Some Applications of Trigonometry

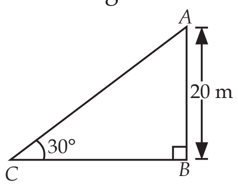
CHAPTER 9

EXAM DRILL

SOLUTIONS

1. **(d) :** Let A be the kite and AC is the string.
In right ΔABC, we have

$$\sin 30° = \frac{BA}{AC}$$



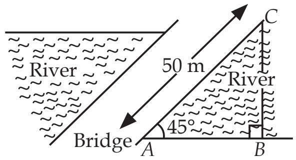
$$\Rightarrow \frac{1}{2} = \frac{20}{AC}$$

$$\Rightarrow AC = 40 \text{ m}$$

Hence, the length of the string is 40 m.

2. **(c) :** From figure, in right ΔABC, we have

$$\sin 45° = \frac{BC}{AC}$$



$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{50}$$

$$\Rightarrow BC = \frac{50}{\sqrt{2}} = 25\sqrt{2} \text{ m}$$

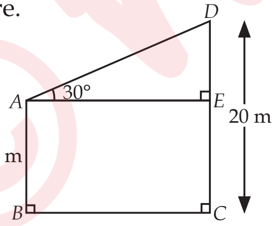
Hence, the width of the river is $25\sqrt{2}$ m.

3. **(a) :** Let $\angle ACB = \theta$
In right ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{4}{4\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow \theta = 30°$$

Now, $\angle ACB = \angle CAD = 30°$ [Alternate angles]
Hence, angle of depression from A is 30°.

4. **(b) :** Let AB and CD are two poles of height 14 m and 20 m respectively. AD is the wire.
Now, $DE = CD - CE$
$= 20 - 14 = 6$ m
In right ΔADE,

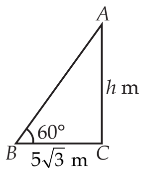


$$\tan 30° = \frac{DE}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{AE} \Rightarrow AE = 6\sqrt{3} \text{ m}$$

Hence, distance between two poles is $6\sqrt{3}$ m.

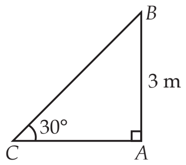
5. Let $AC = h$ m be the height of the pole.
Length of the shadow $= BC = 5\sqrt{3}$ m
It is given that the sun's elevation is 60°.
$\therefore$ $\angle B = 60°$
In right ΔACB, we have



$$\frac{AC}{BC} = \tan 60° \Rightarrow \frac{h}{5\sqrt{3}} = \sqrt{3} \Rightarrow h = 15$$

Hence, the height of the pole is 15 m.

6. Let BC be the length of ramp and AC be the horizontal path.
$AB = 3$ m and $\angle ACB = 30°$
In right ΔABC,

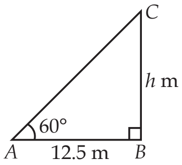


$$\sin 30° = \frac{AB}{BC} \Rightarrow \frac{1}{2} = \frac{3}{BC} \Rightarrow BC = 6 \text{ m}$$

7. $AB = 6$ m, $AD = 2.54$ m (given)
$\therefore$ $BD = AB - AD = 6 - 2.54 = 3.46$ m
Hence, in ΔBDC,

$$\frac{BD}{CD} = \sin 60° \Rightarrow \frac{3.46}{CD} = \frac{\sqrt{3}}{2} \Rightarrow CD = 4 \text{ m}$$

8. Let the height of the tower be h m.
In right ΔABC,

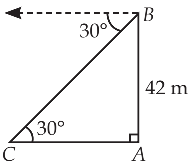


$$\tan 60° = \frac{BC}{AB} = \frac{h}{12.5}$$

$$\Rightarrow \sqrt{3} = h/12.5 \Rightarrow h = 12.5\sqrt{3}$$

$\therefore$ Height of the tower is $12.5\sqrt{3}$ m.

9. Let B be the position of the observer on the bridge and C be the position of boat.
Now, in right ΔABC,

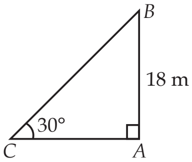


$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{42}{AC} \Rightarrow AC = 42\sqrt{3} \text{ m}$$

$\therefore$ Required distance is $42\sqrt{3}$ m.

10. Let AB is the pillar of height 18 m and AC is the shadow of AB.

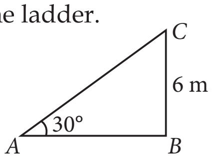


In right ΔABC, $\tan 30° = \dfrac{AB}{AC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{AC} \Rightarrow AC = 18\sqrt{3} \text{ m}$$

$\therefore$ Required length of shadow is $18\sqrt{3}$ m.

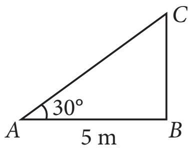
11. **(i)** **(c) :** Let AC be the length of the ladder.



In ΔABC, $\dfrac{BC}{AC} = \sin 30°$

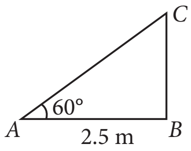
$$\Rightarrow \frac{6}{AC} = \frac{1}{2} \Rightarrow AC = 12 \text{ m}$$

(ii) **(b) :** In ΔABC, $\dfrac{AB}{AC} = \cos 30°$



$$\Rightarrow \frac{5}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{10}{\sqrt{3}} \text{ m}$$

(iii) **(a) :** Let BC be the height of window from ground.

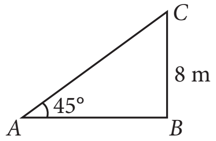


In ΔABC, $\dfrac{BC}{AB} = \tan 60°$

$\Rightarrow \quad \dfrac{BC}{2.5} = \sqrt{3}$

$\Rightarrow \quad BC = 2.5 \times 1.73 = 4.325$ m

(iv) (d) : Let AB be the horizontal distance between the foot of ladder and wall.

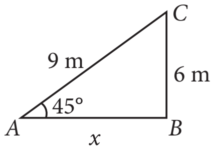


In ΔABC, $\dfrac{BC}{AB} = \tan 45°$

$\Rightarrow \quad \dfrac{8}{AB} = 1 \Rightarrow AB = 8\,\text{m}$

(v) (b) : Let the required distance be x.

In ΔABC, $(9)^2 = x^2 + (6)^2$ [By Pythagoras theorem]



$\Rightarrow \quad 81 - 36 = x^2 \Rightarrow 45 = x^2$

$\Rightarrow \quad x = 3\sqrt{5}$ m

12. (i) Total height of pole = 8 m

$\therefore \quad BD = AD - AB = (8 - 2)\text{m} = 6$ m

(ii) In ΔBDC, $\dfrac{BD}{BC} = \sin 60°$

$\Rightarrow \quad \dfrac{6}{BC} = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad BC = \dfrac{12}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$ m

(iii) In ΔBDC,

$\dfrac{BD}{CD} = \tan 60° \Rightarrow \dfrac{6}{CD} = \sqrt{3} \Rightarrow CD = \dfrac{6}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}\text{m}$

(iv) If ΔBCD,

$\dfrac{BD}{CD} = \tan\theta \Rightarrow 1 = \tan\theta$ [$\because BD = CD$]

$\Rightarrow \quad \theta = 45°$

(v) In ΔBDC, $\angle B + \angle D + \angle C = 180°$

$\therefore \quad \angle B = 180° - 60° - 90° = 30°$

13. (i) $\angle XAC = 45°$ (Given)

$\therefore \quad \angle ACD = 45°$ [Alternate interior angles]

(ii) $\angle ABD = \angle YAB = 30°$ [Alternate interior angles]

(iii) In $\Delta\, ACD$,

$\dfrac{AD}{DC} = \tan 45°$

$\Rightarrow \quad \dfrac{100}{DC} = 1 \Rightarrow DC = 100$ m

(iv) In ΔABD, $\dfrac{AD}{BD} = \tan 30°$

$\Rightarrow \quad \dfrac{100}{BD} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad BD = 100\sqrt{3}$ m

(v) In ΔADC,

$\dfrac{AD}{AC} = \sin 45° \Rightarrow \dfrac{100}{AC} = \dfrac{1}{\sqrt{2}} \Rightarrow AC = 100\sqrt{2}$ m

14. (i) (c) : In ΔOPQ, we have

$\tan 60° = \dfrac{PQ}{PO}$

$\Rightarrow \quad \sqrt{3} = \dfrac{20}{PO} \quad \Rightarrow \quad PO = \dfrac{20}{\sqrt{3}}$ m

(ii) (b) : In ΔORS, we have

$\tan 30° = \dfrac{RS}{OR} \quad \Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{20}{OR} \quad \Rightarrow \quad OR = 20\sqrt{3}$ m

(iii) (d) : Clearly, width of the road = PR

$= PO + OR = \left(\dfrac{20}{\sqrt{3}} + 20\sqrt{3}\right)\text{m}$

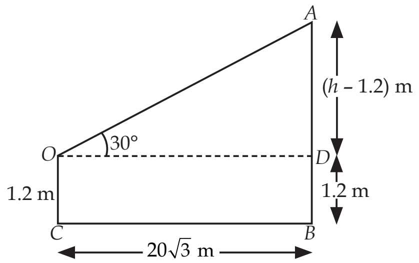
$= 20\left(\dfrac{4}{\sqrt{3}}\right)\text{m} = \dfrac{80}{\sqrt{3}}\text{m} = 46.24$ m

(iv) (a) : In ΔOPQ, if $\angle POQ = 45°$, then

$\tan 45° = \dfrac{PQ}{PO} \quad \Rightarrow \quad 1 = \dfrac{20}{PO} \quad \Rightarrow \quad PO = 20$ m

(v) (b)

15. Let CO be the observer, who is 1.2 m tall.
Let AB be the tower of height h m and $CB = 20\sqrt{3}$ m.
Let O be the point of observation of the angle of elevation of the top of tower such that $\angle AOD = 30°$.



Draw OD parallel to CB such that $OD = CB = 20\sqrt{3}$ m.
In right $\Delta\, AOD$, we have

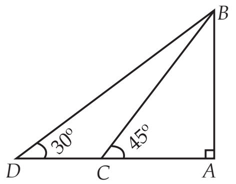
$\tan 30° = \dfrac{AD}{OD} \quad \Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h - 1.2}{20\sqrt{3}}$

$\Rightarrow \quad h - 1.2 = 20 \Rightarrow h = 21.2$

Hence, the height of the tower is 21.2 m.

16. Let AB be the tower and AC and AD are the shadows of tower AB, such that $AC + 14 = AD$

In right ΔABC,



$\tan 45° = \dfrac{AB}{AC}$

$\Rightarrow \quad 1 = \dfrac{AB}{AC} \Rightarrow AB = AC \quad$ ...(i)

In right ΔABD,

$$\tan 30° = \frac{AB}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AC+14} \Rightarrow AC + 14 = AB\sqrt{3}$$

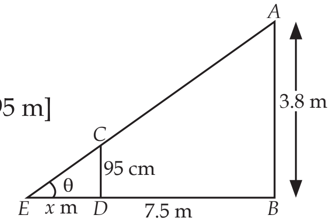
$$\Rightarrow AB\sqrt{3} - AB = 14 \quad \text{[From (i)]}$$

$$\Rightarrow AB = \frac{14}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 7(\sqrt{3}-1) = 5.124 \text{ m}$$

17. Let AB be the lamp-post and CD be the boy after walking 5 seconds. Let $DE = x$ m be the length of his shadow such that $BD = 1.5 \times 5 = 7.5$ m.

Let $\angle AEB = \angle CED = \theta$

Now, in right ΔCED,



$$\tan\theta = \frac{CD}{ED} = \frac{0.95}{x} \quad \ldots\text{(i)}$$

$[\because CD = 95 \text{ cm} = 0.95 \text{ m}]$

In right ΔABE,

$$\tan\theta = \frac{AB}{ED}$$

$$\Rightarrow \tan\theta = \frac{3.8}{7.5+x} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get

$$\frac{0.95}{x} = \frac{3.8}{7.5+x}$$

$$\Rightarrow 7.5 \times 0.95 + 0.95x = 3.8x \Rightarrow 7.125 + 0.95x = 3.8x$$

$$\Rightarrow 7.125 = 3.8x - 0.95x \Rightarrow 7.125 = 2.85x \Rightarrow x = 2.5$$

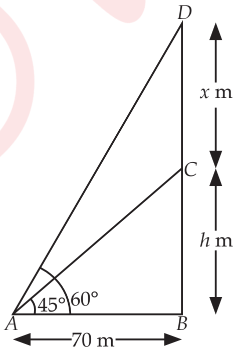
Hence, the length of his shadow after 5 seconds is 2.5 m.

18. Let A be the point on the ground which is 70 m away from the tower. Let BC be the tower of height h m and CD the flagstaff of height x m.

It is given that the angle of elevation of the top of the flagstaff from the point A is 60° and angle of elevation of the bottom of the flagstaff from the point A is 45°.

$\therefore \quad \angle CAB = 45°$ and $\angle DAB = 60°$

In right ΔCBA,



$$\tan 45° = \frac{BC}{AB} \Rightarrow 1 = \frac{h}{70}$$

$$\Rightarrow h = 70 \quad \ldots\text{(i)}$$

Now, in right ΔDBA,

$$\tan 60° = \frac{DB}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h+x}{70}$$

$$\Rightarrow \sqrt{3} = \frac{70+x}{70} \quad \text{[Using (i)]}$$

$$\Rightarrow 70\sqrt{3} = 70 + x \Rightarrow x = 70\sqrt{3} - 70$$

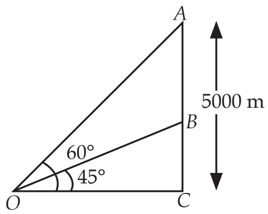
$$\Rightarrow x = 70(\sqrt{3} - 1) \Rightarrow x = 70(0.732) \Rightarrow x = 51.24$$

Hence, the height of the flagstaff is 51.24 m and the height of the tower is 70 m.

19. Let A be the first aeroplane, vertically above another aeroplane B such that $AC = 5000$ m be the height of the first aeroplane from the ground.

Let O be a point on the ground such that $\angle AOC = 60°$ and $\angle BOC = 45°$.

In right ΔAOC,



$$\tan 60° = \frac{AC}{OC}$$

$$\Rightarrow \sqrt{3} = \frac{AC}{OC}$$

$$\Rightarrow OC = \frac{5000}{\sqrt{3}} \quad [\because AC = 5000 \text{ m}]$$

$$\Rightarrow OC = \frac{5000\sqrt{3}}{3} = 2883.3 \text{ m} \quad \ldots\text{(i)}$$

In right ΔBOC,

$$\tan 45° = \frac{BC}{OC} \Rightarrow 1 = \frac{BC}{OC} \Rightarrow BC = OC$$

$$\Rightarrow BC = 2883.3 \text{ m} \quad \text{[Using (i)]} \quad \ldots\text{(ii)}$$

Thus, $AB = AC - BC$

$= 5000 - 2883.3 = 2116.7$ m

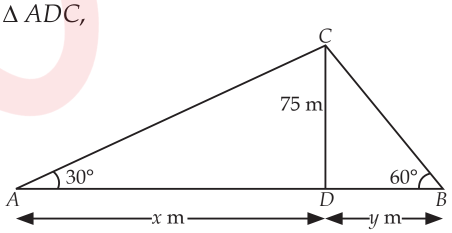
Hence, the vertical distance between the two aeroplanes is 2116.7 m.

20. Let $CD = 75$ m be the height of the building. Let A and B be the points of observations such that the angle of elevation at A is 30° and the angle of elevation at B is 60°.

$\therefore \quad \angle CAD = 30°$ and $\angle CBD = 60°$

Let $AD = x$ m and $DB = y$ m.

In right ΔADC,



$$\tan 30° = \frac{CD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow x = 75\sqrt{3} \text{ m} \quad \ldots\text{(i)}$$

In right ΔBDC,

$$\tan 60° = \frac{CD}{DB} \Rightarrow \sqrt{3} = \frac{75}{y} \Rightarrow y = \frac{75}{\sqrt{3}} \text{ m} \quad \ldots\text{(ii)}$$

The distance between two men is AB, *i.e.*,

$AB = AD + DB = x + y$

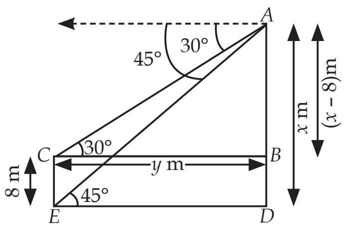
$$\Rightarrow AB = \left(75\sqrt{3} + \frac{75}{\sqrt{3}}\right) \quad \text{[Using (i) and (ii)]}$$

$$\Rightarrow AB = \left(\frac{225+75}{\sqrt{3}}\right) = \frac{300}{\sqrt{3}}$$

$$100\sqrt{3} = 100 \times 1.73 \Rightarrow AB = 173 \text{ m}$$

Hence, the distance between the two men is 173 m.

21. Let CE be the 8 m tall building and AD be the multistoried building of height x m.



Let $BC = DE = y$ m, be the distance between the two buildings.

Then, $AB = AD - BD$

$= AD - CE = (x - 8)$ m

$\therefore \quad \angle BCA = 30°$ and $\angle DEA = 45°$

In right ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x-8}{y}$$

$\Rightarrow \quad y = \sqrt{3}(x-8)$ ...(i)

In right ΔADE,

$$\tan 45° = \frac{AD}{DE} \Rightarrow 1 = \frac{x}{y}$$

$\Rightarrow \quad y = x$ ...(ii)

From (i) and (ii), we get

$$\sqrt{3}(x-8) = x \Rightarrow \sqrt{3}x - 8\sqrt{3} = x \Rightarrow \sqrt{3}x - x = 8\sqrt{3}$$

$$\Rightarrow \quad x = \frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{8(3+\sqrt{3})}{3-1}$$

$\Rightarrow \quad x = 4(3+\sqrt{3})$ ...(iii)

From (ii) and (iii), we get

$y = 4(3+\sqrt{3})$

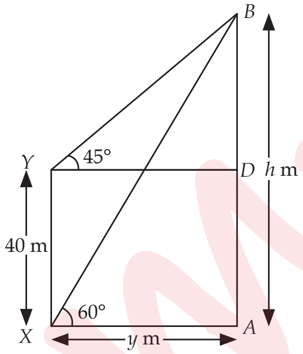
So, the height of the multistoried building is $4(3+\sqrt{3})$ m and the distance between the two buildings is also $4(3+\sqrt{3})$ m.

22. Let AB be the vertical tower of height h m and $YX = 40$ m. Let $YD = XA = y$ m, where X is a point on the ground.

Then, $BD = AB - AD$

$\Rightarrow \quad BD = (h-40)$ m

In right ΔBDY, we have



$$\tan 45° = \frac{BD}{YD}$$

$$\Rightarrow \quad 1 = \frac{h-40}{y}$$

$\Rightarrow \quad y = (h-40)$ ...(i)

In right ΔBAX, we have

$$\tan 60° = \frac{AB}{XA}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{y} \Rightarrow y = \frac{h}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we get

$$h - 40 = \frac{h}{\sqrt{3}} \Rightarrow h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \sqrt{3}h - h = 40\sqrt{3} \Rightarrow h(\sqrt{3}-1) = 40\sqrt{3}$$

$$\Rightarrow \quad h = \frac{40\sqrt{3}}{\sqrt{3}-1} = \frac{40\sqrt{3}\times(\sqrt{3}+1)}{(\sqrt{3}-1)\times(\sqrt{3}+1)} = \frac{40(3+\sqrt{3})}{3-1}$$

$\Rightarrow \quad h = 20(3+\sqrt{3})$ ...(iii)

In right ΔBAX, we have

$$\cos 60° = \frac{AX}{XB} \Rightarrow \frac{1}{2} = \frac{y}{XB}$$

$\Rightarrow XB = 2y \Rightarrow XB = 2(h-40)$ [Using (i)]

$\Rightarrow \quad XB = 2[20(3+\sqrt{3}) - 40]$ [Using (iii)]

$\Rightarrow \quad XB = 2[60 + 20\sqrt{3} - 40] = 2[20 + 20\sqrt{3}]$

$\Rightarrow \quad XB = 40(1+\sqrt{3})$ m

Hence, the height of the tower AB is $20(3+\sqrt{3})$m and the distance XB is $40(1+\sqrt{3})$m.

23. Let AB be the tower of height h m and let the angle of elevation of its top at C be α *i.e.*, $\angle ACB = \alpha$. Let D be a point at a distance of 192 metres from C such that $\angle ADB = \beta$ and $AD = x$ m.

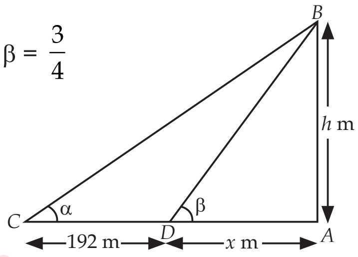
It is given that

$$\tan\alpha = \frac{5}{12} \text{ and } \tan\beta = \frac{3}{4}$$

In right ΔCAB, we have

$$\tan\alpha = \frac{AB}{AC}$$

$\Rightarrow \quad \frac{5}{12} = \frac{h}{x+192}$ ...(i)



In right ΔDAB, we have

$\tan\beta = \frac{AB}{AD} \Rightarrow \tan\beta = \frac{h}{x} \Rightarrow \frac{3}{4} = \frac{h}{x} \Rightarrow x = \frac{4h}{3}$ ...(ii)

$\Rightarrow \quad \frac{5}{12} = \frac{h}{192 + 4h/3}$ [From (i) and (ii)]

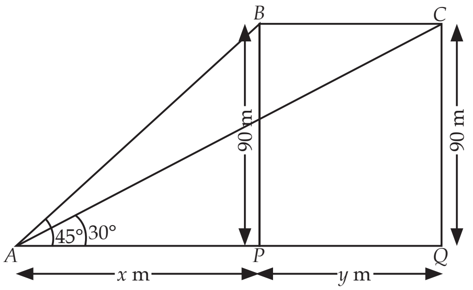
$$\Rightarrow \quad 5\left(192 + \frac{4h}{3}\right) = 12h \Rightarrow 5(576 + 4h) = 36h$$

$$\Rightarrow \quad 2880 + 20h = 36h \Rightarrow 16h = 2880$$

$$\Rightarrow \quad h = \frac{2880}{16} = 180$$

Hence, the height of the tower is 180 m.

24. Let B and C (after 3 seconds) be the two positions of the bird as observed from a point A on the ground.



Given, $\angle BAP = 45°$ and $\angle CAQ = 30°$ and $BP = CQ = 90$ m

Let $AP = x$ m and $PQ = y$ m.

In right ΔAPB, we have

$$\tan 45° = \frac{BP}{AP} \Rightarrow 1 = \frac{90}{x}$$

$\Rightarrow \quad x = 90$ ...(i)

In right ΔAQC, we have

$$\tan 30° = \frac{CQ}{AQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90}{x+y}$$

$\Rightarrow \quad x + y = 90\sqrt{3} \Rightarrow 90 + y = 90\sqrt{3}$ [Using (i)]

$\Rightarrow \quad y = 90(\sqrt{3}-1) = 65.7$

Distance covered by the bird in 3 seconds = 65.7 m

Distance covered by the bird in 1 second = $\frac{65.7}{3}$ m = 21.9 m

Hence, the speed of the bird is 21.9 m/sec.