Areas Related to Circles

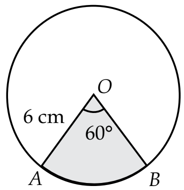
CHAPTER 12

NCERT FOCUS

SOLUTIONS

EXERCISE - 12.2

1. Here $r = 6$ cm and $\theta = 60°$



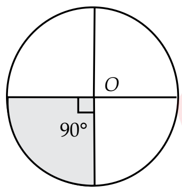
$\therefore$ Area of a sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

$= \frac{22}{7} \times 6 \text{ cm}^2 = \frac{132}{7} \text{cm}^2.$

2. Let radius of the circle = r

Given, circumference of circle = 22 cm



$\therefore$ $2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = 22 \times \frac{7}{22} \times \frac{1}{2} = \frac{7}{2}$ cm

Here, $\theta = 90°$

$\therefore$ Area of quadrant of the circle

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{cm}^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^2 = \frac{77}{8} \text{cm}^2.$

3. Length of minute hand = radius of the circle

$\Rightarrow r = 14$ cm

$\because$ Angle swept by the minute hand in 60 minutes = 360°

$\therefore$ Angle swept by the minute hand in 5 minutes

$= \frac{360°}{60°} \times 5 = 30°$

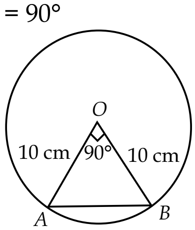
Now, area of the sector with $r = 14$ cm and $\theta = 30°$

$= \frac{\theta}{360°} \times \pi r^2 = \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= \frac{11 \times 14}{3} \text{ cm}^2 = \frac{154}{3} \text{cm}^2$

Thus, the required area swept by the minute hand in 5 minutes $= \frac{154}{3} \text{cm}^2.$

4. Length of the radius $(r) = 10$ cm, $\theta = 90°$



Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{314}{100} \times 10 \times 10 \text{ cm}^2$

$= \frac{1}{4} \times 314 \text{ cm}^2 = \frac{157}{2} \text{cm}^2 = 78.5 \text{ cm}^2$

(i) Area of the minor segment

= [Area of the minor sector] – [Area of right ΔAOB]

$= [78.5 \text{ cm}^2] - \left[\frac{1}{2} \times 10 \times 10 \text{ cm}^2\right]$

$= 78.5 \text{ cm}^2 - 50 \text{ cm}^2 = 28.5 \text{ cm}^2$

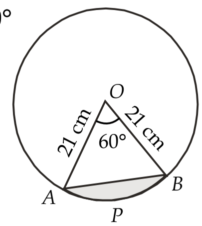
(ii) Area of the major sector

= [Area of the circle] – [Area of the minor sector]

$= \pi r^2 - 78.5 \text{ cm}^2 = \left[\frac{314}{100} \times 10 \times 10 - 78.5\right] \text{cm}^2$

$= (314 - 78.5) \text{ cm}^2 = 235.5 \text{ cm}^2.$

5. Here, radius, $r = 21$ cm and $\theta = 60°$



(i) Length of arc APB

$= \frac{\theta}{360°} \times 2\pi r = \left(\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21\right) \text{cm}$

$= \left(\frac{1}{6} \times 2 \times 22 \times 3\right) \text{cm}$

$= \left(\frac{1}{6} \times 132\right) \text{cm} = 22 \text{ cm}$

(ii) Area of the sector with sector angle 60°

$= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$

$= 11 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$

(iii) Area of the segment APB

= [Area of the sector $AOBP$] – [Area of ΔAOB] ...(1)

In ΔAOB, $OA = OB = 21$ cm

$\therefore$ $\angle A = \angle B = 60°$ $[\because \angle O = 60°]$

$\Rightarrow$ AOB is an equilateral triangle.

$\therefore$ $AB = 21$ cm

$\therefore$ Area of $\Delta AOB = \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 21 \times 21 \text{ cm}^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$ ...(2)

From (1) and (2), we have

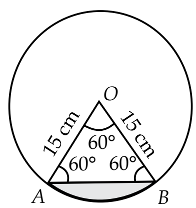
Area of segment $= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$

6. Here, radius $(r) = 15$ cm and

Sector angle $(\theta) = 60°$

$\therefore$ Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{314}{100} \times 15 \times 15 \text{ cm}^2 = \frac{157 \times 3}{4} = 117.75 \text{ cm}^2$



Since $\angle O = 60°$ and $OA = OB = 15$ cm

$\Rightarrow$ $\angle A = \angle B = 60°$

$\therefore$ AOB is an equilateral triangle.

$\therefore$ $AB = 15$ cm

Now, area of $\Delta AOB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 15 \times 15 \text{ cm}^2 = \frac{225\sqrt{3}}{4} \text{cm}^2$

$= \frac{225 \times 1.73}{4} \text{cm}^2 = 97.3125 \text{ cm}^2$

Now, area of the minor segment

= (Area of minor sector) – (Area of ΔAOB)

= (117.75 – 97.3125) cm^2 = 20.4375 cm^2

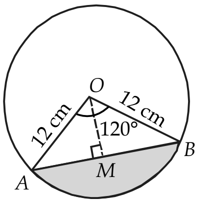
$\therefore$ Area of the major segment

= [Area of the circle] – [Area of the minor segment]

$= \pi r^2 - 20.4375 \text{ cm}^2 = \left[\frac{314}{100} \times 15^2\right] - 20.4375 \text{ cm}^2$

= 706.5 – 20.4375 cm^2 = 686.0625 cm^2.

7. Here, θ = 120° and r = 12 cm



$\therefore$ Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{120°}{360°} \times \frac{314}{100} \times 12 \times 12 \text{ cm}^2$

$= \frac{314 \times 4 \times 12}{100} \text{ cm}^2 = \frac{15072}{100} \text{ cm}^2 = 150.72 \text{ cm}^2$ ...(1)

Draw, $OM \perp AB$

$\Rightarrow$ OM is the perpendicular bisector of AB.

$\therefore \quad AM = BM = \frac{1}{2} AB$

In ΔAOB, $\angle O$ = 120°

$\Rightarrow \angle A + \angle B$ = 180° – 120° = 60°

$\because$ $OB = OA$ = 12 cm $\Rightarrow$ $\angle A = \angle B$ = 30°

So, $\frac{OM}{OA} = \sin 30° = \frac{1}{2} \Rightarrow OM = OA \times \frac{1}{2} = 12 \times \frac{1}{2} = 6 \text{ cm}$

and $\frac{AM}{OA} = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow AM = \frac{\sqrt{3}}{2} OA = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3} \text{ cm}$

$\therefore \quad AB = 2AM = 12\sqrt{3} \text{ cm}$

Now, area of $\Delta AOB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6 \text{ cm}^2 = 36\sqrt{3} \text{ cm}^2$

= 36 × 1.73 cm^2 = 62.28 cm^2 ...(2)

From (1) and (2), we have

Area of the minor segment

= [Area of sector] – [Area of ΔAOB]

= [150.72 cm^2] – [62.28 cm^2] = 88.44 cm^2

8. Here, length of the rope = 5 m

$\therefore$ Radius of the circular region grazed by the horse = 5 m

(i) Area of the circular portion grazed

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{314}{100} \times 5 \times 5 \text{ m}^2 = \frac{1}{4} \times \frac{314}{4} \text{m}^2$

$= \frac{157}{8} \text{m}^2 = 19.625 \text{ m}^2$

(ii) When length of the rope is increased to 10 m

$\therefore \quad r$ = 10 m

$\Rightarrow$ Area of the new circular portion grazed

$= \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{314}{100} \times (10)^2 \text{ m}^2$

$= \frac{1}{4} \times 314 \text{ m}^2 = 78.5 \text{ m}^2$

$\therefore$ Increase in the grazing area

= (78.5 – 19.625) m^2 = 58.875 m^2.

9. Diameter of the circle = 35 mm

$\therefore$ Radius (r) = $\frac{35}{2}$ mm

(i) Circumference of circle = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2} \text{ mm} = 22 \times 5 = 110 \text{ mm}$

Length of 1 piece of wire used to make diameter to divide the circle into 10 equal sectors = 35 mm

$\therefore$ Length of 5 pieces = 5 × 35 = 175 mm

$\therefore$ Total length of the silver wire

= (110 + 175) mm = 285 mm

(ii) Since the circle is divided into 10 equal sectors.

$\therefore$ Sector angle, $\theta = \frac{360°}{10} = 36°$

Now, area of each sector

$= \frac{\theta}{360°} \times \pi r^2 = \frac{36°}{360°} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{mm}^2$

$= \frac{11 \times 35}{4} \text{mm}^2 = \frac{385}{4} \text{mm}^2$.

10. Here, radius (r) = 45 cm

Since circle is divided into 8 equal parts.

$\therefore$ Sector angle corresponding to each part, $\theta = \frac{360°}{8} = 45°$

$\therefore$ Area of a sector (part)

$= \frac{\theta}{360°} \times \pi r^2 = \frac{45°}{360°} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2$

$= \frac{11 \times 45 \times 45}{4 \times 7} \text{cm}^2 = \frac{22275}{28} \text{cm}^2$

$\therefore$ The required area between the two consecutive ribs

$= \frac{22275}{28} \text{cm}^2$

11. Here, radius (r) = 25 cm

Sector angle (θ) = 115°

$\therefore$ Total area cleaned by each sweep of the blades

$= \left[\frac{\theta}{360°} \times \pi r^2\right] \times 2$ [$\because$ There are 2 blades]

$= \left[\frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25\right] \times 2 \text{ cm}^2$

$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7} \text{cm}^2 = \frac{158125}{126} \text{cm}^2$

12. Here, radius (r) = 16.5 km

Sector angle (θ) = 80°

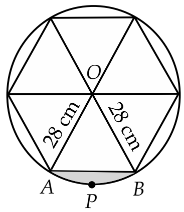
$\therefore$ Area of the sea surface over which the ships are warned $= \frac{\theta}{360°} \times \pi r^2 = \frac{80°}{360°} \times \frac{314}{100} \times \frac{165}{10} \times \frac{165}{10} \text{km}^2$

$= \frac{157 \times 11 \times 11}{100} \text{km}^2 = \frac{18997}{100} \text{km}^2 = 189.97 \text{ km}^2$

13. Here, $r = 28$ cm

Since, the circle is divided into six equal sectors.

$\therefore$ Sector angle, $\theta = \dfrac{360°}{6} = 60°$



$\therefore$ Area of each sector

$= \dfrac{60°}{360°} \times \dfrac{22}{7} \times 28 \times 28\ \text{cm}^2$

$= \dfrac{44 \times 28}{3}\text{cm}^2 = 410.67\text{cm}^2$ ...(1)

Now, area of 1 design = Area of segment APB

= Area of sector $APBO$ – Area of ΔAOB ...(2)

In ΔAOB, $\angle AOB = 60°$, $OA = OB = 28$ cm

$\therefore$ $\angle OAB = 60°$ and $\angle OBA = 60°$

$\Rightarrow$ ΔAOB is an equilateral triangle.

$\therefore$ Area of $\Delta AOB = \dfrac{\sqrt{3}}{4} \times (\text{side})^2$

$= \dfrac{\sqrt{3}}{4} \times 28 \times 28 = 14 \times 14\sqrt{3}\ \text{cm}^2$

$= 14 \times 14 \times 1.7\ \text{cm}^2 = 333.2\ \text{cm}^2$ ...(3)

Now, from (1), (2) and (3), we have

Area of segment $APB = 410.67\ \text{cm}^2 - 333.2\ \text{cm}^2 = 77.47\ \text{cm}^2$

$\Rightarrow$ Area of 1 design = $77.47\ \text{cm}^2$

$\therefore$ Area of the 6 equal designs = $6 \times (77.47)\ \text{cm}^2$

$= 464.82\ \text{cm}^2$

So, cost of making the design at the rate of ₹ 0.35 per cm^2

= ₹ (0.35 × 464.82) = ₹ 162.68

14. (d) : Here, radius (r) = R

Angle of sector (θ) = p

$\therefore$ Area of the sector

$= \dfrac{\theta}{360°} \times \pi r^2 = \dfrac{p}{360} \times \pi R^2 = \dfrac{2}{2} \times \left(\dfrac{p}{360} \times \pi R^2\right) = \dfrac{p}{720} \times 2\pi R^2$

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