Areas Related to Circles

CHAPTER 12

EXAM DRILL

SOLUTIONS

1. **(a) :** We have, $\theta = 60°$ and $r = 6$ cm

$\therefore$ Area of sector $= \frac{\theta}{360°} \times \pi r^2$

$\Rightarrow$ Area $= \frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

$\Rightarrow$ Area $= \frac{1}{6} \times \frac{22}{7} \times 36 = 18.86 \text{ cm}^2$

2. **(c) :** The sum of the angles subtended by the minor and major sectors of the circle at the centre of the circle is 360°. So, the sum of their area is equal to area of the circle *i.e.*, πr^2, where r is the radius of the circle.

3. **(b) :** Here, diameter = 8 cm and $\theta = 120°$

$\therefore$ Area of sector $= \frac{\theta}{360°} \times \pi r^2 = \frac{120°}{360°} \times \pi \left(\frac{8}{2}\right)^2 = \frac{16}{3} \pi \text{ cm}^2$

4. **(a) :** Since the minute hand rotates through 6° in one minute.

$\therefore$ Required area = Area of sector with sector angle 6° and radius 7 cm

$= \frac{6°}{360°} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{60} \times 22 \times 7 = 2.57 \text{ cm}^2$

5. Circumference of circle = 44 cm [Given]

$\Rightarrow 2\pi r = 44 \Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7$ cm

$\therefore$ Area of sector of circle $= \frac{90°}{360°} \times \frac{22}{7} \times 7 \times 7 = 38.5 \text{ cm}^2$

6. Here, $\theta = x°$ and radius, $R = 2r$

Area of sector $= \frac{\theta}{360°} \times \pi R^2 = \frac{x°}{360°} \times \pi (2r)^2 = \frac{x°}{90°} \pi r^2$

7. Length of the minute hand = radius of the circle $(r) = \sqrt{21}$ cm

Area swept by minute hand in 60 minutes = 360°

$\therefore$ Area swept by minute hand in 10 minutes

$= \frac{360°}{60} \times 10 = 60°$

Now, area of sector with $r = \sqrt{21}$ cm and $\theta = 60°$ is

$\frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times (\sqrt{21})^2 = \frac{1}{6} \times \frac{22}{7} \times 21 = 11 \text{ cm}^2$

Hence, the area swept by the minute hand in 10 minutes is 11 cm^2.

8. Let radius of circle be r and length of arc be l.

Perimeter of a sector of a circle = 24.4 cm [Given]

$\Rightarrow 2r + l = 24.4$

$\Rightarrow 2 \times 4.3 + l = 24.4 \Rightarrow l = 24.4 - 8.6 = 15.8$ cm

Area of sector $= \frac{1}{2} \times lr = \frac{1}{2} \times 15.8 \times 4.3 = 33.97 \text{ cm}^2$

9. Let r be the radius of circle. It is given that

Area of sector of circle $= \frac{3}{18} \times$ Area of the same circle

$\Rightarrow \frac{\theta}{360°} \times \pi r^2 = \frac{3}{18} \times \pi r^2 \Rightarrow \theta = \frac{3}{18} \times 360° = 60°$

10. **(i)** **(a) :** In a week, Kartik drives his motorbike 3 days to go to college

$\therefore$ Total distance travelled by Kartik through motorbike = 2 × 4.2 × 6 = 50.4 km

(ii) **(b) :** In a week Kartik rides his bicycle 3 days to go to college.

$\therefore$ Total distance travelled by Kartik through bicycle

= Length of arc $\overparen{ACB} \times 6$

$= \frac{\theta}{360°} \times 2\pi r \times 6 = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 4.2 \times 6 = 26.4$ km

(iii) **(c) :** Area of sector $AOB = \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times (4.2)^2 = 9.24 \text{ km}^2$

(iv) **(a) :** Total cost of fuel used for a week

= ₹ (20 × 50.4) = ₹ 1008

(v) **(d) :** Total length of available paths

$= 4.2 + 4.2 + \frac{\theta}{360°} \times 2\pi r$

$= 8.4 + \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 4.2 = 8.4 + 6.6 = 15$ km

11. **(i)** **(b) :** Angle made by minute hand in 60 minutes = 360°

$\therefore$ Angle made by minute hand in 10 minutes

$= \frac{360°}{60} \times 10 = 60°$

Length of minute hand = 9 cm

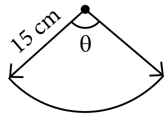
$\therefore$ Area swept by minute hand in 10 minutes

= Area of sector having central angle 60°

$= \pi r^2 \left(\frac{60°}{360°}\right) = \frac{22}{7} \times 9 \times 9 \times \frac{1}{6}$

$= \frac{297}{7} = 42.42 \text{ cm}^2$

(ii) (b) : We have, $r = 15$ cm



and $l = \frac{1}{2}(22) = 11$ cm

We known that $l = 2\pi r\left(\frac{\theta}{360°}\right)$

$$\Rightarrow \theta = \frac{11\times 360°}{2\times\frac{22}{7}\times 15} = \frac{90°\times 7}{15} = 6° \times 7 = 42°$$

(iii) (a) **:** Angle made by hour hand in 12 hours = 360°

$\therefore$ Angle made by hour hand in 10 minutes

$= \left(\frac{360°}{12}\times\frac{1}{6}\right) = 5°$

(iv) (d) : Angle made by hour hand in 1 hour

$= \frac{360°}{12} = 30°$

Also, $r = 6$ cm

$\therefore$ Area swept by hour hand in 1 hour

= Area of sector having central angle 30°

$= \pi r^2 \times \left(\frac{30°}{360°}\right) = \frac{22}{7} \times 6 \times 6 \times \frac{1}{12} = \frac{66}{7} = 9.428 \text{ cm}^2$

(v) (a) : Number of hours from 11 a.m. to 5 p.m. = 6

Area swept by hour hand in 1 hour = 9.428 cm^2

$\therefore$ Area swept by hour hand in 6 hours = 9.428 × 6

= 56.568 cm^2

12. (i) Area of sector $ODCO = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\times\frac{22}{7}\times 14\times 14 = 154 \text{ cm}^2$

(ii) Area of $\Delta AOB = \frac{1}{2}\times OA\times OB = \frac{1}{2}(20\times 20)$

= 200 cm^2

(iii) Area of major sector = area of circle − area of minor sector

$= \pi r^2 - \frac{1}{4}\pi r^2 = \frac{3\pi r^2}{4} = \frac{3}{4}\times\frac{22}{7}\times 14\times 14 = 462 \text{ cm}^2$

(iv) Length of arc $DC = \frac{90°}{360°}\times 2\times\frac{22}{7}\times 14$

= 22 cm

(v) Area of glass trophy = Area of circle + Area of triangle - area of sector of circle.

$= \pi r^2 + 200 - 154 = 616 + 200 - 154$

= 662 cm^2

13. (i) (c) : Area of the region containing blue colour

$= \frac{22}{7}\times 5\times 5\times\frac{80°}{360°} - \frac{22}{7}\times 3\times 3\times\frac{80°}{360°}$

$= \frac{22}{7}\times\frac{2}{9}\times[25-9] = \frac{44}{63}(16) = 11.17 \text{ cm}^2$

(ii) (b) : Area of the region containing green colour

$= \frac{22}{7}\times\frac{60°}{360°}[5\times 5 - 3\times 3] = \frac{22}{7}\times\frac{1}{6}\times 16 = 8.38 \text{ cm}^2$

(iii) (d) : Perimeter of the region containing red colour = 2 + 2 + length of arc of sector having radius 3 cm + length of arc of sector having radius 5 cm

$= 4 + 2\times\frac{22}{7}\times 3\times\frac{20°}{360°} + 2\times\frac{22}{7}\times 5\times\frac{20°}{360°}$

$= 4 + \frac{44}{7}\times\frac{1}{18}\times 8 = 4 + \frac{176}{63} = 4 + 2.79 = 6.79$ cm

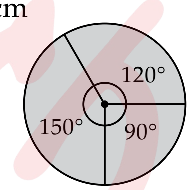
(iv) (a) : Required area $= \frac{22}{7}\times 3\times 3\times\frac{160°}{360°}$

$= \frac{88}{7} = 12.57 \text{ cm}^2$

(v) (a) : Angle of given sector = 80° + 60° + 20° = 160°

Thus, the given region represents minor sector of a circle.

14. Given $r = 6$ cm



Area of sector with central angle $\theta = \frac{\pi r^2\theta}{360°}$

$\therefore$ Ratio of the areas of the three sectors

$= \pi r^2 \times \frac{120°}{360°} : \pi r^2 \times \frac{150°}{360°} : \pi r^2 \times \frac{90°}{360°}$

= 120° : 150° : 90° = 4 : 5 : 3

15. Given that, radius of the sector (r) = 5 cm

and length of an arc (l) = 3.5 cm

Now, length of an arc with sector angle θ and

$\text{radius}(r) = \frac{\theta}{360°}\times 2\pi r$

$3.5 = \frac{\theta}{360°}\times 2\times\frac{22}{7}\times 5 \Rightarrow \frac{35}{10}\times\frac{7}{22}\times\frac{360°}{2\times 5} = \theta$

$\Rightarrow \theta = \frac{49\times 9}{11} = \left(\frac{441}{11}\right)^{\circ}$

Area of the sector $= \frac{\theta}{360°}\times\pi r^2$

$= \left(\frac{441}{11}\right)^{\circ}\times\frac{1}{360°}\times\frac{22}{7}\times 5\times 5 = \frac{5\times 7}{4} = \frac{35}{4} = 8.75 \text{ cm}^2$

Hence, the required area of the sector of a circle is 8.75 cm^2.

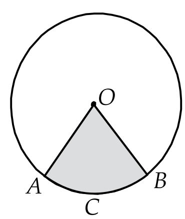
16. Let O be the centre of a circle of radius 5.6 cm, and let $OACBO$ be its sector with perimeter 27.2 cm.

Then, $OA + OB$ + arc ACB = 27.2 cm

$\Rightarrow$ 5.6 cm + 5.6 cm + arc ACB = 27.2 cm

$\Rightarrow$ arc ACB = 16 cm.

Area of the sector $OACBO$

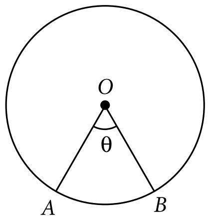


$= \left(\frac{1}{2}\times\text{radius}\times\text{arc}\right)\text{sq. units}$

$= \left(\frac{1}{2}\times 5.6\times 16\right)\text{cm}^2 = 44.8 \text{ cm}^2$

17. Given, length of arc $AB = 5\pi$ cm

Let $\angle AOB = \theta$



We know that, $l = \dfrac{\theta}{360^\circ} \times 2\pi r$

$\Rightarrow \quad 5\pi = \dfrac{\theta}{180^\circ} \times \pi r \Rightarrow \dfrac{900^\circ}{r} = \theta$

Now, area of sector $= \dfrac{\theta}{360^\circ} \times \pi r^2$

$\Rightarrow \quad 20\pi = \dfrac{900^\circ}{r(360^\circ)} \times \pi r^2 \Rightarrow 20 = \dfrac{900^\circ}{360^\circ} r$

$\Rightarrow \quad r = 8$ cm

18. Perimeter of sector of circle

$= \dfrac{\theta}{360^\circ} \times 2\pi r + 2r \Rightarrow 19.5 = \dfrac{45^\circ}{360^\circ} \times 2 \times \dfrac{22}{7} r + 2r$

$\Rightarrow \quad 19.5 = r\left[\dfrac{11}{14} + 2\right]$

$\Rightarrow \quad r = \dfrac{19.5 \times 14}{39} = 7\text{ cm}$

19. Since, ΔABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60^\circ$ and $AB = BC = CA = 10$ cm

E, F and D are mid-points of the given sides.

$\therefore \quad AE = EC = CD = DB = BF = FA = 5$ cm

Radius of a sector $(r) = 5$ cm

Now, area of sector $CDE = \dfrac{60^\circ}{360^\circ} \times 3.14 \times (5)^2$

$= \dfrac{78.5}{6} = 13.0833\text{ cm}^2$

$\therefore$ Area of shaded region = 3 × (Area of sector CDE)

$= (3 \times 13.0833)\text{ cm}^2 = 39.25\text{ cm}^2$

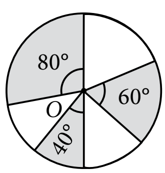
20. Radius (r) of circle = 7 cm

Area of shaded region

$= \dfrac{\pi(7)^2 \cdot 40^\circ}{360^\circ} + \dfrac{\pi(7)^2 \cdot 60^\circ}{360^\circ} + \dfrac{\pi(7)^2 \cdot 80^\circ}{360^\circ}$

$\left[\because \text{ Area of sector} = \dfrac{\theta}{360^\circ}\pi r^2\right]$

$= \dfrac{\pi(7)^2}{9} + \dfrac{\pi(7)^2}{6} + \dfrac{\pi(7)^2 \cdot 2}{9}$



$= \pi(7)^2\left[\dfrac{1}{9} + \dfrac{1}{6} + \dfrac{2}{9}\right]$

$= \dfrac{22}{7} \times 7 \times 7 \times \dfrac{9}{18} = 77\text{ cm}^2$

21. We have, radius $(r) = 10$ cm and $\theta = 90^\circ$

So, area of sector $OAPB = \dfrac{\theta}{360^\circ}\pi r^2$

$= \dfrac{90^\circ}{360^\circ} \times 3.14 \times 10^2 = 78.5\text{ cm}^2$

Area of $\Delta OAB = \dfrac{1}{2} \times 10 \times 10 = 50\text{ cm}^2$

$\therefore$ Area of the minor segment $AQBP$ = Area of sector $OAPB$ – Area of $\Delta OAB = (78.5 - 50)\text{ cm}^2 = 28.5\text{ cm}^2$

Area of circle $= \pi r^2 = 3.14 \times 10^2 = 314\text{ cm}^2$

$\therefore$ Area of major segment $ALBQA$

= Area of circle – Area of minor segment $AQBP$

$= (314 - 28.5)\text{ cm}^2 = 285.5\text{ cm}^2$

22. Area of region $ABDC$ = (Area of sector OAB) – (Area of sector OCD)

$= \dfrac{\pi(OA)^2 \times 60^\circ}{360^\circ} - \dfrac{\pi(OC)^2 \times 60^\circ}{360^\circ}$

$= \dfrac{\pi}{6} \times (42)^2 - \dfrac{\pi}{6} \times (21)^2$ [$\because$ $OA = 42$ cm and $OC = 21$ cm]

$= \dfrac{\pi}{6}(1764 - 441) = \dfrac{1323\pi}{6}\text{ cm}^2$

Area of circular ring $= \pi[(42)^2 - (21)^2]$

$= \pi(1764 - 441) = 1323\,\pi\text{ cm}^2$

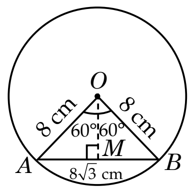
$\therefore$ Area of shaded region = Area of circular ring – Area of region $ABDC$

$= 1323\pi - \dfrac{1323\pi}{6} = 1323\pi\left(1 - \dfrac{1}{6}\right)$

$= 1323 \times \dfrac{22}{7} \times \dfrac{5}{6} = 3465\text{ cm}^2$

23. Given diameter of circle = 16 cm

$\therefore$ Radius of circle $= \dfrac{16}{2} = 8\text{ cm}$



Draw, $OM \perp AB$

Then, OM bisects the side AB and also bisects $\angle AOB$ $(\because OA = OB = 8\text{ cm})$

$\Rightarrow AM = BM = \dfrac{1}{2}AB = \dfrac{8\sqrt{3}}{2} = 4\sqrt{3}\text{ cm}$

In right ΔAOM, $\sin\angle AOM = \dfrac{AM}{OA} = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \angle AOM = 60^\circ \quad \left(\because \sin 60^\circ = \dfrac{\sqrt{3}}{2}\right)$

$\therefore \quad \angle AOB = 2\angle AOM = 2 \times 60^\circ = 120^\circ$

and $\cos 60^\circ = \dfrac{OM}{OA} \Rightarrow \dfrac{1}{2} = \dfrac{OM}{8} \Rightarrow OM = 4$ cm

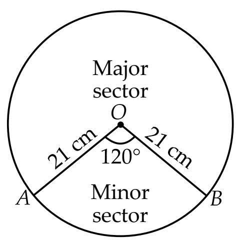
Area of minor segment = Area of sector AOB – Area of ΔAOB

$= \dfrac{\theta}{360^\circ} \times \pi r^2 - \dfrac{1}{2} \times AB \times OM$

$= \dfrac{120^\circ}{360^\circ} \times \dfrac{22}{7} \times 8 \times 8 - \dfrac{1}{2} \times 8\sqrt{3} \times 4$

$= 67.04 - 16 \times 1.732 = 39.328\text{ cm}^2$

24. Given that, radius of the circle $(r) = 21$ cm and central angle of the sector $AOBA(\theta) = 120^\circ$



So, area of the circle $= \pi r^2$

$= \frac{22}{7} \times (21)^2 = \frac{22}{7} \times 21 \times 21$

$= 1386 \text{ cm}^2$

Now, area of the minor sector *AOBA*

$= \frac{\pi r^2}{360°} \times \theta = \frac{22}{7} \times \frac{21 \times 21}{360°} \times 120°$

$= 22 \times 21 = 462 \text{ cm}^2$

$\therefore$ Area of the major sector *ABOA*

= Area of the circle – Area of the minor sector *AOBA*

$= 1386 - 462 = 924 \text{ cm}^2$

$\therefore$ Difference of the areas of a minor sector *AOBA* and its corresponding major sector *ABOA*

$= (924 - 462) = 462 \text{ cm}^2$

Hence, the required difference of two sectors is 462 cm^2.

OR

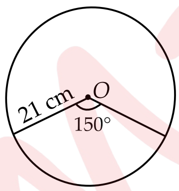
The arc length l and area A of a sector of angle θ in a circle of radius r are given by

$l = \frac{\theta}{360°} \times 2\pi r$ and $A = \frac{\theta}{360°} \times \pi r^2$ respectively.

Here, $r = 21$ cm and $\theta = 150°$

$\therefore \quad l = \left\{\frac{150°}{360°} \times 2 \times \frac{22}{7} \times 21\right\} \text{cm} = 55 \text{ cm}$

and $A = \left\{\frac{150°}{360°} \times \frac{22}{7} \times (21)^2\right\} \text{cm}^2$



$= \frac{1155}{2} \text{cm}^2 = 577.5 \text{ cm}^2$

25. **(d) :** Let r be the radius of the circle.

Area of the sector is given by $\frac{\theta}{360°} \times \pi r^2$.

$\therefore$ Area of sector $S_1 = \frac{120°}{360°}(\pi r^2) = \frac{\pi r^2}{3}$

Area of sector $S_2 = \frac{150°}{360°}(\pi r^2) = \frac{5\pi r^2}{12}$

$\therefore$ Required ratio $= \frac{\pi r^2}{3} : \frac{5\pi r^2}{12} = 4 : 5$

26. **(a) :** Since ΔABC is an equilateral triangle.

$\therefore \quad \angle A = 60°$

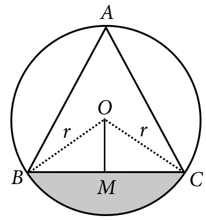
We know, angle subtended by an arc at centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle BOC = 2 \times \angle A = 2 \times 60° = 120°$

In ΔBOC, $OB = OC = r$

$\Rightarrow \quad \angle OBC = \angle OCB = \frac{1}{2}(180° - 120°) = 30°$

Draw $OM \perp BC$



In ΔOMC, $\frac{OM}{OC} = \sin 30° = \frac{1}{2}$

$\Rightarrow \quad OM = \frac{1}{2} \times OC = \frac{r}{2}$

Also, $\frac{MC}{OC} = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad MC = \frac{\sqrt{3}}{2}r \quad \Rightarrow \quad 2MC = \sqrt{3}r$

$\Rightarrow \quad BC = \sqrt{3}r \qquad (\because BM = MC$, by construction)

$\therefore \quad$ Area of minor segment $= \frac{\pi r^2\theta}{360°} - \frac{1}{2} \times OM \times BC$

$= \left(\pi r^2 \times \frac{120°}{360°}\right) - \left(\frac{1}{2} \times \frac{r}{2} \times \sqrt{3}r\right)$

$= \frac{\pi r^2}{3} - \frac{\sqrt{3}}{4}r^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$

27. **(a) :** We have, $OA = OB$ (Radii of circle)

$\Rightarrow \quad \angle A = \angle B = 60° \qquad (\because \angle AOB = 60°)$

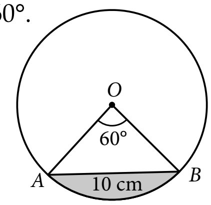
$\Rightarrow \quad \Delta AOB$ is an equilateral triangle.

$\therefore \quad OA = OB = AB = 10$ cm and $\theta = 60°$.

Area of the minor segment

= Area of minor sector

– Area of ΔAOB



$= \left(\frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}r^2\right)$

$= \left\{\left(3.14 \times 10 \times 10 \times \frac{60°}{360°}\right) - \left(\frac{\sqrt{3}}{4} \times 10 \times 10\right)\right\}$

$= \left(\frac{157}{3} - 25 \times \sqrt{3}\right) = \left(\frac{157}{3} - \frac{433}{10}\right)$

$= \frac{(1570 - 1299)}{30} = \frac{271}{30} = 9.03 \text{ cm}^2$

Area of major segment $= \pi r^2$ – Area of minor segment

$= \{(\pi \times 10 \times 10) - 9.03\} \text{ cm}^2$

$= \{(3.14 \times 100) - 9.03\} \text{ cm}^2 = (314 - 9.03) \text{ cm}^2$

$= 304.97 \text{ cm}^2$

28. Perimeter of the shaded region = Length of $\overparen{APB}$ + Length of $\overparen{ARC}$ + Length of $\overparen{CQD}$ + Length of $\overparen{DSB}$

Now, perimeter of $\overparen{APB}$ = Perimeter of $\overparen{CQD}$

$= \frac{1}{2} \times 2\pi\left(\frac{7}{2}\right) = \frac{22}{7} \times \frac{7}{2} = 11 \text{ cm}$

Perimeter of $\overparen{ARC}$ = Perimeter of $\overparen{DSB}$

$= \frac{1}{2} \times 2\pi(7) = \frac{22}{7} \times 7 = 22 \text{ cm}$

Thus, perimeter of the shaded region

$= 2 \times (11) + 2 \times (22) = 66$ cm