(4-0)^2 + (7-7)^2} = \sqrt{16+0} = 4 \text{ units}$$

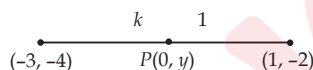
4. (d) : Let a point on x -axis be $(x_1, 0)$, then its distance

$$\text{from the point } (2, 3) = \sqrt{(x_1-2)^2 + 9} = c$$

$$\Rightarrow (x_1-2)^2 = c^2 - 9 \Rightarrow x_1 - 2 = \sqrt{c^2 - 9}$$

$$\text{But } c < 3 \Rightarrow c^2 - 9 < 0 \therefore \text{No real point exist.}$$

5. (c) : Let the point $P(0, y)$ on the y -axis divides the line segment joining the points $(-3, -4)$ and $(1, -2)$ in the ratio $k : 1$.



Using section formula, we have $(0, y) = \left(\frac{k-3}{k+1}, \frac{-2k-4}{k+1} \right)$

$$\Rightarrow \frac{k-3}{k+1} = 0 \Rightarrow k-3=0 \Rightarrow k=3$$

Hence, the required ratio is $3 : 1$.

6. (c) : Let $A(3, -5)$, $B(-7, 4)$ and $C(10, -k)$ be the given points.

Given, centroid of $\triangle ABC = (k, -1)$

$$\Rightarrow \left(\frac{3-7+10}{3}, \frac{-5+4-k}{3} \right) = (k, -1)$$

$$\Rightarrow \left(\frac{6}{3}, \frac{-1-k}{3} \right) = (k, -1) \Rightarrow \frac{6}{3} = k \text{ and } \frac{-1-k}{3} = -1$$

$$\Rightarrow k = 2 \text{ and } k = 3 - 1 = 2$$

7. Let $A(0, 0)$, $B(2, 0)$, $C(0, 3)$ and $D(x, y)$ be the vertices of rectangle $ABCD$.

Since, diagonals of rectangle bisect each other.

\therefore Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{0+0}{2}, \frac{0+3}{2} \right) = \left(\frac{2+x}{2}, \frac{0+y}{2} \right)$$

$$\Rightarrow \left(0, \frac{3}{2} \right) = \left(\frac{2+x}{2}, \frac{y}{2} \right)$$

$$\Rightarrow \frac{2+x}{2} = 0 \text{ and } \frac{y}{2} = \frac{3}{2}$$

$$\Rightarrow x = -2 \text{ and } y = 3$$

8. Coordinate of the centroid G of $\triangle ABC$

$$= \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3} \right) = \left(\frac{-6}{3}, \frac{9}{3} \right) = (-2, 3)$$

Since, G lies on the median, $x - 2y + k = 0$

$$\therefore -2 - 2(3) + k = 0 \Rightarrow -2 - 6 + k = 0 \Rightarrow k = 8$$

9. Let coordinates of P be $(0, y)$ and $A(-5, -2)$, $B(3, 2)$

We have $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (-5-0)^2 + (-2-y)^2 = (3-0)^2 + (2-y)^2$$

$$\Rightarrow 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$$

$$\Rightarrow 8y = -16 \Rightarrow y = -2$$

Coordinates of P are $(0, -2)$

$$PA = \sqrt{25 + 4 + (-2)^2 + (-2)^2} = \sqrt{25} = 5 \text{ cm}$$

10. Required distance

$$= \sqrt{[c+a-(b+c)]^2 + [a+b-(c+a)]^2}$$

$$= \sqrt{[c+a-b-c]^2 + [a+b-c-a]^2}$$

$$= \sqrt{(a-b)^2 + (b-c)^2} = \sqrt{a^2 + 2b^2 + c^2 - 2ab - 2bc} \text{ units}$$

11. Here, point A lies on Y -axis, so its abscissa is zero and given its ordinate is 5, therefore, its coordinates are $A(0, 5)$.

$$\text{Now, } AB = \sqrt{(-5-0)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29} \text{ units}$$

12. Diameter of circle, $d = \sqrt{(2-24)^2 + (23-1)^2}$

$$= \sqrt{(-22)^2 + (22)^2} = \sqrt{(22)^2(1+1)} = 22\sqrt{2} \text{ units}$$

$$\therefore \text{Radius of a circle, } r = \frac{d}{2} = \frac{22\sqrt{2}}{2} = 11\sqrt{2} \text{ units}$$

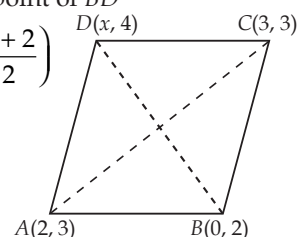
13. As the diagonals of parallelogram bisect each other.

\therefore Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{2+3}{2}, \frac{3+3}{2} \right) = \left(\frac{x+0}{2}, \frac{4+2}{2} \right)$$

$$\Rightarrow \left(\frac{5}{2}, 3 \right) = \left(\frac{x}{2}, 3 \right)$$

$$\Rightarrow \frac{x}{2} = \frac{5}{2} \Rightarrow x = 5$$



14. Let coordinates of P and Q be $(x, 0)$ and $(0, y)$ respectively.

Let $M(-2, -6)$ be the mid-point of PQ .

\therefore By mid-point formula, we have

$$-2 = \frac{x+0}{2} \text{ and } -6 = \frac{0+y}{2} \Rightarrow -4 = x \text{ and } -12 = y$$

\therefore Points are $P(-4, 0)$ and $Q(0, -12)$.

15. Let $A(1, 2)$, $B(4, 3)$ and $C(6, 6)$ and $D(x, y)$ be the vertices of parallelogram.

Since diagonals AC and BD bisect each other.

\therefore Mid-point of BD = Mid-point of AC

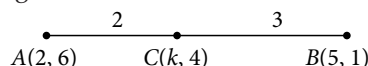
$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2} \right) = \left(\frac{1+6}{2}, \frac{2+6}{2} \right)$$

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2} \right) = \left(\frac{7}{2}, \frac{8}{2} \right)$$

$$\Rightarrow 4+x=7 \text{ and } 3+y=8 \Rightarrow x=3 \text{ and } y=5$$

\therefore Coordinates of D are $(3, 5)$.

16. We have, $A(2, 6)$, $B(5, 1)$ and $C(k, 4)$ divides the given line segment in the ratio $2 : 3$.



Using section formula, we have

$$(k, 4) = \left(\frac{2 \times 5 + 3 \times 2}{2+3}, \frac{2 \times 1 + 3 \times 6}{2+3} \right) = \left(\frac{16}{5}, \frac{20}{5} \right) \Rightarrow k = \frac{16}{5}$$

17. (i) (a) : We have, $OA = 2\sqrt{2}$ km

$$\Rightarrow \sqrt{2^2 + y^2} = 2\sqrt{2}$$

$$\Rightarrow 4 + y^2 = 8 \Rightarrow y^2 = 4$$

$$\Rightarrow y = 2 \quad (\because y = -2 \text{ is not possible})$$

(ii) (c) : We have, $OB = 8\sqrt{2}$

$$\Rightarrow \sqrt{x^2 + 8^2} = 8\sqrt{2}$$

$$\Rightarrow x^2 + 64 = 128 \Rightarrow x^2 = 64$$

$$\Rightarrow x = 8 \quad (\because x = -8 \text{ is not possible})$$

(iii) (c) : Coordinates of A and B are $(2, 2)$ and $(8, 8)$ respectively, therefore coordinates of point M are

$$\left(\frac{2+8}{2}, \frac{2+8}{2} \right), \text{ i.e., } (5, 5)$$

(iv) (d) : Let A divides OM in the ratio $k : 1$.

$$\text{Then, } 2 = \frac{5k+0}{k+1} \Rightarrow 2k+2=5k \Rightarrow 3k=2 \Rightarrow k = \frac{2}{3}$$

\therefore Required ratio = $2 : 3$

(v) (b) : Since M is the mid-point of A and B therefore $AM = MB$. Hence, he should try his luck moving towards B .

18. Consider the house is at origin $(0, 0)$, then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively $(2, 3)$, $(-4, -6)$, $(6, -8)$ and $(-6, 8)$

(i) (d) : Since, grocery store is at $(2, 3)$ and food cart is at $(6, -8)$

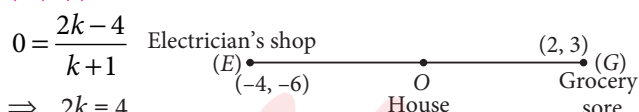
$$\therefore \text{ Required distance} = \sqrt{(6-2)^2 + (-8-3)^2}$$

$$= \sqrt{4^2 + 11^2} = \sqrt{16+121} = \sqrt{137} \text{ cm}$$

(ii) (b) : Required distance

$$= \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ cm}$$

(iii) (c) : Let O divides EG in the ratio $k : 1$, then



$$\Rightarrow 2k = 4$$

$$\Rightarrow k = 2$$

Thus, O divides EG in the ratio $2 : 1$

Hence, required ratio = $OG : OE$ i.e., $1 : 2$

(iv) (c) : Since, $(0, 0)$ is the mid-point of $(-6, 8)$ and $(6, -8)$, therefore both bus stand and food cart are at equal distances from the house.

Hence, required ratio is $1 : 1$.

(v) (d) : Mid-point of grocery store and electrician's

$$\text{shop is } \left(\frac{2-4}{2}, \frac{3-6}{2} \right), \text{ i.e., } \left(-1, \frac{-3}{2} \right)$$

Thus, the diagonals does not bisect each other

[\because Mid-point are not same]

Hence, they form a quadrilateral.

19. (i) The coordinates of point A are $(9, 27)$, therefore its distance from x -axis = 27 units.

(ii) Coordinates of B and C are $(4, 19)$ and $(14, 19)$

$$\therefore \text{ Required distance} = \sqrt{(14-4)^2 + (19-19)^2} \\ = \sqrt{10^2} = 10 \text{ units}$$

(iii) Coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively.

$$\therefore \text{ Required distance} = \sqrt{(16-2)^2 + (6-6)^2} \\ = \sqrt{14^2} = 14 \text{ units}$$

(iv) Since the coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively therefore mid-point of FG is

$$\left(\frac{2+16}{2}, \frac{6+6}{2} \right) = (9, 6)$$

Thus, the mid-point of FG will lie on the line represented by $x = 9$.

(v) Coordinates of L and N are $(6, 4)$ and $(7, 1)$ respectively.

$$\begin{aligned}\text{Length of } LN &= \sqrt{(7-6)^2 + (1-4)^2} \\ &= \sqrt{1+9} = \sqrt{10} \text{ units}\end{aligned}$$

$$\Rightarrow \text{Length of } MP = \sqrt{10} \text{ units}$$

Now, perimeter of $LMPN = LN + LM + MP + NP$

$$= \sqrt{10} + 6 + \sqrt{10} + 4 = (2\sqrt{10} + 10) \text{ units}$$

$$[\because LM = 12 - 6 = 6 \text{ units and } NP = 11 - 7 = 4 \text{ units}]$$

20. (i) Coordinates of Q are $(9, 5)$.

\therefore Distance of point Q from y -axis = 9 units

(ii) Coordinates of point U are $(8, 2)$.

(iii) We have, $P(2, 5)$ and $Q(9, 5)$

$$\therefore PQ = \sqrt{(2-9)^2 + (5-5)^2} = \sqrt{49+0} = 7 \text{ units}$$

(iv) Point $A(x, y)$ is equidistant from $R(3, 8)$ and $T(3, 2)$.

$$\therefore AR = AT \Rightarrow AR^2 = AT^2$$

$$\Rightarrow (x-3)^2 + (y-8)^2 = (x-3)^2 + (y-2)^2$$

$$\Rightarrow y^2 + 64 - 16y = y^2 + 4 - 4y$$

$$\Rightarrow 16y - 4y = 64 - 4 \Rightarrow 12y = 60 \Rightarrow y = 5$$

(v) Length of $TU = 5$ units and of $TL = 2$ units

\therefore Perimeter of image of a rectangular face

$$= 2(5+2) = 14 \text{ units}$$

21. We have, $P(x, y)$, $A(5, 1)$ and $B(-1, 5)$

$$\text{Given, } AP = BP \Rightarrow AP^2 = BP^2$$

$$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$\Rightarrow -10x - 2y = 2x - 10y \Rightarrow -12x = -8y \Rightarrow 3x = 2y$$

22. Point on the x -axis is in the form $(x, 0)$.

$$\therefore (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

$$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$\Rightarrow -8x = 85 - 29 \Rightarrow x = -7$$

\therefore The required point is $(-7, 0)$.

23. Let $A(6, -2)$ and $B(-2, y)$ be the given points.

Length of the line segment $AB = 10$ units (Given)

$$\Rightarrow \sqrt{(-2-6)^2 + (y+2)^2} = 10$$

$$\Rightarrow \sqrt{64 + y^2 + 4y + 4} = 10$$

$$\Rightarrow y^2 + 4y + 68 = 100 \Rightarrow y^2 + 4y - 32 = 0$$

$$\Rightarrow y^2 + 8y - 4y - 32 = 0 \Rightarrow y(y+8) - 4(y+8) = 0$$

$$\Rightarrow (y+8)(y-4) = 0 \Rightarrow y = 4 \text{ or } y = -8$$

So, ordinate of B will be 4 or -8.

OR

The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

Since, A is equidistant from B and C .

$$\therefore AB = AC \Rightarrow AB^2 = AC^2$$

$$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$$

$$\Rightarrow 9 + p^2 + 4 - 4p = p^2 + 9 \Rightarrow 4 - 4p = 0$$

$$\Rightarrow 4p = 4 \Rightarrow p = 1$$

24. Let the coordinates of B be (x, y) .

Using section formula, we have

coordinates of C are $A(2, 7)$ $C(-2, 4)$ $B(x, y)$

$$\left(\frac{4x+5(2)}{4+5}, \frac{4y+5(7)}{4+5} \right) = \left(\frac{4x+10}{9}, \frac{4y+35}{9} \right)$$

Also, the coordinates of C are $(-2, 4)$.

(Given)

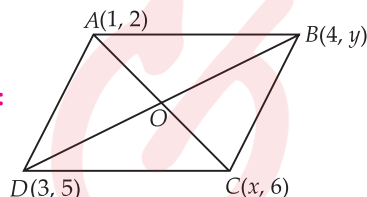
$$\therefore \frac{4x+10}{9} = -2 \text{ and } \frac{4y+35}{9} = 4$$

$$\Rightarrow 4x+10 = -18 \text{ and } 4y+35 = 36$$

$$\Rightarrow 4x = -18 - 10 \text{ and } 4y = 36 - 35$$

$$\Rightarrow 4x = -28 \text{ and } 4y = 1$$

$$\Rightarrow x = -7 \text{ and } y = \frac{1}{4}$$



25. (18) :

Coordinates of O is the mid-point of AC and BD

$$\therefore \frac{x+1}{2} = \frac{3+4}{2} \text{ and } \frac{6+2}{2} = \frac{y+5}{2}$$

$$\Rightarrow x+1 = 7 \text{ and } 8 = y+5 \Rightarrow x = 6 \text{ and } y = 3$$

$$\therefore xy = 6 \times 3 = 18$$

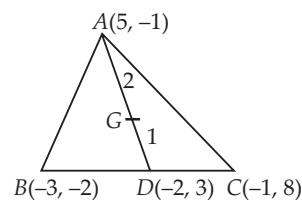
26. Let AD be the median

from the vertex A of $\triangle ABC$.

Then, D is the mid-point of

BC . So, coordinates of D are

$$\left(\frac{-3-1}{2}, \frac{-2+8}{2} \right) \text{ i.e., } (-2, 3).$$



$$\therefore AD = \sqrt{(5+2)^2 + (-1-3)^2} = \sqrt{49+16} = \sqrt{65} \text{ units}$$

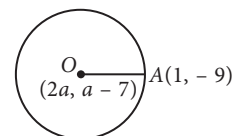
Let G be the centroid of $\triangle ABC$.

$$\text{Coordinates of } G \text{ are } \left(\frac{5-3-1}{3}, \frac{-1-2+8}{3} \right) = \left(\frac{1}{3}, \frac{5}{3} \right).$$

27. (d) : Let $O(2a, a-7)$ be the

centre and $A(1, -9)$ be any point

through which circle passes.



$$\therefore 2(OA) = 10\sqrt{2}$$

[\because Diameter = $2 \times$ radius]

$$\Rightarrow OA = 5\sqrt{2} \Rightarrow OA^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow (2a-1)^2 + (a-7+9)^2 = 50$$

$$\Rightarrow 4a^2 + 1 - 4a + a^2 + 4 + 4a = 50$$

$$\Rightarrow 5a^2 = 50 - 5 = 45 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$$

28. Using distance formula, we have

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(5-7)^2 + (-2+2)^2}$$

$$= \sqrt{4+0} = 2 \text{ units}$$

Now, $AB = AC \neq BC$.

So, $\triangle ABC$ is an isosceles triangle.

Let $D(x, y)$ be the mid-point of BC .

Using mid-point formula, we have

$$(x, y) = \left(\frac{5+7}{2}, \frac{-2-2}{2} \right) = \left(\frac{12}{2}, \frac{-4}{2} \right) = (6, -2)$$

\therefore Coordinates of D are $(6, -2)$

$$\begin{aligned} \therefore \text{Length of median, } AD &= \sqrt{(6-6)^2 + (-2-4)^2} \\ &= \sqrt{0+36} = 6 \text{ units} \end{aligned}$$

OR

Since C is equidistant from A and B .

$$\therefore AC = CB \Rightarrow AC^2 = CB^2$$

$$\Rightarrow (3+2)^2 + (-1-3)^2 = (x+2)^2 + (8-3)^2$$

$$\Rightarrow 25 + 16 = x^2 + 4 + 4x + 25$$

$$\Rightarrow x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x(x+6) - 2(x+6) = 0 \Rightarrow (x+6)(x-2) = 0$$

$$\Rightarrow x = -6 \text{ or } x = 2$$

Now, using distance formula

$$BC = \sqrt{(-6+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$\text{or } \sqrt{(2+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

$$AB = \sqrt{(-6-3)^2 + (8+1)^2} = \sqrt{81+81} = 9\sqrt{2} \text{ units}$$

$$\text{or } \sqrt{(2-3)^2 + (8+1)^2} = \sqrt{1+81} = \sqrt{82} \text{ units}$$

29. Let the given points are $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$.

We know, diagonals of a parallelogram bisect each other.

\therefore Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{-2+4}{2}, \frac{1+b}{2} \right) = \left(\frac{a+1}{2}, \frac{0+2}{2} \right)$$

$$\Rightarrow \left(\frac{2}{2}, \frac{1+b}{2} \right) = \left(\frac{a+1}{2}, \frac{2}{2} \right) \Rightarrow \frac{2}{2} = \frac{a+1}{2}$$

$$\Rightarrow 1+a=2 \Rightarrow a=1 \text{ and } \frac{1+b}{2} = \frac{2}{2}$$

$$\Rightarrow b+1=2 \Rightarrow b=1$$

Hence, $a=1$, $b=1$

Now, $AB = CD$ and $BC = AD$

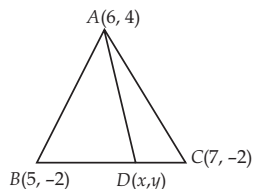
(\because Opposite sides of a parallelogram are equal)

$$\therefore AB = CD = \sqrt{(-2-1)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$$\text{and } BC = AD = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

30. $\therefore O$ is the mid-point of the base BC .

\therefore Coordinates of point B are $(0, 3)$



$$\text{So, } BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{6^2} = 6 \text{ units.}$$

Let the coordinates of point A be $(x, 0)$.

$$\begin{aligned} \therefore AB &= \sqrt{(0-x)^2 + (3-0)^2} \\ &= \sqrt{x^2+9} \end{aligned}$$

Also, $AB = BC$

($\because \triangle ABC$ is an equilateral triangle)

$$\Rightarrow \sqrt{x^2+9} = 6 \Rightarrow x^2+9 = 36$$

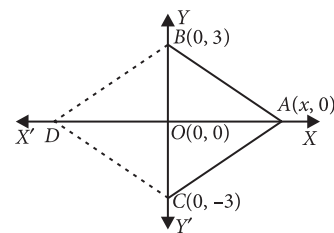
$$\Rightarrow x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

\therefore Coordinates of point $A = (x, 0) = (3\sqrt{3}, 0)$

Since, $BACD$ is a rhombus.

$\therefore AB = AC = CD = DB$

\therefore Coordinates of point $D = (-3\sqrt{3}, 0)$



31. We have $ABCD$ is a rectangle, where AC and BD are its diagonal.

$$\text{Now, } AC = \sqrt{[6-(-4)]^2 + (3-5)^2}$$

$$= \sqrt{(10)^2 + (-2)^2}$$

$$= \sqrt{100+4} = \sqrt{104} \text{ units}$$

$$BD = \sqrt{(-4-6)^2 + (3-5)^2} = \sqrt{(-10)^2 + (-2)^2}$$

$$= \sqrt{100+4} = \sqrt{104} \text{ units}$$

$$\Rightarrow AC = BD$$

Hence, diagonals of rectangle $ABCD$ are equal.

Let O is the mid-point of both AC and BD .

Using mid-point formula, we have

$$\text{coordinates of } O \text{ from } AC = \left(\frac{6+(-4)}{2}, \frac{3+5}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

$$\text{Coordinates of } O \text{ from } BD = \left(\frac{-4+6}{2}, \frac{3+5}{2} \right)$$

$$= \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

$\Rightarrow AC$ and BD bisect each other at O .

OR

Let D , E and F be the

mid-points of the

sides AC , BC and AB

respectively.

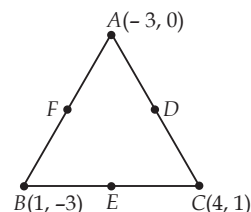
Then the coordinates

of D are

$$\left(\frac{-3+4}{2}, \frac{0+1}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right)$$

$$\text{Coordinates of } E \text{ are } \left(\frac{1+4}{2}, \frac{-3+1}{2} \right) = \left(\frac{5}{2}, \frac{-2}{2} \right) = \left(\frac{5}{2}, -1 \right)$$

$$\text{Coordinates of } F \text{ are } \left(\frac{-3+1}{2}, \frac{0-3}{2} \right) = \left(\frac{-2}{2}, \frac{-3}{2} \right) = \left(-1, \frac{-3}{2} \right)$$



Using distance formula, lengths of medians are

$$\therefore AE = \sqrt{\left[\frac{5}{2} - (-3)\right]^2 + (-1 - 0)^2}$$

$$= \sqrt{\left(\frac{11}{2}\right)^2 + 1} = \sqrt{\frac{121}{4} + 1} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2} \text{ units}$$

$$BD = \sqrt{\left(\frac{1}{2} - 1\right)^2 + \left[\frac{1}{2} - (-3)\right]^2} = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{\sqrt{50}}{2} \text{ units}$$

$$CF = \sqrt{(-1 - 4)^2 + \left(\frac{-3}{2} - 1\right)^2} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100 + 25}{4}}$$

$$= \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2} \text{ units}$$

32. We have, points $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$.

$$\therefore SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2} = \sqrt{a^2t^4 + a^2 - 2a^2t^2 + 4a^2t^2}$$

$$= \sqrt{a^2t^4 + a^2 + 2a^2t^2} = \sqrt{(at^2 + a)^2} = at^2 + a \quad \dots(i)$$

$$SQ = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{-2a}{t} - 0\right)^2} = \sqrt{\frac{a^2}{t^4} + a^2 - \frac{2a^2}{t^2} + \frac{4a^2}{t^2}}$$

$$= \sqrt{\frac{a^2}{t^4} + a^2 + \frac{2a^2}{t^2}} = \sqrt{\left(\frac{a}{t^2} + a\right)^2} = \frac{a}{t^2} + a \quad \dots(ii)$$

$$\text{Now, } \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{at^2 + a} + \frac{1}{\frac{a}{t^2} + a} \quad [\text{Using (i) and (ii)}]$$

$$= \frac{1}{at^2 + a} + \frac{t^2}{a + at^2} = \frac{1 + t^2}{a + at^2} = \frac{1 + t^2}{a(1 + t^2)} = \frac{1}{a}, \text{ which is independent of } t.$$

33. Let $P(1, 1)$, $Q(2, -3)$, $R(3, 4)$ be the mid-points of sides AB , BC and CA respectively of triangle ABC . Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC . Then

P is the mid-point of AB

$$\Rightarrow \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow x_1 + x_2 = 2 \text{ and } y_1 + y_2 = 2$$

Q is the mid-point of BC

$$\Rightarrow \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$$

$$\Rightarrow x_2 + x_3 = 4 \text{ and } y_2 + y_3 = -6$$

R is the mid-point of AC

$$\Rightarrow \frac{x_1 + x_3}{2} = 3 \text{ and } \frac{y_1 + y_3}{2} = 4$$

$$\Rightarrow x_1 + x_3 = 6 \text{ and } y_1 + y_3 = 8$$

From (1), (2) and (3), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$$

$$\text{and } y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$$

$$\Rightarrow x_1 + x_2 + x_3 = 6 \text{ and } y_1 + y_2 + y_3 = 2 \quad \dots(4)$$

The coordinates of the centroid of $\triangle ABC$ are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right) \quad [\text{Using (4)}]$$

34. Let the vertices of an equilateral triangle are $A(-6, 5)$, $B(6, 5)$ and $C(x, y)$.

We know that, in an equilateral triangle all three sides are equal.

$$\therefore AB = BC = CA \Rightarrow AB^2 = BC^2 = CA^2$$

Consider, $AB^2 = BC^2$

$$\Rightarrow (6 + 6)^2 + (5 - 5)^2 = (x - 6)^2 + (y - 5)^2$$

$$\Rightarrow 144 + 0 = x^2 + 36 - 12x + y^2 + 25 - 10y$$

$$\Rightarrow x^2 + y^2 - 12x - 10y + 61 = 144$$

$$\Rightarrow x^2 + y^2 - 12x - 10y = 83 \quad \dots(i)$$

Now, consider $AB^2 = CA^2$

$$\Rightarrow (6 + 6)^2 + (5 - 5)^2 = (x + 6)^2 + (y - 5)^2$$

$$\Rightarrow 144 = 36 + x^2 + 12x + 25 + y^2 - 10y$$

$$\Rightarrow x^2 + y^2 + 12x - 10y + 61 = 144$$

$$\Rightarrow x^2 + y^2 + 12x - 10y = 83 \quad \dots(ii)$$

From (i) and (ii) we get

$$x^2 + y^2 - 12x - 10y = x^2 + y^2 + 12x - 10y$$

$$\Rightarrow 12x + 12x = 0 \Rightarrow 24x = 0 \Rightarrow x = 0$$

Putting $x = 0$ in (i), we get

$$0 + y^2 - 12(0) - 10y = 83$$

$$\Rightarrow y^2 - 10y - 83 = 0$$

$$\therefore y = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-83)}}{2 \times 1}$$

$$\Rightarrow y = \frac{10 \pm \sqrt{100 + 332}}{2}$$

$$\Rightarrow y = \frac{10 \pm \sqrt{432}}{2}$$

$$\therefore y = \frac{10 + \sqrt{432}}{2} \text{ or } y = \frac{10 - \sqrt{432}}{2}$$

$$\text{Hence, the third vertex is } \left(0, \frac{10 + \sqrt{432}}{2}\right) \text{ or } \left(0, \frac{10 - \sqrt{432}}{2}\right).$$

OR

35. Let $PQRS$ be a square and let $P(3, 4)$ and $R(1, -1)$ be the given opposite angular points.

Let $Q(x, y)$ be the unknown vertex. Since, all sides of square are equal.

$$\therefore PQ = QR$$

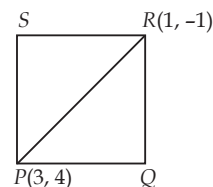
$$\Rightarrow PQ^2 = QR^2$$

$$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 1)^2 + (y + 1)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 + 2y + 1$$

$$\Rightarrow -6x - 8y + 25 = -2x + 2y + 2$$

$$\Rightarrow -6x + 2x - 8y - 2y = 2 - 25$$



$$\Rightarrow -4x - 10y = -23 \Rightarrow 4x + 10y = 23$$

$$\Rightarrow x = \frac{23 - 10y}{4} \quad \dots(i)$$

In right angled triangle PQR , we have

$$PQ^2 + QR^2 = PR^2$$

$$\Rightarrow (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 = (3-1)^2 + (4+1)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 + x^2 - 2x + 1 + y^2 + 2y + 1 = 4 + 25$$

$$\Rightarrow 2x^2 + 2y^2 - 8x - 6y + 27 = 29$$

$$\Rightarrow 2x^2 + 2y^2 - 8x - 6y = 2$$

$$\Rightarrow x^2 + y^2 - 4x - 3y - 1 = 0 \quad \dots(ii)$$

Substitute the value of x from (i) into (ii), we get

$$\left(\frac{23-10y}{4}\right)^2 + y^2 - 4\left(\frac{23-10y}{4}\right) - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 - 23 + 10y - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529+100y^2-460y}{16}\right) + y^2 + 7y - 24 = 0$$

$$\Rightarrow \frac{529+100y^2-460y+16y^2+112y-384}{16} = 0$$

$$\Rightarrow 116y^2 - 348y + 145 = 0$$

$$\Rightarrow 4y^2 - 12y + 5 = 0 \quad [\text{On dividing by 29}]$$

$$\Rightarrow 4y^2 - 10y - 2y + 5 = 0$$

$$\Rightarrow 2y(2y-5) - 1(2y-5) = 0$$

$$\Rightarrow (2y-5)(2y-1) = 0$$

$$\Rightarrow y = \frac{5}{2} \text{ or } y = \frac{1}{2}$$

On putting $y = \frac{1}{2}$ in (i), we get

$$x = \frac{23-10\left(\frac{1}{2}\right)}{4} = \frac{23-5}{4} = \frac{18}{4} = \frac{9}{2}$$

On putting $y = \frac{5}{2}$ in (i) we get

$$x = \frac{23-10\left(\frac{5}{2}\right)}{4} = \frac{23-25}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Hence, the other two vertices are $\left(\frac{9}{2}, \frac{1}{2}\right)$ and $\left(\frac{-1}{2}, \frac{5}{2}\right)$.

35. Given, $\triangle AOB$ is a right angle triangle right angled at O and AB is hypotenuse and C is the mid-point of AB .

Let the coordinates of B and A are $(0, b)$ and $(a, 0)$ respectively.

$$\Rightarrow OA = a \text{ and } OB = b$$

$$\text{So, the coordinates of } C = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$$\text{Now, } CO = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2}$$

$$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units}$$

$$CA = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units}$$

$$CB = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$= \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2} \text{ units}$$

$$\Rightarrow CO = CA = CB$$

Therefore, C is equidistant from the three vertices of $\triangle AOB$.

OR

Let the point $P(x, 2)$ divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $k : 1$.

$$\begin{array}{c} k \qquad \qquad 1 \\ \bullet \text{---} \text{---} \text{---} \bullet \text{---} \bullet \\ A(12, 5) \qquad P(x, 2) \qquad B(4, -3) \end{array}$$

Using section formula, we have

$$\text{coordinates of } P \text{ are } \left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$$

Now, the coordinates of P are $(x, 2)$. (Given)

$$\therefore \frac{4k+12}{k+1} = x \text{ and } \frac{-3k+5}{k+1} = 2$$

$$\Rightarrow -3k+5 = 2k+2 \Rightarrow 5k = 3 \Rightarrow k = 3/5$$

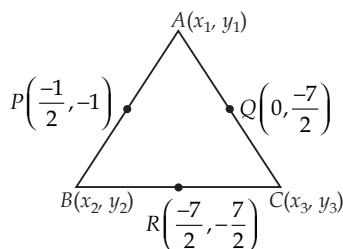
$$\text{Substituting, } k = \frac{3}{5} \text{ in } \frac{4k+12}{k+1} = x, \text{ we get}$$

$$x = \frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1} = \frac{12 + 60}{3 + 5} = \frac{72}{8} = 9$$

Thus, the value of x is 9.

Also, the point P divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $\frac{3}{5} : 1$ i.e., $3 : 5$.

36. Let the coordinates of the vertices A , B and C of $\triangle ABC$ be (x_1, y_1) , (x_2, y_2) and (x_3, y_3) respectively.



As, $P\left(-\frac{1}{2}, -1\right)$ is the mid-point of AB .

$$\therefore \frac{x_1 + x_2}{2} = -\frac{1}{2} \text{ and } \frac{y_1 + y_2}{2} = -1$$

$$\Rightarrow x_1 + x_2 = -1 \text{ and } y_1 + y_2 = -2$$

Point R is the mid-point of BC.

$$\therefore \frac{x_2 + x_3}{2} = \frac{-7}{2} \text{ and } \frac{y_2 + y_3}{2} = \frac{-7}{2}$$

$$\Rightarrow x_2 + x_3 = -7 \text{ and } y_2 + y_3 = -7$$

Point Q is the mid-point of AC.

$$\therefore \frac{x_1 + x_3}{2} = 0 \text{ and } \frac{y_1 + y_3}{2} = \frac{-7}{2}$$

$$\Rightarrow x_1 + x_3 = 0 \text{ and } y_1 + y_3 = -7$$

Adding (i), (ii) and (iii), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = -1 - 7 + 0 \text{ and } y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = -2 - 7 - 7$$

$$\Rightarrow 2(x_1 + x_2 + x_3) = -8 \text{ and } 2(y_1 + y_2 + y_3) = -16$$

$$\dots(i) \Rightarrow x_1 + x_2 + x_3 = -4 \text{ and } y_1 + y_2 + y_3 = -8$$

From (i) and (iv), we get

$$-1 + x_3 = -4 \text{ and } -2 + y_3 = -8$$

$$\Rightarrow x_3 = -3 \text{ and } y_3 = -6$$

$$\dots(ii) \text{ So, coordinates of C are } (-3, -6).$$

From (ii) and (iv) we get

$$x_1 + (-7) = -4 \text{ and } y_1 + (-7) = -8$$

$$\Rightarrow x_1 = 3 \text{ and } y_1 = -1$$

$$\dots(iii) \therefore \text{Coordinates of A are } (3, -1)$$

From (iii) and (iv), we get

$$x_2 + 0 = -4 \text{ and } y_2 + (-7) = -8$$

$$\Rightarrow x_2 = -4 \text{ and } y_2 = -1$$

$$\therefore \text{Coordinates of B are } (-4, -1).$$

$$\dots(iv)$$

