

Introduction to Trigonometry

CHAPTER 8



SOLUTIONS

EXERCISE - 8.1

1. In right angle $\triangle ABC$, we have

$$AB = 24 \text{ cm}, BC = 7 \text{ cm}$$

Using Pythagoras theorem, we have

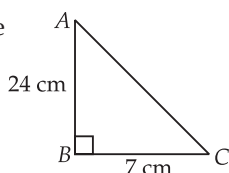
$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 24^2 + 7^2 = 576 + 49 = 625 = 25^2$$

$$\Rightarrow AC = 25 \text{ cm}$$

$$(i) \sin A = \frac{BC}{AC} = \frac{7}{25}, \cos A = \frac{AB}{AC} = \frac{24}{25}$$

$$(ii) \sin C = \frac{AB}{AC} = \frac{24}{25}, \cos C = \frac{BC}{AC} = \frac{7}{25}$$



2. In right angle $\triangle PQR$

Using Pythagoras theorem, we have

$$QR^2 = PR^2 - PQ^2$$

$$\Rightarrow QR^2 = 13^2 - 12^2 = (13 - 12)(13 + 12) = 1 \times 25 = 25$$

$$\therefore QR = \sqrt{25} = 5 \text{ cm}$$

$$\text{Now, } \tan P = \frac{QR}{PQ} = \frac{5}{12}, \cot R = \frac{QR}{PQ} = \frac{5}{12}$$

$$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

3. In right angle $\triangle ABC$, we have

$$\sin A = \frac{BC}{AC} = \frac{3}{4}$$

Let $BC = 3k$ units and $AC = 4k$ units

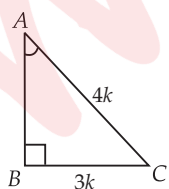
Using Pythagoras theorem, we have

$$AB^2 = AC^2 - BC^2 = (4k)^2 - (3k)^2 = (4k - 3k)(4k + 3k) = k(7k) = 7k^2$$

$$\Rightarrow AB = \sqrt{7k^2} = \sqrt{7}k$$

$$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\text{Also, } \tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$



4. In right angle $\triangle ABC$, we have

$$15 \cot A = 8 \Rightarrow \cot A = 8/15$$

$$\Rightarrow \cot A = \frac{AB}{BC} = \frac{8}{15}$$

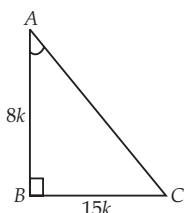
Let $AB = 8k$ units and $BC = 15k$ units

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2 = (17k)^2$$

$$\Rightarrow AC = \sqrt{(17k)^2} = 17k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17} \text{ and } \sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$



5. Consider a right angled $\triangle ABC$ with $\angle B = 90^\circ$

Let $\angle A = \theta$ and $\sec \theta = 13/12$

$$\Rightarrow \frac{AC}{AB} = \frac{13}{12}$$

Let $AC = 13k$ units and

$AB = 12k$ units

Using Pythagoras theorem, we have

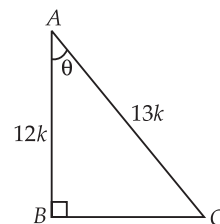
$$BC^2 = AC^2 - AB^2 \Rightarrow BC^2 = (13k)^2 - (12k)^2 = (13k - 12k)(13k + 12k) = k(25k) = 25k^2 = (5k)^2$$

$$\Rightarrow BC = \sqrt{(5k)^2} = 5k$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cot \theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}, \tan \theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}, \operatorname{cosec} \theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$



6. Let us consider a right $\triangle ABC$, $\angle C = 90^\circ$

Now, $\cos A = \frac{AC}{AB}$ and

$$\cos B = \frac{BC}{AB}$$

Since, $\cos A = \cos B$

$$\therefore \frac{AC}{AB} = \frac{BC}{AB} \Rightarrow AC = BC$$

Now, in $\triangle ABC$, two sides AC and BC are equal.

$\therefore$ Their opposite angles are also equal. Hence, $\angle A = \angle B$

7. Let in right $\triangle ABC$, $\angle B = 90^\circ$ and $\angle A = \theta$.

$$\text{Given, } \cot \theta = \frac{7}{8} \Rightarrow \frac{AB}{BC} = \frac{7}{8}$$

Now, let $AB = 7k$ units

and $BC = 8k$ units

By Pythagoras theorem, we have

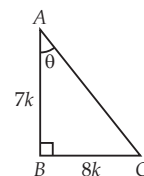
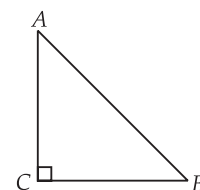
$$AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2 \Rightarrow AC = \sqrt{49k^2 + 64k^2} = \sqrt{113k^2} = \sqrt{113}k$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}} \text{ and}$$

$$\cos \theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

Now,

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$



$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

$$(ii) \cot^2 \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

8. Let in a right-angled $\triangle ABC$, $\angle B = 90^\circ$.

Now, $3 \cot A = 4$ (Given)

$$\Rightarrow \cot A = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$$

Now, let $AB = 4k$ units

and $BC = 3k$ units

Using Pythagoras theorem, we have,

$$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 \\ \Rightarrow AC = \sqrt{16k^2 + 9k^2} = \sqrt{25k^2} = \sqrt{(5k)^2} = 5k \text{ units}$$

$$\text{Now, } \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}, \cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\text{Also, } \tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

Now, to check the given equation,

$$\text{L.H.S.} = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\text{R.H.S.} = \cos^2 A - \sin^2 A$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25} = \text{L.H.S.}$$

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

9. In right $\triangle ABC$, $\angle B = 90^\circ$

$$\therefore \tan A = \frac{1}{\sqrt{3}} \quad (\text{Given})$$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Now, let $AB = \sqrt{3}k$ units and $BC = k$ units

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (\sqrt{3}k)^2 + k^2$$

$$\Rightarrow AC = \sqrt{3k^2 + k^2} = \sqrt{4k^2} = 2k$$

$$\text{Now, } \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}, \cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

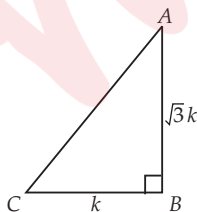
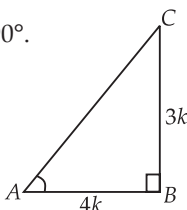
$$\text{Also, } \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}, \cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$(i) \sin A \cos C + \cos A \sin C$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$



10. In right $\triangle PQR$, $\angle Q = 90^\circ$

$PR + QR = 25$ cm and $PQ = 5$ cm

Let QR be x cm $\Rightarrow PR = (25 - x)$ cm

Using Pythagoras theorem, we have

$$PR^2 = QR^2 + PQ^2$$

$$\Rightarrow (25 - x)^2 = x^2 + 5^2$$

$$\Rightarrow 625 - 50x + x^2 = x^2 + 25$$

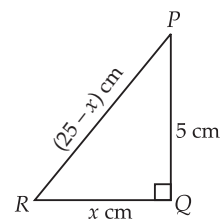
$$\Rightarrow -50x = -600$$

$$\Rightarrow x = \frac{-600}{-50} = 12 \text{ i.e., } QR = 12 \text{ cm}$$

$$\Rightarrow PR = 25 - 12 = 13 \text{ cm}$$

$$\text{Now, } \sin P = \frac{RQ}{RP} = \frac{12}{13}, \cos P = \frac{PQ}{RP} = \frac{5}{13} \text{ and}$$

$$\tan P = \frac{RQ}{PQ} = \frac{12}{5}$$



11. (i) False

$\therefore$ A tangent of an angle is the ratio of perpendicular to base which may be equal or unequal to each other.

(ii) True

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$ and hypotenuse is the greatest side of the triangle.

$\therefore \cos A$ is always less than 1.

$\therefore \frac{1}{\cos A}$ i.e., $\sec A$ will always be greater than 1.

(iii) False

$\therefore$ 'cosine A' is abbreviated as 'cos A'.

(iv) False

$\therefore$ 'cot A' is a single and meaningful term whereas 'cot' alone has no meaning.

(v) False

$\therefore 4/3$ is greater than 1 and $\sin \theta$ cannot be greater than 1.

EXERCISE - 8.2

1. (i) We have, $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) We have, $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 \times 1 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) We have, $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1 + 2\sqrt{2}}{\sqrt{2}}} = \frac{1}{1 + 2\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{2(1 + \sqrt{3})} = \frac{\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1 + \sqrt{3})} = \frac{1 - \sqrt{3}}{2(1 + \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{\sqrt{6}}{4} \times \frac{(1 - \sqrt{3})}{1 - 3} = \frac{\sqrt{6}(1 - \sqrt{3})}{4(-2)} = \frac{\sqrt{6}(\sqrt{3} - 1)}{8}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) We have, $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$\begin{aligned} &= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{1+2}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1+2}{2}} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}} \\ &= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} \\ &= \frac{(3\sqrt{3})^2 + (4)^2 - 2 \times 4 \times 3\sqrt{3}}{(3\sqrt{3})^2 - (4)^2} = \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11} \end{aligned}$$

(v) We have, $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$\begin{aligned} &= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{1}{12}(15+64-12)}{\frac{1+3}{4}} = \frac{\frac{1}{12} \times 67}{\frac{4}{4}} = \frac{67}{12} \end{aligned}$$

2. (i) (a) : $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{\sqrt{3}} \times \frac{1}{2} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3 \times \sqrt{3}}{3 \times 2} = \frac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) (d) : $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - (1)^2}{1 + (1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

(iii) (a) : When $A = 0^\circ$, then
 $\sin 2A = \sin 2(0^\circ) = \sin 0^\circ = 0$,
 $2 \sin A = 2 \sin 0^\circ = 2 \times 0 = 0$
i.e., $\sin 2A = 2 \sin A$ for $A = 0^\circ$

(iv) (c) : $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} \times \sqrt{3} = \sqrt{3} = \tan 60^\circ$

3. We have,

$\tan 60^\circ = \sqrt{3}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$... (i)

Also, $\tan (A+B) = \sqrt{3}$ and $\tan (A-B) = \frac{1}{\sqrt{3}}$... (ii)

From (i) and (ii), we get

$A+B = 60^\circ$... (iii)

and $A-B = 30^\circ$... (iv)

On adding (iii) and (iv), we get

$2A = 90^\circ \Rightarrow A = 45^\circ$

On subtracting (iv) from (iii), we get

$2B = 30^\circ \Rightarrow B = 15^\circ$

4. (i) False :

Let us take $A = 30^\circ$ and $B = 60^\circ$, then

L.H.S. = $\sin (30^\circ + 60^\circ) = \sin 90^\circ = 1$

R.H.S. = $\sin 30^\circ + \sin 60^\circ$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$, since $1 \neq \frac{1+\sqrt{3}}{2}$

$\therefore$ L.H.S. $\neq$ R.H.S.

(ii) True :

Since, the value of $\sin \theta$ increases from 0 to 1 as θ increases from 0° to 90° .

(iii) False :

Since, the value of $\cos \theta$ decreases from 1 to 0 as θ increases from 0° to 90° .

(iv) False :

Let us take $\theta = 30^\circ$

$\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin 30^\circ \neq \cos 30^\circ$

(v) True :

We have, $\cot 0^\circ =$ not defined

EXERCISE - 8.3

1. (i) $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{\operatorname{cosec}^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}}$

(ii) $\sec A = \sqrt{\sec^2 A} = \sqrt{1 + \tan^2 A}$
 $= \sqrt{1 + \frac{1}{\cot^2 A}} = \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$

(iii) $\tan A = \frac{1}{\cot A}$

2. (i) $\sin A = \frac{\sin A}{1} = \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\tan A}{\sec A} = \frac{\sqrt{\tan^2 A}}{\sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

(ii) $\cos A = \frac{1}{\sec A}$

(iii) $\tan A = \sqrt{\tan^2 A} = \sqrt{\sec^2 A - 1}$

(iv) $\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

(v) $\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$

3. (i) (b) : We have, $9 \sec^2 A - 9 \tan^2 A$
 $= 9 (\sec^2 A - \tan^2 A) = 9 (1) = 9$ [$\because \sec^2 A - \tan^2 A = 1$]

(ii) (c) : Here, $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

$$= (1 + \tan \theta + \sec \theta) \left[1 + \frac{1}{\tan \theta} - \operatorname{cosec} \theta \right]$$

$$= (1 + \tan \theta + \sec \theta) \left[\frac{\tan \theta + 1 - \tan \theta \cdot \operatorname{cosec} \theta}{\tan \theta} \right]$$

$$= \frac{(1 + \tan \theta + \sec \theta) [\tan \theta + 1 - \sec \theta]}{\tan \theta}$$

$$\left[\because \tan \theta \cdot \operatorname{cosec} \theta = \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} = \frac{1}{\cos \theta} = \sec \theta \right]$$

$$= \frac{(1 + \tan \theta)^2 - \sec^2 \theta}{\tan \theta} = \frac{1 + \tan^2 \theta + 2 \tan \theta - \sec^2 \theta}{\tan \theta}$$

$$= \frac{\sec^2 \theta + 2 \tan \theta - \sec^2 \theta}{\tan \theta} \quad (\because 1 + \tan^2 \theta = \sec^2 \theta)$$

$$= \frac{2 \tan \theta}{\tan \theta} = 2$$

(iii) (d) : We have, $(\sec A + \tan A) (1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A) = \frac{(1 + \sin A) (1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

(iv) (d) : Here, $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}}$

$$= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}} = (1 + \tan^2 A) \frac{\tan^2 A}{(1 + \tan^2 A)} = \tan^2 A$$

4. (i) L.H.S. $= (\operatorname{cosec} \theta - \cot \theta)^2$

$$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} \quad [\because \sin^2 \theta = 1 - \cos^2 \theta]$$

$$= \frac{(1 - \cos \theta) \times (1 - \cos \theta)}{(1 - \cos \theta) \times (1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} = \text{R.H.S.}$$

(ii) L.H.S. $= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) \cos A}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2 \sin A}{(1 + \sin A) \cos A}$$

$$= \frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A} \quad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{2 + 2 \sin A}{(1 + \sin A) \cos A} = \frac{2(1 + \sin A)}{\cos A (1 + \sin A)}$$

$$= \frac{2}{\cos A} = 2 \sec A = \text{R.H.S.} \quad \left[\because \frac{1}{\cos A} = \sec A \right]$$

(iii) L.H.S. $= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cdot \cos \theta (\sin \theta - \cos \theta)}$$

$$= \frac{(\sin \theta - \cos \theta) (\sin^2 \theta + \sin \theta \cdot \cos \theta + \cos^2 \theta)}{\sin \theta \cdot \cos \theta (\sin \theta - \cos \theta)}$$

$$= \frac{(1 + \sin \theta \cdot \cos \theta)}{\sin \theta \cos \theta} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} + 1$$

$$= 1 + \sec \theta \operatorname{cosec} \theta = \text{R.H.S.}$$

(iv) L.H.S. $= \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\cos A + 1}{\frac{1}{\cos A}} = \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} = \cos A + 1$$

$$= (1 + \cos A) \times \frac{(1 - \cos A)}{(1 - \cos A)}$$

[Multiplying and dividing by $(1 - \cos A)$]

$$= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A} \quad [\because 1 - \cos^2 A = \sin^2 A]$$

$$= \text{R.H.S.}$$

(v) L.H.S. $= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$

[Dividing numerator and denominator by $\sin A$]

$$\begin{aligned}
&= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\
&= \frac{(\cot A - 1 + \operatorname{cosec} A)(\cot A + \operatorname{cosec} A)}{(\cot A + 1 - \operatorname{cosec} A)(\cot A + \operatorname{cosec} A)} \\
&\quad [\text{Multiplying and dividing by } (\cot A + \operatorname{cosec} A)] \\
&= \frac{[(\cot A + \operatorname{cosec} A) - 1](\cot A + \operatorname{cosec} A)}{[(\cot A - \operatorname{cosec} A) + 1](\cot A + \operatorname{cosec} A)} \\
&= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{(\cot A - \operatorname{cosec} A)(\cot A + \operatorname{cosec} A) + (\cot A + \operatorname{cosec} A)} \\
&= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{[\cot^2 A - \operatorname{cosec}^2 A] + (\cot A + \operatorname{cosec} A)} \\
&= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{[-1 + \cot A + \operatorname{cosec} A]} \\
&\quad [\because \cot^2 A - \operatorname{cosec}^2 A = -1] \\
&= \cot A + \operatorname{cosec} A = \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(vi) L.H.S.} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
&= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
&\quad [\text{Multiplying and dividing by } \sqrt{(1 + \sin A)}] \\
&= \sqrt{\frac{(1 + \sin A)^2}{(1 - \sin^2 A)}} \\
&= \frac{\sqrt{(1 + \sin A)^2}}{\sqrt{\cos^2 A}} \quad [\because 1 - \sin^2 A = \cos^2 A] \\
&= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
&= \sec A + \tan A = \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(vii) L.H.S.} &= \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\
&= \frac{\sin \theta [(\sin^2 \theta + \cos^2 \theta) - 2 \sin^2 \theta]}{\cos \theta [2 \cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)]} \\
&= \frac{\sin \theta [\cos^2 \theta - \sin^2 \theta]}{\cos \theta [\cos^2 \theta - \sin^2 \theta]} = \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{R.H.S.}
\end{aligned}$$

$$\text{(viii) L.H.S.} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$\begin{aligned}
&= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A \\
&\quad + \sec^2 A + 2 \cos A \sec A \\
&= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2 \\
&\quad [\because \sin A \operatorname{cosec} A = 1 \text{ and } \sec A \cos A = 1] \\
&= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4 \quad [\because \sin^2 A + \cos^2 A = 1] \\
&= 5 + (1 + \cot^2 A) + (1 + \tan^2 A) \\
&= 7 + \cot^2 A + \tan^2 A = \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(ix) L.H.S.} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
&= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \\
&= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) = \frac{\cos^2 A \sin^2 A}{\sin A \cos A} \\
&\quad [\because 1 - \sin^2 A = \cos^2 A \text{ and } 1 - \cos^2 A = \sin^2 A] \\
&= \sin A \cos A \\
&= \frac{\sin A \cos A}{1} = \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A} \quad [\because 1 = \sin^2 A + \cos^2 A] \\
&= \frac{\sin A \cos A}{\sin A \cos A} \\
&= \frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A} \\
&\quad [\text{Dividing num. and den. by } \sin A \cos A] \\
&= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\tan A + \cot A} = \text{R.H.S.}
\end{aligned}$$

$$\begin{aligned}
\text{(x) We have, } \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}} \\
&= \frac{1 + \tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}} = \frac{1 + \tan^2 A}{1} \times \frac{\tan^2 A}{1 + \tan^2 A} = \tan^2 A \quad \dots \text{(i)}
\end{aligned}$$

$$\begin{aligned}
\text{Also, } \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 &= \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2 \\
&= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2 = \left(\frac{1 - \tan A}{-(1 - \tan A)} \right)^2 \\
&= \left(\frac{(1 - \tan A)}{1} \times \frac{-\tan A}{(1 - \tan A)} \right)^2 = (-\tan A)^2 = \tan^2 A \quad \dots \text{(ii)}
\end{aligned}$$

$\therefore$ From (i) and (ii), we get

$$\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

