# Introduction to Trigonometry

CHAPTER 8

EXAM DRILL

## SOLUTIONS

**1.** **(b) :** We have, $\frac{\sin A}{\tan A}+\frac{\cos A\cot A}{\operatorname{cosec}A}$

$=\frac{\sin A}{\frac{\sin A}{\cos A}}+\cos A\times\frac{\cos A}{\sin A}\times\sin A$

$\left[\because \tan A=\frac{\sin A}{\cos A},\cot A=\frac{\cos A}{\sin A},\frac{1}{\operatorname{cosec}A}=\sin A\right]$

$=\cos A+\cos^2 A$

**2.** **(a) :** We have, $\sin\theta=\cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta}=1 \Rightarrow \tan\theta=1$ $\quad\left[\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \tan\theta=\tan 45°$ $\quad[\because \tan 45°=1]$

$\Rightarrow \theta=45°$

**3.** **(c) :** Given, $\sec(2x+17)°=\sqrt{2}$

$\Rightarrow \sec(2x+17)°=\sec 45°$ $\quad[\because \sec 45°=\sqrt{2}]$

$\Rightarrow 2x+17=45 \Rightarrow 2x=45-17$

$\Rightarrow 2x=28 \Rightarrow x=14$

**4.** **(b) :** We have, $\cos1°\cdot\cos2°\cdot\cos3°\cdot\cos4°\cdot \ldots \cos100°$

$=\cos1°\cdot\cos2°\cdot\cos3°\ldots\cos90°\ldots\cos100°$

$=0$ $\quad[\because \cos 90°=0]$

**5.** **(b) :** $\sin(45°+\theta)-\cos(45°-\theta)$

$=\cos[90°-(45°+\theta)]-\cos(45°-\theta)$ $\quad[\because \cos(90°-\theta)=\sin\theta]$

$=\cos(45°-\theta)-\cos(45°-\theta)=0$

**6.** We have, $\sin^2\theta+\frac{1}{1+\tan^2\theta}$

$=\sin^2\theta+\frac{1}{\sec^2\theta}$ $\quad[\because 1+\tan^2\theta=\sec^2\theta]$

$=\sin^2\theta+\cos^2\theta$ $\quad[\because \sec\theta=1/\cos\theta]$

$=1$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

**7.** Given, $\sin\theta=\frac{a}{b}$

$\therefore \cos\theta=\sqrt{1-\sin^2\theta}$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

$=\sqrt{1-\left(\frac{a}{b}\right)^2}=\sqrt{1-\frac{a^2}{b^2}}=\frac{\sqrt{b^2-a^2}}{b}$

**8.** $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}=\frac{2}{1-\sin^2\theta}=\frac{2}{\cos^2\theta}=2\sec^2\theta$

**9.** In $\Delta OPQ$, we have

$OQ^2=OP^2+PQ^2$

$\Rightarrow (1+PQ)^2=OP^2+PQ^2$

$\Rightarrow 1+PQ^2+2PQ=OP^2+PQ^2 \Rightarrow 1+2PQ=7^2$

$\Rightarrow PQ=24$ cm and $OQ=1+PQ=25$ cm

$\therefore \sin Q=\frac{7}{25}$ and $\cos Q=\frac{24}{25}$

**10.** L.H.S. $=\sqrt{\cos\alpha\operatorname{cosec}\beta-\cos\alpha\sin\beta}$

$=\sqrt{\cos\alpha\operatorname{cosec}(90°-\alpha)-\cos\alpha\sin(90°-\alpha)}$

[Given $\alpha+\beta=90°$]

$=\sqrt{\cos\alpha\sec\alpha-\cos\alpha\cos\alpha}=\sqrt{1-\cos^2\alpha}=\sin\alpha$ = R.H.S.

$[\because \sin^2\theta+\cos^2\theta=1]$

**11.** $24\cot A=7 \Rightarrow \cot A=\frac{7}{24}$

$\therefore \sin A=\frac{1}{\operatorname{cosec}A}=\frac{1}{\sqrt{1+\cot^2 A}}$

$=\frac{1}{\sqrt{1+\left(\frac{7}{24}\right)^2}}=\frac{1}{\sqrt{1+\frac{49}{576}}}$

$=\frac{1}{\sqrt{\frac{576+49}{576}}}=\frac{1}{\sqrt{\frac{625}{576}}}=\frac{1}{\left(\frac{25}{24}\right)}=\frac{24}{25}$

**12.** Given, $x=b\sec^3\theta$ and $y=a\tan^3\theta$

$\Rightarrow \sec^3\theta=\frac{x}{b}$ and $\tan^3\theta=\frac{y}{a}$ $\quad$ ...(i)

Now consider, $\left(\frac{x}{b}\right)^{2/3}-\left(\frac{y}{a}\right)^{2/3}$

$=(\sec^3\theta)^{2/3}-(\tan^3\theta)^{2/3}$ $\quad$ (Using (i))

$=\sec^2\theta-\tan^2\theta=1$

**13.** We know that $\sin^2\theta+\cos^2\theta=1$

$\Rightarrow (\sin^2\theta+\cos^2\theta)^3=1^3$ $\quad$ [Cubing both the sides]

$\Rightarrow (\sin^2\theta)^3+(\cos^2\theta)^3+3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)=1$

$\Rightarrow \sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta=1$

**14.** L.H.S. $=(\sin^4\theta-\cos^4\theta+1)\operatorname{cosec}^2\theta$

$=[(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)+1]\operatorname{cosec}^2\theta$

$=(\sin^2\theta-\cos^2\theta+1)\operatorname{cosec}^2\theta$ $\quad[\because \sin^2\theta+\cos^2\theta=1]$

$=2\sin^2\theta\operatorname{cosec}^2\theta$ $\quad[\because 1-\cos^2\theta=\sin^2\theta]$

$=2$ = R.H.S.

**15.** We have, $2(\text{cosec}^2\theta - 1)\tan^2\theta$

$= 2(\cot^2\theta)\tan^2\theta$ $[\because \text{cosec}^2\theta = 1 + \cot^2\theta]$

$= 2$ $[\because \tan\theta \cdot \cot\theta = 1]$

**16.** L.H.S. $= \dfrac{\tan^2 A(1+\cot^2 A)}{(1+\tan^2 A)} = \dfrac{\tan^2 A(\text{cosec}^2 A)}{\sec^2 A}$

$[\because 1 + \cot^2 A = \text{cosec}^2 A\ ;\ 1 + \tan^2 A = \sec^2 A]$

$= \dfrac{\sin^2 A}{\cos^2 A} \cdot \dfrac{\cos^2 A}{\sin^2 A} = 1 =$ R.H.S.

**17.** **(i)** In $\Delta APQ$, $\tan\theta = \dfrac{AQ}{PQ} = \dfrac{1.2}{1.6} = \dfrac{3}{4}$

**(ii)** In $\Delta PBQ$, $\cot B = \dfrac{QB}{PQ} = \dfrac{3}{1.6} = \dfrac{15}{8}$ ...(1)

**(iii)** In $\Delta APQ$, $\tan A = \dfrac{PQ}{AQ} = \dfrac{1.6}{1.2} = \dfrac{4}{3}$ ...(2)

**(iv)** We have, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sec A = \sqrt{\left(\dfrac{4}{3}\right)^2 + 1}$ [Using (2)]

$= \sqrt{\dfrac{16}{9} + 1} = \sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$

**(v)** Since, $\text{cosec} B = \sqrt{\cot^2 B + 1}$

$= \sqrt{\left(\dfrac{15}{8}\right)^2 + 1}$ [Using (1)]

$= \dfrac{17}{8}$

**18.** $\because$ $\Delta PQR$ is a right angled triangle.

$\therefore$ $PR^2 + RQ^2 = PQ^2$

$\Rightarrow$ $PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5$ cm

**(i)** **(c) :** $\cos\theta = \dfrac{QR}{PQ} = \dfrac{12}{13}$

**(ii)** **(c) :** $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{13}{12}$

**(iii)** **(c) :** $\tan\theta = \dfrac{PR}{RQ} = \dfrac{5}{12}$ ...(1)

$\therefore \dfrac{\tan\theta}{1+\tan^2\theta} = \dfrac{\frac{5}{12}}{1+\frac{25}{144}} = \dfrac{\frac{5}{12}}{\frac{169}{144}} = \dfrac{60}{169}$

**(iv)** **(a) :** $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{12}{5}$ [Using (1)]

$\text{cosec}\theta = \dfrac{PQ}{PR} = \dfrac{13}{5}$

$\therefore \cot^2\theta - \text{cosec}^2\theta = \dfrac{144}{25} - \dfrac{169}{25} = -1$

**(v)** **(b) :** $\sin^2\theta + \cos^2\theta = 1$ (Using identity)

**19.** We have, $KL = 4$ cm, $ML = 4\sqrt{3}$ cm, $KM = 8$ cm

**(i)** **(a) :** $\tan M = \dfrac{KL}{LM} = \dfrac{4}{4\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan M = \tan 30° \Rightarrow \angle M = 30°$

**(ii)** **(c) :** $\tan K = \dfrac{ML}{KL} = \dfrac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60°$

$\Rightarrow$ $\angle K = 60°$

**(iii)** **(b)** **(iv)** **(c)**

**(v)** **(a) :** $\dfrac{\tan^2 45° - 1}{\tan^2 45° + 1} = \dfrac{(1)^2 - 1}{1^2 + 1} = \dfrac{0}{2} = 0$

**20.** We have, $AB = BC = 6\sqrt{2}$ m and $AC = 12$ m.

**(i)** $\because$ $D$ is mid point of $AC$.

$\therefore$ $AD = DC = 6$ m

Now, $AB^2 = BD^2 + AD^2$ ($\because$ $\Delta ABD$ is a right triangle)

$\Rightarrow$ $BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$

$\Rightarrow$ $BD = 6$ m ...(1)

**(ii)** In $\Delta ABD$, $\sin A = \dfrac{BD}{AB} = \dfrac{6}{6\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ [Using (1)]

$\Rightarrow$ $\sin A = \sin 45° \Rightarrow \angle A = 45°$

**(iii)** In $\Delta BDC$, $\tan C = \dfrac{BD}{DC} = \dfrac{6}{6}$ [Using (1)]

$\Rightarrow$ $\tan C = 1 = \tan 45° \Rightarrow \angle C = 45°$

**(iv)** $\sin A = \dfrac{1}{\sqrt{2}}, \cos C = \cos 45° = \dfrac{1}{\sqrt{2}}$

$\therefore \sin A + \cos C = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

**(v)** $\tan C = 1$, $\tan A = \tan 45° = 1$

$\Rightarrow$ $\tan^2 C + \tan^2 A = 1 + 1 = 2$

**21.** Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \text{ and } \tan 45° = 1\right]$

$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45° = \cos 45° = \dfrac{1}{\sqrt{2}}\right]$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

**22.** Given that, $A = 60°$ and $B = 30°$

$\therefore$ $\cos A = \cos 60° = \frac{1}{2}$; $\cos B = \cos 30° = \frac{\sqrt{3}}{2}$

and $\cos(A + B) = \cos(60° + 30°) = \cos 90° = 0$

Now, $\cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \neq 0$

$\therefore$ $\cos(A + B) \neq \cos A + \cos B$.

**23.** L.H.S. $= \operatorname{cosec}^2 60° \sec^2 30° \cos^2 0° \sin 45° \cot^2 60° \tan^2 60°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2 (1)^2 \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)^2 (\sqrt{3})^2 = \frac{4}{3} \times \frac{4}{3} \times \frac{1}{\sqrt{2}} \times \frac{1}{3} \times 3$$

$$= \frac{16}{9} \times \frac{1}{\sqrt{2}} = \frac{8\sqrt{2}}{9} = \text{R.H.S.}$$

**OR**

We have, $\sin(A + B) = 1$

$\Rightarrow$ $\sin(A + B) = \sin 90° \Rightarrow A + B = 90°$ ...(i)

Also, $\sin(A - B) = \frac{1}{2}$

$\Rightarrow$ $\sin(A - B) = \sin 30° \Rightarrow A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$A + B + A - B = 120° \Rightarrow 2A = 120° \Rightarrow A = 60°$

From (i), we have $60° + B = 90° \Rightarrow B = 30°$

**24.** We have, $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = k + \tan^2\theta + \cot^2\theta$

$\Rightarrow$ $\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta = k + \tan^2\theta + \cot^2\theta$

$\Rightarrow$ $\sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2 = k + \tan^2\theta + \cot^2\theta$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta - \cot^2\theta + \sec^2\theta - \tan^2\theta + 4 = k$

$\Rightarrow$ $1 + 1 + 1 + 4 = k \Rightarrow k = 7$

**25.** Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow$ $2\sin^2\theta - (1 - \sin^2\theta) = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow$ $3\sin^2\theta = 3 \Rightarrow \sin^2\theta = 1$

$\Rightarrow$ $\sin\theta = 1 = \sin 90°$ $[\because \sin 90° = 1]$

$\therefore$ $\theta = 90°$

**26.** In $\Delta OPQ$, we have $OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $(1 + PQ)^2 = OP^2 + PQ^2$ $[\because OQ - PQ = 1]$

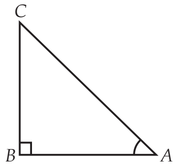
$\Rightarrow$ $1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$\Rightarrow$ $1 + 2PQ = 7^2 \Rightarrow 2PQ = 48$

$\Rightarrow$ $PQ = 24$ cm and $OQ = 1 + PQ = 25$ cm

So, $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$ and $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

**27.** In $\Delta ABC$, $\tan A = \frac{BC}{AB} = 1$




$\Rightarrow$ $BC = AB$

Let $AB = BC = k$ units

$\therefore$ By Pythagoras theorem, we have

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2}$ units

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AB}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$

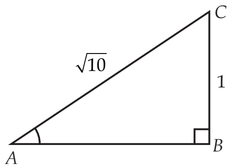
So, $2\sin A\cos A = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 1$

Hence verified.

**28.** We have, $\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$

So, we draw a $\Delta ABC$, right-angled at $B$ such that

$BC = 1$ unit and $AC = \sqrt{10}$ units.




By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $(\sqrt{10})^2 = AB^2 + 1^2$

$\Rightarrow$ $AB^2 = 10 - 1 = 9$

$\Rightarrow$ $AB = \sqrt{9} = 3$ units

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{10}}$, $\cos A = \frac{AB}{AC} = \frac{3}{\sqrt{10}}$,

$\tan A = \frac{BC}{AB} = \frac{1}{3}$, $\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$

and $\cot A = \frac{1}{\tan A} = \frac{3}{1} = 3$

**29.** Since, $\tan R = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow R = 30°$

Again, $\cos P = \frac{1}{2} = \cos 60° \Rightarrow P = 60°$

(i) Now, $\cos P\cos R + \sin P\sin R$

$= \cos 60° \cos 30° + \sin 60° \sin 30°$

$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

(ii) $\cos(P - R) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

From (i) and (ii), we get

$\cos(P - R) = \cos P\cos R + \sin P\sin R$

**30.** Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both the sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow$ $\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow \quad 1 = 2\cos^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow \quad 1 - 2\cos^2\theta = -2\cos\theta\sin\theta$

$\Rightarrow \quad 1 + 1 - 2\cos^2\theta = 1 - 2\cos\theta\sin\theta$

$\Rightarrow \quad 2 - 2\cos^2\theta = \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta$

$\Rightarrow \quad 2(1 - \cos^2\theta) = (\cos\theta - \sin\theta)^2$

$\Rightarrow \quad 2\sin^2\theta = (\cos\theta - \sin\theta)^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

**31.** We have, $\sin\theta = \dfrac{24}{25}$

$$\Rightarrow \sin^2\theta = \left(\frac{24}{25}\right)^2 \Rightarrow 1 - \cos^2\theta = \frac{576}{625}$$

$$\Rightarrow \cos^2\theta = 1 - \frac{576}{625} = \frac{625-576}{625} = \frac{49}{625}$$

$$\Rightarrow \cos\theta = \frac{7}{25} \Rightarrow \sec\theta = \frac{25}{7}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{24/25}{7/25} = \frac{24}{7}$$

$$\therefore 100(\sec\theta + \tan\theta) = 100\left(\frac{25}{7} + \frac{24}{7}\right) = 100\left(\frac{49}{7}\right) = 700$$

**OR**

$$\text{L.H.S.} = \sqrt{\sec^2\theta + \text{cosec}^2\theta} = \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$\left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \text{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cdot\cos^2\theta}} = \sqrt{\frac{1}{\sin^2\theta\cdot\cos^2\theta}} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta\cdot\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cdot\cos\theta} \quad [\because 1 = \sin^2\theta + \cos^2\theta]$$

$$= \frac{\sin^2\theta}{\sin\theta\cdot\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cdot\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

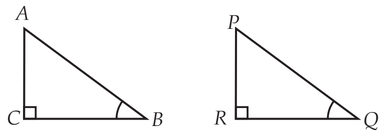
$$= \tan\theta + \cot\theta \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

= R.H.S.

**32.** Let us consider two triangles *ABC* and *PQR*, right angled at *C* and *R* respectively such that $\sin B = \sin Q$.

We have, $\sin B = \dfrac{AC}{AB}$ and $\sin Q = \dfrac{PR}{PQ}$

Then $\dfrac{AC}{AB} = \dfrac{PR}{PQ} \Rightarrow \dfrac{AC}{PR} = \dfrac{AB}{PQ} = k$ (say) ...(i)




Now, by using Pythagoras theorem, we have

$BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$$\text{So, } \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$$

$$= \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k \quad \text{...(ii)}$$

From (i) and (ii), we have, $\dfrac{AC}{PR} = \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

So, $\Delta ACB \sim \Delta PRQ$ and therefore, $\angle B = \angle Q$.

**33.** Given, $\sin(A + C - B) = \dfrac{\sqrt{3}}{2}$ and $\cot(B + C - A) = \sqrt{3}$

$\Rightarrow \quad \sin(A + C - B) = \sin 60°$ and $\cot(B + C - A) = \cot 30°$

$\Rightarrow \quad A + C - B = 60°$ ...(i)

and $B + C - A = 30°$ ...(ii)

Adding (i) and (ii), we have $2C = 90° \Rightarrow C = 45°$

Putting this value of *C* in (i), we get $A - B = 15°$ ...(iii)

Also, by angle sum property of a triangle

$A + B + C = 180°$

$\Rightarrow \quad A + B = 135°$ ...(iv)

Adding (iii) and (iv), we have

$2A = 150° \Rightarrow A = 75°$

From (iii), we have $B = 75° - 15° = 60°$

**OR**

We are given that, $\text{cosec}\,\theta = \dfrac{17}{12} \Rightarrow \sin\theta = \dfrac{12}{17}$

$$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{12}{17}\right)^2} = \frac{\sqrt{145}}{17}$$

$$\text{So, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{17} \times \frac{17}{\sqrt{145}} = \frac{12}{\sqrt{145}} \quad \text{...(i)}$$

$$\text{Now, } \sqrt{\frac{(\sec\theta - 1)(\sec\theta + 1)}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}} = \sqrt{\frac{(\sec^2\theta - 1)}{(\text{cosec}^2\theta - 1)}}$$

$$= \sqrt{\frac{\tan^2\theta}{\cot^2\theta}} = \sqrt{\left(\frac{\tan\theta}{\cot\theta}\right)^2} = \frac{\tan\theta}{\cot\theta} = \tan\theta \times \tan\theta = \tan^2\theta$$

$$= \left(\frac{12}{\sqrt{145}}\right)^2 \quad \text{[Using (i)]}$$

$$= \frac{144}{145}$$

**34.** We have, $a\sin\theta = b\cos\theta$ ...(i)

Also, $a\sin^3\theta + b\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \quad (a\sin\theta)\sin^2\theta + b\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \quad b\cos\theta\,.\,\sin^2\theta + b\cos^3\theta = \sin\theta\cos\theta$ [Using (i)]

$\Rightarrow \quad b\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$

$\Rightarrow \quad b\cos\theta \times 1 = \sin\theta\cos\theta$

$\Rightarrow \quad b = \sin\theta$ ...(ii)

From (i) and (ii), we have

$a \cdot b = b\cos\theta \Rightarrow a = \cos\theta$ ...(iii)

Squaring and adding (ii) and (iii), we get

$a^2 + b^2 = \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad a^2 + b^2 = 1$

Hence proved.

**35.** Given, $m = \cos A - \sin A$, $n = \cos A + \sin A$

Now, $\frac{m}{n} - \frac{n}{m} = \frac{m^2 - n^2}{mn}$

$$= \frac{(\cos A - \sin A)^2 - (\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$$

$$= \frac{\cos^2 A + \sin^2 A - 2\cos A \sin A - \cos^2 A - \sin^2 A - 2\sin A\cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{-4\sin A\cos A}{\cos^2 A - \sin^2 A} \quad \text{...(i)}$$

Dividing numerator and denominator by sin $A$ cos $A$, we get

$$\frac{-4}{\frac{\cos^2 A}{\sin A\cos A} - \frac{\sin^2 A}{\sin A\cos A}} = \frac{-4}{\cot A - \tan A} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{m}{n} - \frac{n}{m} = \frac{-4\sin A\cos A}{\cos^2 A - \sin^2 A} = \frac{-4}{\cot A - \tan A}$$