Coordinate Geometry

CHAPTER 7

NCERT FOCUS SOLUTIONS

EXERCISE - 7.1

1. (i) Here, $x_1 = 2, y_1 = 3$ and $x_2 = 4, y_2 = 1$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-2)^2 + (1-3)^2}$

$= \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) Here, $x_1 = -5, y_1 = 7$ and $x_2 = -1, y_2 = 3$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[-1-(-5)]^2 + (3-7)^2}$

$= \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{16+16} = 4\sqrt{2}$ units

(iii) Here $x_1 = a, y_1 = b$ and $x_2 = -a, y_2 = -b$

$\therefore$ The required distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-a-a)^2 + (-b-b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2+b^2)}$

$= 2\sqrt{(a^2+b^2)}$ units

2. Part-I

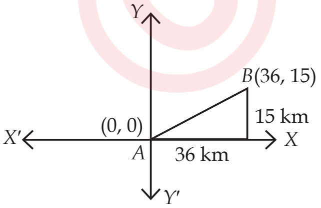
Let the given points be $A(0, 0)$ and $B(36, 15)$.

Then, $AB = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2}$

$= \sqrt{1296 + 225} = \sqrt{1521} = \sqrt{39^2} = 39$ units

Part-II

The given situation can be represented graphically as shown in the figure given below.



We have $A(0, 0)$ and $B(36, 15)$ as the positions of two towns.

Now, $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(36-0)^2 + (15-0)^2} = 39$ km.

3. Let the given points be $A(1, 5)$, $B(2, 3)$ and $C(-2, -11)$. Clearly, A, B and C will be collinear, if

$AB + BC = AC$ or $AC + CB = AB$ or $BA + AC = BC$

Here, $AB = \sqrt{(2-1)^2 + (3-5)^2}$

$= \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5} = 2.24$ units (Approx.)

$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$

$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

$= 14.56$ units (Approx.)

and, $AC = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$ units

$= 16.28$ units (Approx.)

Since, $AB + BC \neq AC$, $AC + CB \neq AB$ and $BA + AC \neq BC$

$\therefore$ A, B and C are not collinear.

4. Let the given points be $A(5, -2)$, $B(6, 4)$ and $C(7, -2)$.

Then, $AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$

$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$ units

$BC = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$ units

and $AC = \sqrt{(7-5)^2 + [-2-(-2)]^2}$

$= \sqrt{(2)^2 + (0)^2} = \sqrt{4+0} = 2$ units

Since, $AB = BC$

$\therefore$ ΔABC is an isosceles triangle.

5. The coordinates of given points are $A(3, 4)$, $B(6, 7)$, $C(9, 4)$ and $D(6, 1)$

$\therefore$ $AB = \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$AD = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{(6)^2 + (0)^2} = 6$ units

and $BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{(0)^2 + (-6)^2} = 6$ units

Since, $AB = BC = CD = AD$ *i.e.*, all the four sides are equal. and also, $BD = AC$ *i.e.*, both the diagonals are also equal.

$\therefore$ $ABCD$ is a square.

Thus, Champa is correct.

6. (i) Let the given points be $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$.

Now, $AB = \sqrt{(1+1)^2 + (0+2)^2}$

$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$ units

$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8}$ units

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$ units

$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8}$ units

$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+4^2} = 4$ units

$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{(-4)^2} = 4$ units

Since, $AB = BC = CD = DA$ *i.e.*, all the sides are equal, and also, $AC = BD$ *i.e.*, the diagonals are also equal.

$\therefore$ $ABCD$ is a square.

(ii) Let the given points be $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$ and $D(-1, -4)$.

Now, $AB = \sqrt{[3-(-3)]^2 + (1-5)^2}$

$= \sqrt{6^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$ units

$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$ units

$DA = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2} = \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{4+81} = \sqrt{85}$ units

$AC = \sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9+4} = \sqrt{13}$ units

and $BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{16+25} = \sqrt{41}$ units

Here, we can see that $\left[\because \sqrt{13} + \sqrt{13} = 2\sqrt{13}\right]$

$AC + BC = AB$

$\Rightarrow$ A, B and C are collinear points. Hence, $ABCD$ is not a quadrilateral.

(iii) Let the given points be $A(4, 5)$, $B(7, 6)$, $C(4, 3)$ and $D(1, 2)$.

Now, $AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ units

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$ units

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$ units

$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18}$ units

$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+(-2)^2} = 2$ units

and $BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52}$ units

Since, $AB = CD$, $BC = DA$ *i.e.*, opposite sides of the given quadrilateral are equal, and also, $AC \neq BD$, *i.e.*, diagonals are unequal.

$\therefore$ $ABCD$ is a parallelogram.

7. We know that any point on x-axis is of the form $(x, 0)$.

$\therefore$ Let the required point be $P(x, 0)$.

Also, let the given points be $A(2, -5)$ and $B(-2, 9)$.

Now, $AP = \sqrt{(x-2)^2 + [0-(-5)]^2}$

$= \sqrt{(x-2)^2 + 5^2} = \sqrt{x^2 - 4x + 4 + 25} = \sqrt{x^2 - 4x + 29}$

and $BP = \sqrt{[x-(-2)]^2 + (0-9)^2}$

$= \sqrt{(x+2)^2 + (-9)^2} = \sqrt{x^2 + 4x + 4 + 81} = \sqrt{x^2 + 4x + 85}$

Since, A and B are equidistant from P.

$\therefore$ $AP = BP$

$\Rightarrow$ $\sqrt{x^2 - 4x + 29} = \sqrt{x^2 + 4x + 85}$

$\Rightarrow$ $x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow$ $-8x = 56$ $\Rightarrow$ $x = \frac{56}{-8} = -7$

$\therefore$ The required point is $(-7, 0)$.

8. The given points are $P(2, -3)$ and $Q(10, y)$.

$\therefore$ $PQ = \sqrt{(10-2)^2 + [y-(-3)]^2}$

$= \sqrt{8^2 + (y+3)^2} = \sqrt{64 + y^2 + 6y + 9} = \sqrt{y^2 + 6y + 73}$

But $PQ = 10$ [Given]

$\therefore$ $\sqrt{y^2 + 6y + 73} = 10$

On squaring both sides, we get $y^2 + 6y + 73 = 100$

$\Rightarrow$ $y^2 + 6y - 27 = 0$

$\Rightarrow$ $y^2 - 3y + 9y - 27 = 0$ $\Rightarrow$ $(y-3)(y+9) = 0$

$\Rightarrow$ $y = 3$ or $y = -9$

$\therefore$ The required values of y are 3 and -9.

9. Here, $QP = \sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{5^2 + (-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$ units

and $QR = \sqrt{(x-0)^2 + (6-1)^2}$

$= \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ units

$\because$ $QP = QR$

$\therefore$ $\sqrt{41} = \sqrt{x^2 + 25}$

On squaring both sides, we get $x^2 + 25 = 41$

$\Rightarrow$ $x^2 = 16$ $\Rightarrow$ $x = \pm 4$

Thus, the point R is $(4, 6)$ or $(-4, 6)$

Now, $QR = \sqrt{[(\pm 4)-(0)]^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

and $PR = \sqrt{(4-5)^2 + (6+3)^2}$ or $\sqrt{(-4-5)^2 + (6+3)^2}$

$\Rightarrow$ $PR = \sqrt{1+81}$ or $\sqrt{81+81}$

$\Rightarrow$ $PR = \sqrt{82}$ units or $9\sqrt{2}$ units

10. Let $A(x, y)$, $B(3, 6)$ and $C(-3, 4)$ be the given points. Now let, the point $A(x, y)$ is equidistant from $B(3, 6)$ and $C(-3, 4)$.
Then, we get $AB = AC$

$$\Rightarrow \sqrt{(3-x)^2+(6-y)^2} = \sqrt{(-3-x)^2+(4-y)^2}$$

On squaring both sides, we get

$$(3-x)^2 + (6-y)^2 = (-3-x)^2 + (4-y)^2$$
$$\Rightarrow 9 + x^2 - 6x + 36 + y^2 - 12y = 9 + x^2 + 6x + 16 + y^2 - 8y$$
$$\Rightarrow -6x - 6x + 36 - 12y - 16 + 8y = 0$$
$$\Rightarrow -12x - 4y + 20 = 0 \Rightarrow -3x - y + 5 = 0$$

$\Rightarrow$ $3x + y - 5 = 0$, which is the required relation between x and y.

EXERCISE - 7.2

1. Let the required point be $P(x, y)$.
Here, the end points are $(-1, 7)$ and $(4, -3)$

$\because$ Ratio $= 2 : 3 = m_1 : m_2$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1+m_2} = \frac{(2\times4)+3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$\text{and } y = \frac{m_1y_2 + m_2y_1}{m_1+m_2} = \frac{2\times(-3)+(3\times7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Thus, the required point is $(1, 3)$.

2. Let the given points be $A(4, -1)$ and $B(-2, -3)$.

Let the points P and Q trisect AB.
i.e., $AP = PQ = QB$
i.e., P divides AB in the ratio of $1 : 2$ and Q divides AB in the ratio of $2 : 1$.
Let the coordinates of P be (x, y).

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1+m_2} = \frac{1(-2)+2(4)}{1+2} = \frac{-2+8}{3} = 2 \text{ and}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1+m_2} = \frac{1(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

$\therefore$ The required coordinates of P are $\left(2, \frac{-5}{3}\right)$.

Let the coordinates of Q be (X, Y).

$$\therefore \quad X = \frac{m_1x_2 + m_2x_1}{m_1+m_2} = \frac{2(-2)+1(4)}{2+1} = \frac{-4+4}{3} = 0$$

$$Y = \frac{m_1y_2 + m_2y_1}{m_1+m_2} = \frac{2(-3)+1(-1)}{2+1} = \frac{-6-1}{3} = \frac{-7}{3}$$

$\therefore$ The required coordinates of Q are $\left(0, \frac{-7}{3}\right)$.

3. Let us consider 'A' as origin, then AB is the x-axis and AD is the y-axis.

Now, the position of green flag-post is $\left(2, \frac{100}{4}\right)$ or $(2, 25)$.

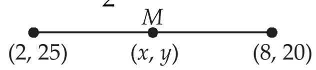
and, the position of red flag-post is $\left(8, \frac{100}{5}\right)$ or $(8, 20)$.

$\therefore$ Distance between both the flags

$$= \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

Let the mid-point of the line segment joining the two flags be $M(x, y)$.

$$\therefore \quad x = \frac{2+8}{2} \text{ and } y = \frac{25+20}{2}$$

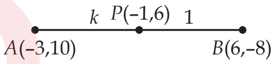


$\Rightarrow$ $x = 5$ and $y = 22.5$

Thus, the blue flag is on the 5^{th} line at a distance 22.5 m above AB.

4. Given, points are $A(-3, 10)$ and $B(6, -8)$
Let the point $P(-1, 6)$ divides AB in the ratio $k : 1$.
Using section formula, we have

$$(-1, 6) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$$



$$\Rightarrow \frac{6k-3}{k+1} = -1 \text{ and } \frac{-8k+10}{k+1} = 6$$
$$\Rightarrow 6k - 3 = -k - 1 \text{ and } -8k + 10 = 6k + 6$$
$$\Rightarrow 7k = 2 \text{ and } 14k = 4$$
$$\Rightarrow k = \frac{2}{7}$$

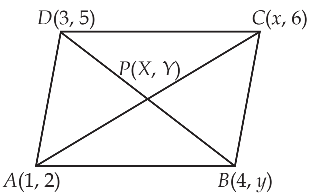
$\therefore$ Required ratio is $\frac{2}{7} : 1$ *i.e.*, $2 : 7$.

5. The given points are $A(1, -5)$ and $B(-4, 5)$.
Let the required ratio be $k : 1$ and the required point be $P(x, y)$.
Since the point P lies on x-axis,
$\therefore$ Its y-coordinate is 0.

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1+m_2} \text{ and } 0 = \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$
$$\Rightarrow x = \frac{k(-4)+1(1)}{k+1} \text{ and } 0 = \frac{5k-5}{k+1}$$
$$\Rightarrow x = \frac{-4k+1}{k+1} \text{ and } 0 = \frac{5k-5}{k+1}$$
$$\Rightarrow x(k+1) = -4k + 1 \text{ and } 5k - 5 = 0 \Rightarrow k = 1$$
$$\Rightarrow x(1+1) = -4 + 1 \qquad [\because k = 1]$$
$$\Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$$

$\therefore$ The required ratio is $1 : 1$ and coordinates of P are $\left(\frac{-3}{2}, 0\right)$.

6. Let the given points are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$.



Since, the diagonals of a parallelogram bisect each other.
$\therefore$ The coordinates of P are

$$X = \frac{x+1}{2} = \frac{3+4}{2}$$

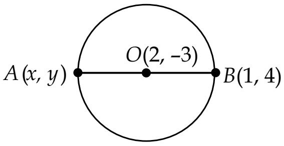
$\Rightarrow \quad x + 1 = 7 \Rightarrow x = 6$ and $Y = \frac{5+y}{2} = \frac{6+2}{2}$

$\Rightarrow \quad 5 + y = 8 \Rightarrow y = 3$

$\therefore$ The required values of x and y are 6 and 3 respectively.

7. Here, centre of the circle is $O(2, -3)$.

Let the end points of the diameter be $A(x, y)$ and $B(1, 4)$.



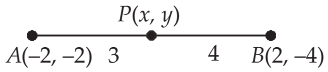
The centre of a circle bisects the diameter.

$\therefore \quad 2 = \frac{x+1}{2} \Rightarrow x + 1 = 4 \Rightarrow x = 3$

And, $-3 = \frac{y+4}{2} \Rightarrow y + 4 = -6 \Rightarrow y = -10$

Hence, the coordinates of A are $(3, -10)$.

8.



Here, the given points are $A(-2, -2)$ and $B(2, -4)$.

Let the coordinates of P are (x, y).

Since, the point P lies on AB such that

$AP = \frac{3}{7}AB \Rightarrow \frac{AP}{AB} = \frac{3}{7} \Rightarrow \frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \quad \frac{AP + BP}{AP} = \frac{7}{3}$ $\quad (\because AB = AP + BP)$

$\Rightarrow \quad 1 + \frac{BP}{AP} = \frac{3+4}{3} = 1 + \frac{4}{3} \Rightarrow \frac{BP}{AP} = \frac{4}{3}$

$\Rightarrow \quad AP : PB = 3 : 4$ *i.e.*, $P(x, y)$ divides AB in the ratio $3 : 4$.

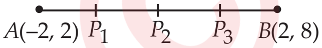
$\therefore \quad x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$ and

$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$

Thus, the coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

9. Here, the given points are $A(-2, 2)$ and $B(2, 8)$.

Let P_1, P_2 and P_3 divide AB in four equal parts.



Since, $AP_1 = P_1P_2 = P_2P_3 = P_3B$

$\therefore \quad P_2$ is the mid-point of AB

$\therefore$ Coordinates of P_2 are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = (0, 5)$

Again, P_1 is the mid-point of AP_2.

$\therefore$ Coordinates of P_1 are

$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(-1, \frac{7}{2}\right)$

Also, P_3 is the mid-point of P_2B.

$\therefore$ Coordinates of P_3 are

$\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

Thus, the coordinates of P_1, P_2 and P_3 are $\left(-1, \frac{7}{2}\right)$, $(0, 5)$ and $\left(1, \frac{13}{2}\right)$ respectively.

10. Let the vertices of the given rhombus are $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$.

$\because \quad AC$ and BD are the diagonals of rhombus $ABCD$.

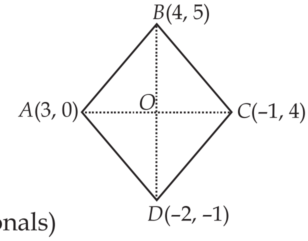
$AC = \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = 4\sqrt{2}$ units

$BD = \sqrt{(-2-4)^2 + (-1-5)^2}$

$= \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36} = 6\sqrt{2}$ units



$\therefore$ Area of a rhombus

$= \frac{1}{2} \times$ (Product of diagonals)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 4 \times 6 = 24$ square units.

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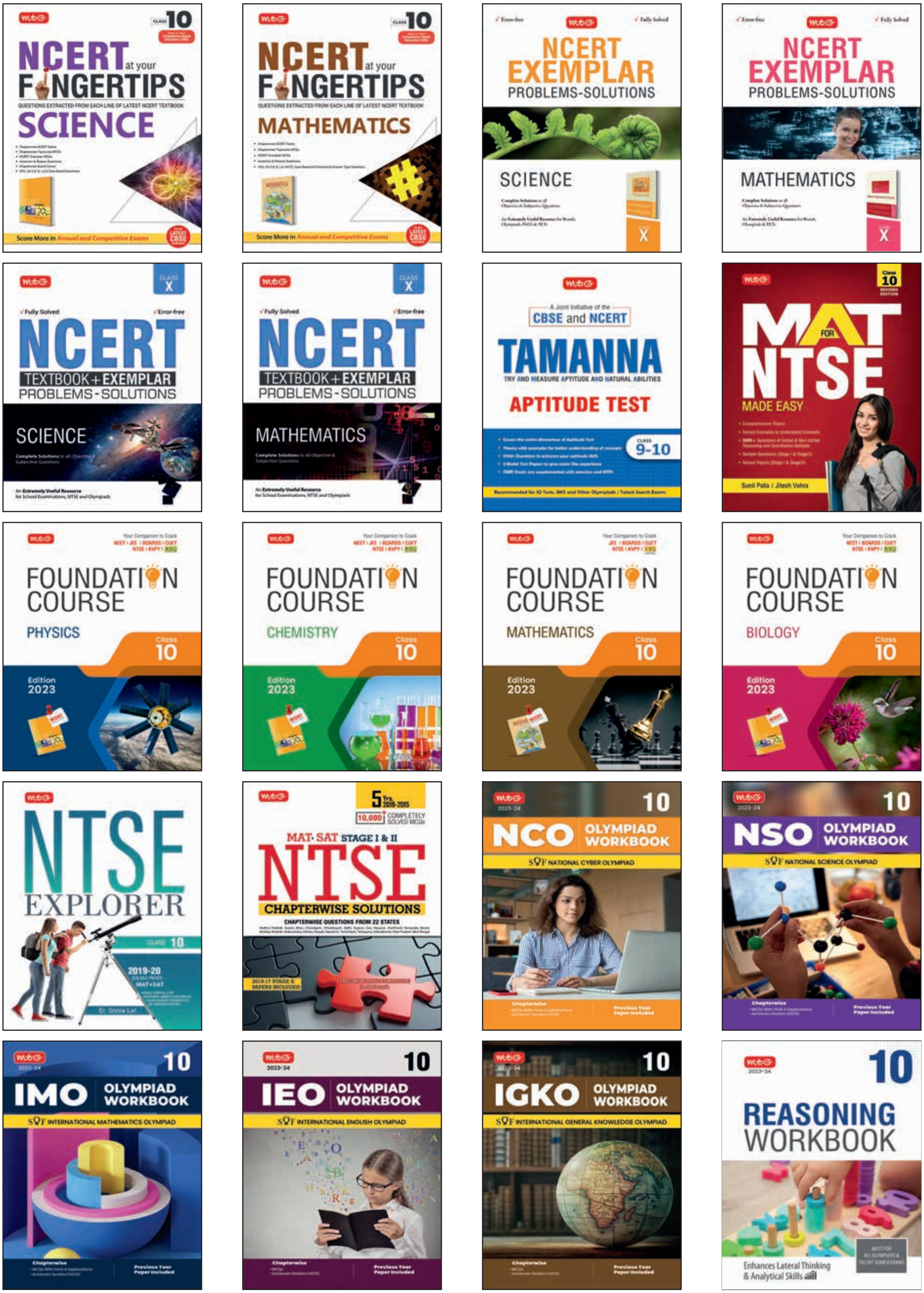

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