Areas Related to Circles

CHAPTER 12

TRY YOURSELF SOLUTIONS

1. Given that, radius of a circle (r) = 28 cm and measure of central angle $(\theta) = 45°$

$\therefore$ Area of a sector of a circle $= \dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{45°}{360°} \times \dfrac{22}{7} \times (28)^2 = 22 \times 4 \times 28 \times \dfrac{1}{8}$

$= 22 \times 14 = 308 \text{ cm}^2$

Area of major sector $= \pi r^2$ – Area of minor sector

$= \dfrac{22}{7} \times 28 \times 28 - 308 = 2464 - 308 = 2156 \text{ cm}^2$

2. Angle described by hour hand in 1 hour = 30°

Time covered from 8 a.m. to 11 a.m. = 3 hours

$\therefore$ Angle described by hour hand in 3 hours = 3 × 30° = 90°

Area swept by hour hand in 3 hours = Area of sector of angle 90° in a circle of radius 5.6 cm

$= \dfrac{90°}{360°} \times \dfrac{22}{7} \times (5.6)^2 = \dfrac{1}{4} \times \dfrac{22}{7} \times 5.6 \times 5.6 = 24.64 \text{ cm}^2$

3. Given, length of arc = 26.4 cm

$\Rightarrow \dfrac{\theta°}{360°} \times 2\pi r = 26.4 \Rightarrow \dfrac{80°}{360°} \times 2\pi r = 26.4 \quad [\because \theta = 80°]$

$\Rightarrow r = \dfrac{26.4 \times 360°}{2\pi \times 80°} \text{ cm}$

$\therefore$ Area of minor sector $= \dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{80°}{360°} \times \pi \times \left(\dfrac{26.4 \times 360°}{2\pi \times 80°}\right)^2$

$= \dfrac{360°}{80°} \times \dfrac{7}{4 \times 22} \times 26.4 \times 26.4 = 249.48 \text{ cm}^2$

4. Length of arc = 22 m [Given]

$\Rightarrow \dfrac{\theta}{360°} \times 2\pi r = 22 \Rightarrow \dfrac{\theta}{360°} \times 2 \times \dfrac{22}{7} \times 8.4 = 22$

$\Rightarrow \theta = \dfrac{22 \times 7 \times 360°}{2 \times 22 \times 8.4} = 150°$

$\therefore$ Area of sector $= \dfrac{\theta}{360°} \times \pi r^2 = \dfrac{150°}{360°} \times \dfrac{22}{7} \times (8.4)^2$

$= \dfrac{5}{12} \times 22 \times 1.2 \times 8.4 = 92.4 \text{ cm}^2$

5. Let r = 7.5 cm and R = 11 cm.

$\therefore$ Area of shaded region = Area of sector of radius 11 cm – Area of sector of radius 7.5 cm

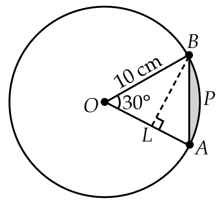
$= \dfrac{\theta}{360°} \times \pi R^2 - \dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{\theta}{360°} \pi(11^2 - 7.5^2) = \dfrac{120°}{360°} \times \dfrac{22}{7}(11 + 7.5)(11 - 7.5)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 18.5 \times 3.5 = 67.83 \text{ cm}^2$

6. Let O be the centre and AB is the chord of the circle having radius 10 cm.

Draw $BL \perp OA$



In right angled ΔOLB,

$\dfrac{BL}{OB} = \sin 30° = \dfrac{1}{2}$

$\Rightarrow \dfrac{BL}{10} = \dfrac{1}{2} \quad (\because OB = 10 \text{ cm})$

$\Rightarrow BL = \dfrac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

Area of the segment APB = Area of the sector $OAPB$ – Area of the ΔAOB

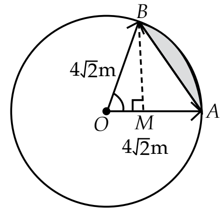
$= \dfrac{30°}{360°} \times \pi \times (r)^2 - \dfrac{1}{2} \times OA \times BL$

$= \left\{\dfrac{1}{12} \pi \times (10)^2 - \dfrac{1}{2} \times 10 \times 5\right\} \text{cm}^2$

$= \left(\dfrac{25\pi}{3} - 25\right) \text{cm}^2 = 25 \times \left(\dfrac{\pi}{3} - 1\right) \text{cm}^2$

$= 25 \times \left(\dfrac{3.14}{3} - 1\right) \text{cm}^2 = \left(\dfrac{25 \times 0.14}{3}\right) \text{cm}^2 = 1\dfrac{1}{6} \text{ cm}^2$

7. Draw $BM \perp OA$



In right ΔOMB, $\dfrac{BM}{OB} = \sin 45°$

$\Rightarrow \dfrac{BM}{4\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow BM = 4 \text{ m}$

Area of shaded region = Area of sector OAB – Area of ΔOAB

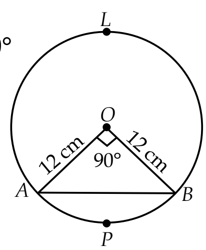
$= \dfrac{\theta}{360°} \pi r^2 - \dfrac{1}{2} \times OA \times BM$

$= \dfrac{45°}{360°} \times \dfrac{22}{7} \times 4\sqrt{2} \times 4\sqrt{2} - \dfrac{1}{2} \times 4\sqrt{2} \times 4$

$= \dfrac{1}{8} \times \dfrac{22}{7} \times 16 \times 2 - 2\sqrt{2} \times 4 = 12.57 - 11.28 = 1.29 \text{ m}^2$

8. Let O be the centre of circle.

We have, radius (r) = 12 cm and $\theta = 90°$



So, area of sector $OAPB = \dfrac{\theta}{360°} \pi r^2$

$= \dfrac{90°}{360°} \times 3.14 \times 12^2 = 113.04 \text{ cm}^2$

Now, area of ΔAOB

$= \dfrac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$

$\therefore$ Area of the minor segment APB = Area of sector $OAPB$ – Area of ΔAOB = (113.04 – 72) cm^2 = 41.04 cm^2

Area of circle = $\pi r^2 = 3.14 \times 12^2 = 452.16$ cm^2

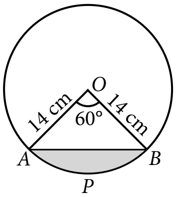
$\therefore$ Area of major segment ALB = Area of circle – Area of minor segment APB

= (452.16 – 41.04) cm^2 = 411.12 cm^2

9. We have, radius (r) = 14 cm and $\theta = 60°$

Area of minor segment = Area of sector $OAPB$ – Area of ΔAOB

$$= \frac{\theta \pi r^2}{360°} - \frac{1}{2} r^2 \sin\theta$$



$$= \frac{60° \times 22 \times 14 \times 14}{7 \times 360°} - \frac{1}{2} \times 14 \times 14 \times \sin 60°$$

$$= \frac{22 \times 14}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2} = 102.67 - 84.87 = 17.8 \text{ cm}^2$$