

EXERCISE - 10.1

1. A circle can have an infinite number of tangents.

2. (i) exactly one (ii) secant
(iii) two (iv) point of contact

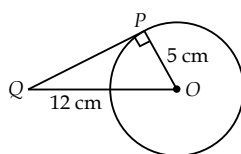
3. (d) : PQ is a tangent to the circle.

$\therefore OP \perp PQ$

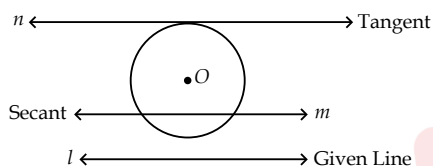
In right $\triangle POQ$, $OQ^2 = OP^2 + PQ^2$

$$\Rightarrow PQ = \sqrt{OQ^2 - OP^2}$$

$$= \sqrt{12^2 - 5^2} = \sqrt{144 - 25} = \sqrt{119} \text{ cm}$$



4. We have the required figure, as shown



Here, l is the given line and a circle with centre O is drawn.

The line n is drawn which is parallel to l and tangent to the circle. Also, m is drawn parallel to line l and is a secant to the circle.

EXERCISE - 10.2

1. (a) $\because QT$ is a tangent to the circle at T and OT is radius.

$\therefore OT \perp QT$

Also, $OQ = 25$ cm

and $QT = 24$ cm

\therefore Using Pythagoras theorem in $\triangle QTO$, we get

$$OQ^2 = QT^2 + OT^2$$

$$\Rightarrow OT^2 = OQ^2 - QT^2 = 25^2 - 24^2 = 49 \Rightarrow OT = 7 \text{ cm}$$

Thus, the required radius is 7 cm.

2. (b) $\because TQ$ and TP are tangents to a circle with centre O .

$\therefore OP \perp PT$ and $OQ \perp QT \Rightarrow \angle OPT = \angle OQT = 90^\circ$

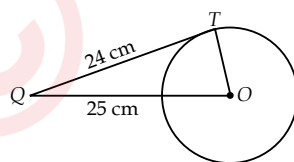
Now, in the quadrilateral $TPOQ$,

$$\angle PTQ + 90^\circ + 110^\circ + 90^\circ = 360^\circ$$

[By angle sum property of a quadrilateral]

$$\Rightarrow \angle PTQ + 290^\circ = 360^\circ \Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$$

3. (a) : Since, O is the centre of the circle and two tangents from P to the circle are PA and PB .



$\therefore OA \perp AP$ and $OB \perp BP$

$\Rightarrow \angle OAP = \angle OBP = 90^\circ$

Now, in quadrilateral

$PAOB$, we have

$$\angle APB + \angle PAO + \angle AOB + \angle PBO = 360^\circ$$

$$\Rightarrow 80^\circ + 90^\circ + \angle AOB + 90^\circ = 360^\circ$$

$$\Rightarrow 260^\circ + \angle AOB = 360^\circ$$

$$\Rightarrow \angle AOB = 360^\circ - 260^\circ$$

$$\Rightarrow \angle AOB = 100^\circ$$

In right $\triangle OAP$ and right $\triangle OBP$, we have

$OP = OP$ [Common]

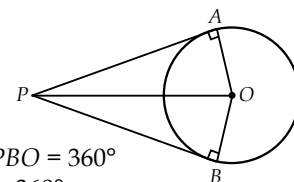
$\angle OAP = \angle OBP$ [Each 90°]

$OA = OB$ [Radii of the same circle]

$\therefore \triangle OAP \cong \triangle OBP$ [By RHS congruence criterion]

$\Rightarrow \angle POA = \angle POB$ [By CPCT]

$$\therefore \angle POA = \frac{1}{2} \angle AOB = \frac{1}{2} \times 100^\circ = 50^\circ.$$



4. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ .

Since, the tangent at a point to a circle is perpendicular to the radius through the point of contact.

$\therefore PQ \perp AB$

$\Rightarrow \angle APQ = 90^\circ$

And $PQ \perp CD$

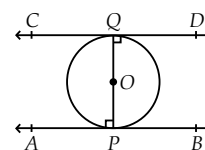
$\Rightarrow \angle PQD = 90^\circ$

$\Rightarrow \angle APQ = \angle PQD$

But they form a pair of alternate angles.

$\therefore AB \parallel CD$.

Hence, the two tangents are parallel.



5. In the figure, the centre of the circle is O and tangent AB touches the circle at P . If possible, let PQ be perpendicular to AB such that it is not passing through O . Join OP .

Since tangent at a point to a circle is perpendicular to the radius through that point.

$\therefore OP \perp AB$

$\Rightarrow \angle OPB = 90^\circ$... (1)

But by construction,

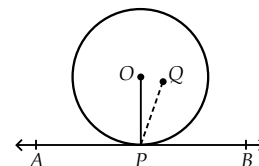
$PQ \perp AB$

$\Rightarrow \angle QPB = 90^\circ$... (2)

From (1) and (2),

$\angle QPB = \angle OPB$,

which is possible only when O and Q coincide. Thus, the perpendicular at the point of contact to the tangent to a circle passes through the centre.



6. \therefore The tangent to a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OTA = 90^\circ$$

Now, in the right $\triangle OTA$, we have

$$OA^2 = OT^2 + AT^2 \quad [\text{By Pythagoras theorem}]$$

$$\Rightarrow 5^2 = OT^2 + 4^2$$

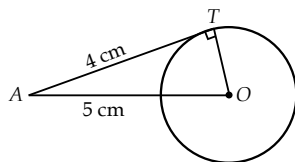
$$\Rightarrow OT^2 = 5^2 - 4^2$$

$$\Rightarrow OT^2 = 25 - 16$$

$$\Rightarrow OT^2 = 9 = 3^2$$

$$\Rightarrow OT = 3 \text{ cm}$$

Thus, the radius of the circle is 3 cm.



7. In the figure, O is the common centre, of the given concentric circles.

AB is a chord of the bigger circle such that it is a tangent to the smaller circle at P.

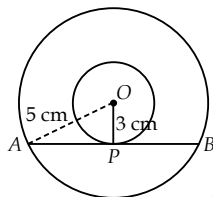
Since OP is the radius of the smaller circle.

$$\therefore OP \perp AB \Rightarrow \angle APO = 90^\circ$$

Also, radius perpendicular to a chord bisects the chord.

$$\therefore OP \text{ bisects } AB$$

$$\Rightarrow AP = \frac{1}{2} AB$$



Now, in right $\triangle APO$, $OA^2 = AP^2 + OP^2$

$$\Rightarrow 5^2 = AP^2 + 3^2 \Rightarrow AP^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\Rightarrow AP^2 = 4^2 \Rightarrow AP = 4 \text{ cm}$$

$$\Rightarrow \frac{1}{2} AB = 4 \Rightarrow AB = 2 \times 4 = 8 \text{ cm}$$

Hence, the required length of the chord AB is 8 cm.

8. Since, the sides of quadrilateral ABCD, i.e., AB, BC, CD and DA touch the circle at P, Q, R and S respectively, and the lengths of two tangents to a circle from an external point are equal.

$$\therefore AP = AS, BP = BQ, CR = CQ \text{ and } DR = DS$$

Adding all these equations, we get

$$(AP + BP) + (CR + DR) = (BQ + CQ) + (DS + AS)$$

$$\Rightarrow AB + CD = BC + DA$$

9. Join OC.

\therefore The tangents drawn to a circle from an external point are equal.

$$\therefore AP = AC \quad \dots(1)$$

In $\triangle PAO$ and $\triangle CAO$,

we have,

$$AO = AO$$

$$OP = OC$$

$$AP = AC$$

[Common]

[Radii of the same circle]

[Using (1)]

$$\therefore \triangle PAO \cong \triangle CAO \quad [\text{By SSS congruency criterion}]$$

$$\Rightarrow \angle PAO = \angle CAO$$

[By CPCT]

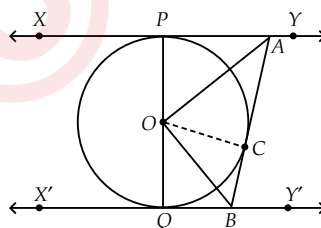
$$\Rightarrow \angle PAC = 2\angle CAO \quad \dots(2)$$

$$\text{Similarly, } \angle CBQ = 2\angle CBO \quad \dots(3)$$

Again, we know that sum of internal angles on the same side of a transversal is 180° .

$$\therefore \angle PAC + \angle CBQ = 180^\circ$$

$$\Rightarrow 2\angle CAO + 2\angle CBO = 180^\circ \quad [\text{From (2) and (3)}]$$



$$\Rightarrow \angle CAO + \angle CBO = \frac{180^\circ}{2} = 90^\circ \quad \dots(4)$$

$$\text{Also, in } \triangle AOB, \angle BAO + \angle ABO + \angle AOB = 180^\circ$$

[Sum of angles of a triangle]

$$\Rightarrow \angle CAO + \angle CBO + \angle AOB = 180^\circ$$

$$\Rightarrow 90^\circ + \angle AOB = 180^\circ$$

[Using (4)]

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ \Rightarrow \angle AOB = 90^\circ.$$

10. Let PA and PB be two tangents drawn from an external point P to a circle with centre O.

Now, in right $\triangle OAP$

and right $\triangle OBP$, we have

$$PA = PB \quad [\text{Tangents to a circle from an external point}]$$

$$OA = OB$$

[Radii of the same circle]

$$OP = OP$$

[Common]

$$\therefore \triangle OAP \cong \triangle OBP \quad [\text{By SSS congruency criterion}]$$

$$\Rightarrow \angle OPA = \angle OPB \text{ and } \angle AOP = \angle BOP \quad [\text{By CPCT}]$$

$$\Rightarrow \angle APB = 2\angle OPA \text{ and } \angle AOB = 2\angle AOP \quad \dots(1)$$

$$\text{But } \angle AOP = 90^\circ - \angle OPA \Rightarrow 2\angle AOP = 180^\circ - 2\angle OPA$$

$$\Rightarrow \angle AOB = 180^\circ - \angle APB$$

[Using (1)]

$$\Rightarrow \angle AOB + \angle APB = 180^\circ.$$

11. We have ABCD, a parallelogram which circumscribes a circle (i.e., its sides touch the circle) with centre O.

Since, tangents to a circle from an external point are equal in length.

$$\therefore AP = AS \Rightarrow BP = BQ$$

$$CR = CQ \Rightarrow DR = DS$$

Adding above all, we get

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\text{But } AB = CD$$

[\therefore Opposite sides of parallelogram]

$$\text{and } BC = AD$$

$$\therefore AB + CD = AD + BC \Rightarrow 2AB = 2BC \Rightarrow AB = BC$$

$$\text{Similarly, } AB = DA \text{ and } DA = CD$$

$$\text{Thus, } AB = BC = CD = AD$$

Hence, ABCD is a rhombus.

12. Join OA, OE, OF, OB and OC.

Here, $\triangle ABC$ circumscribe the circle with centre O. Also, radius = 4 cm

\therefore The sides BC, CA and AB touch the circle at D, E and F respectively.

$$\therefore BF = BD = 8 \text{ cm}$$

$$CD = CE = 6 \text{ cm}$$

$$AF = AE = x \text{ cm (say)}$$

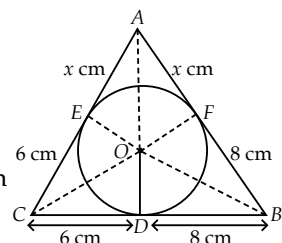
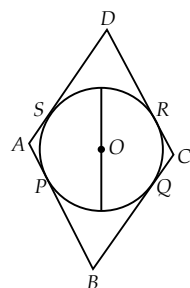
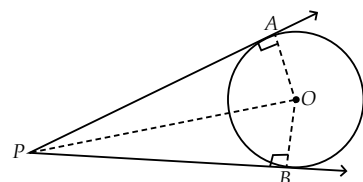
So, the sides of the triangle are

$$14 \text{ cm, } (x + 6) \text{ cm and } (x + 8) \text{ cm}$$

$$\text{Perimeter of } \triangle ABC$$

$$= [14 + (x + 6) + (x + 8)] \text{ cm}$$

$$= [14 + 6 + 8 + 2x] \text{ cm} = (28 + 2x) \text{ cm}$$



⇒ Semi perimeter of $\triangle ABC$ (s)

$$= \frac{1}{2}[28 + 2x] \text{ cm} = (14 + x) \text{ cm},$$

where $a = BC$, $b = AC$, $c = AB$

$$\therefore s - a = (14 + x) - (14) = x$$

$$s - b = (14 + x) - (6 + x) = 8$$

$$s - c = (14 + x) - (8 + x) = 6$$

$$\therefore \text{Area of } \triangle ABC = \sqrt{(14+x)(x)(8)(6)} \text{ cm}^2$$

$$= \sqrt{(14+x)48x} \text{ cm}^2 \quad \dots(1)$$

$$\text{Now, } ar(\triangle OBC) = \frac{1}{2} \times BC \times OD$$

$$= \frac{1}{2} \times 14 \times 4 = 28 \text{ cm}^2 \quad [\because OD = \text{Radius}]$$

$$ar(\triangle OCA) = \frac{1}{2} CA \times OE = \frac{1}{2} \times (x+6) \times 4 = (2x+12) \text{ cm}^2$$

$$ar(\triangle OAB) = \frac{1}{2} AB \times OF = \frac{1}{2} \times (x+8) \times 4 = (2x+16) \text{ cm}^2$$

$$\therefore ar(\triangle ABC) = ar(\triangle OBC) + ar(\triangle OCA) + ar(\triangle OAB)$$

$$= 28 \text{ cm}^2 + (2x+12) \text{ cm}^2 + (2x+16) \text{ cm}^2$$

$$= (28+12+16+4x) \text{ cm}^2 = (56+4x) \text{ cm}^2 \quad \dots(2)$$

From (1) and (2), we have

$$56 + 4x = \sqrt{(14+x)48x}$$

$$\Rightarrow 4(14+x) = 4\sqrt{(14+x) \times 3x}$$

$$\Rightarrow 14+x = \sqrt{(14+x)3x}$$

Squaring both sides, we get $(14+x)^2 = (14+x)3x$

$$\Rightarrow 196 + x^2 + 28x = 42x + 3x^2$$

$$\Rightarrow 2x^2 + 14x - 196 = 0$$

$$\Rightarrow x^2 + 7x - 98 = 0$$

$$\Rightarrow (x-7)(x+14) = 0$$

$$\Rightarrow x-7=0 \text{ or } x+14=0 \Rightarrow x=7 \text{ or } x=-14$$

But $x = -14$ is rejected.

$$\therefore x = 7 \text{ cm}$$

Thus, $AB = 8 + 7 = 15 \text{ cm}$

and $CA = 6 + 7 = 13 \text{ cm}$.

13. We have a circle with centre O . A quadrilateral $ABCD$ is such that the sides AB , BC , CD and DA touch the circle at P , Q , R and S respectively.

Let us join OP , OQ , OR and OS .

We know that tangents drawn from an external point to a circle subtend equal angles at the centre.

$$\therefore \angle 1 = \angle 2$$

$$\angle 3 = \angle 4$$

$$\angle 5 = \angle 6$$

$$\text{and } \angle 7 = \angle 8$$

Also, the sum of all the angles around a point is 360° .

$$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$\therefore 2(\angle 1 + \angle 8 + \angle 5 + \angle 4) = 360^\circ$$

$$\Rightarrow (\angle 1 + \angle 8) + (\angle 5 + \angle 4) = 180^\circ \quad \dots(1)$$

$$\text{And } 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ$$

$$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ \quad \dots(2)$$

Since, $\angle 2 + \angle 3 = \angle AOB$,

$$\angle 6 + \angle 7 = \angle COD$$

$$\angle 1 + \angle 8 = \angle AOD,$$

$$\angle 4 + \angle 5 = \angle BOC$$

\therefore From (1) and (2), we have

$$\angle AOD + \angle BOC = 180^\circ$$

$$\text{and } \angle AOB + \angle COD = 180^\circ$$

