Surface Areas and **Volumes**



SOLUTIONS

EXERCISE - 13.1

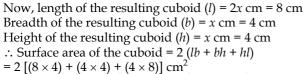
Let the edge of each cube = xGiven, volume of each cube

$$= 64 \text{ cm}^3$$

$$x^3 = 64 \text{ cm}^3$$

$$\Rightarrow x = 4 \text{ cm}$$

∴.



- = $2 [(8 \times 4) + (4 \times 4) + (4 \times 8)] \text{ cm}^2$ = $2 [32 + 16 + 32] \text{ cm}^2 = 2 [80] \text{ cm}^2 = 160 \text{ cm}^2$
- For hemispherical part, radius $(r) = \frac{14}{2} = 7 \text{ cm}$

6 cm

Curved surface area of hemisphere = $2\pi r^2$

$$=2\times\frac{22}{7}\times7\times7\text{cm}^2=308\text{ cm}^2$$

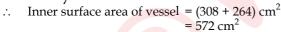
Total height of vessel = 13 cm

Height of cylinder

=
$$(13 - 7)$$
 cm = 6 cm
and radius (r) = 7 cm

Curved surface area of cylinder

$$= 2 \times \frac{22}{7} \times 7 \times 6 \text{ cm}^2 = 264 \text{ cm}^2$$



Given radius of cone (r)

= radius of hemisphere (r)

= 3.5 cm

Height of cone (h) =
$$(15.5 - 3.5)$$
 cm
= 12 cm

Also, slant height (*l*) = $\sqrt{h^2 + r^2}$

$$=\sqrt{12^2 + (3.5)^2} = \sqrt{156.25}$$

= 12.5 cm

Total surface area of the toy = curved surface area of conical part + curved surface area of hemispherical part

=
$$\pi rl + 2\pi r^2 = \pi r (l + 2r) = \frac{22}{7} \times \frac{35}{10} (12.5 + 2 \times 3.5) \text{ cm}^2$$

=
$$11 \times (12.5 + 7) \text{ cm}^2 = 11 \times 19.5 \text{ cm}^2 = 214.5 \text{ cm}^2$$

Let side of the block, (a) = 7 cm

The greatest diameter of the hemisphere = 7 cm

Radius of hemisphere, (r)

= 7/2 cm



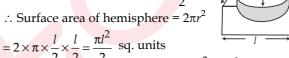
Surface area of the solid

- = [Total surface area of the cubical block]
 - + [Curved surface of the hemisphere]
- [Base area of the hemisphere]

$$= (6 \times a^2) + 2\pi r^2 - \pi r^2$$

$$= 6a^2 + \pi r^2 = (6 \times 7^2) + \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) = \left(294 + \frac{77}{2}\right) = 332.5 \text{ cm}^2$$

- Given, side of the cube = diameter of the hemisphere
- \Rightarrow Radius of the hemisphere $=\frac{l}{2}$
- ∴ Surface area of hemisphere = $2\pi r^2$



Base area of the hemisphere = $\pi \left(\frac{l}{2}\right)^2 = \frac{\pi l^2}{4}$ sq. units

Surface area of the cube = $6 \times l^2 = 6l^2$ sq. units

:. Surface area of the remaining solid

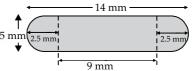
$$= 6l^{2} + \frac{\pi l^{2}}{2} - \frac{\pi l^{2}}{4} = \frac{24l^{2} + 2\pi l^{2} - \pi l^{2}}{4} = \frac{24l^{2} + \pi l^{2}}{4}$$

$$=\frac{l^2}{4}(24+\pi)$$
 sq.units.

13 cm

15.5 cm

Radius of the hemispherical part $(r) = \frac{5}{2}$ mm



Curved surface area of one hemispherical part = $2\pi r^2$

Surface area of both hemispherical parts

$$= 2(2\pi r^2) = 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{25}{10}\right)^2 \text{ mm}^2$$

$$=4\times\frac{22}{7}\times\frac{25}{10}\times\frac{25}{10}$$
mm²

Entire length of capsule = 14 mm

- Length of cylindrical part = $14 2 \times 2.5 = 9 \text{ mm}$
- Area of cylindrical part = $2\pi rh$

$$=2\times\frac{22}{7}\times2.5\times9 \text{ mm}^2=2\times\frac{22}{7}\times\frac{25}{10}\times9 \text{ mm}^2$$

Total surface area of capsul

$$= \left[2 \times \frac{22}{7} \times \frac{25}{10} \times 9\right] + \left[4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\right] mm^2$$

$$= \left(2 \times \frac{22}{7} \times \frac{25}{10}\right) \left[9 + \frac{50}{10}\right] \text{mm}^2 = \frac{44 \times 25}{70} \times 14 \text{ mm}^2 = 220 \text{ mm}^2$$

For cylindrical part:

Radius
$$(r) = \frac{4}{2}m = 2m$$
 and

height (h) = 2.1 m

:. Curved surface area

$$=2\pi rh=2\times\frac{22}{7}\times2\times\frac{21}{10}\text{m}^2$$



Slant height (l) = 2.8 m and base radius (r) = 2 m

$$\therefore$$
 Curved surface area = $\pi rl = \frac{22}{7} \times 2 \times \frac{28}{10} \text{m}^2$

Total surface area

= [Curved surface area of the cylindrical part] +

[Curved surface area of conical part]

0.7 cm

1.4 cm

2.4 cm

$$= \left[2 \times \frac{22}{7} \times 2 \times \frac{21}{10}\right] + \left[\frac{22}{7} \times 2 \times \frac{28}{10}\right] m^2$$
$$= 2 \times \frac{22}{7} \left[\frac{42}{10} + \frac{28}{10}\right] m^2 = 2 \times \frac{22}{7} \times \frac{70}{10} m^2 = 44 m^2$$

Cost of 1 m² of canvas = ₹500

Cost of 44 m² of canvas = ₹ (500×44) = ₹ 22000.

For cylindrical part:

Height (h) = 2.4 cm and diameter = 1.4 cm

$$\Rightarrow$$
 Radius (r) = 0.7 cm

Total surface area of

the cylindrical part

$$= 2\pi rh + 2\pi r^2 = 2\pi r [h + r]$$

$$=2\times\frac{22}{7}\times\frac{7}{10}[2.4+0.7]$$

$$= \frac{44}{10} \times 3.1 = \frac{44 \times 31}{100} = \frac{1364}{100} \text{ cm}^2$$

For conical part:

Base radius (r) = 0.7 cm and height (h) = 2.4 cm

:. Slant height (l) =
$$\sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2}$$

= $\sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$ cm

Curved surface area of the conical part

$$= \pi r l = \frac{22}{7} \times 0.7 \times 2.5 \text{ cm}^2 = \frac{550}{100} \text{cm}^2$$

Base area of the conical par

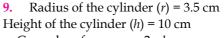
$$= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{10}\right)^2 \text{cm}^2 = \frac{22 \times 7}{100} \text{cm}^2 = \frac{154}{100} \text{cm}^2$$

Total surface area of the remaining solid = [(Total surface area of cylindrical part)

> + (Curved surface area of conical part) - (Base area of the conical part)]

$$= \left[\frac{1364}{100} + \frac{550}{100} - \frac{154}{100} \right] \text{ cm}^2 = \frac{1760}{100} \text{ cm}^2 = 17.6 \text{ cm}^2.$$

Hence, total surface area to the nearest cm² is 18 cm².



 \therefore Curved surface area = $2\pi rh$

$$=2\times\frac{22}{7}\times\frac{35}{10}\times10\text{cm}^2=220\text{cm}^2$$

Curved surface area of a hemisphere = $2\pi r^2$

:. Curved surface area of both hemispheres

$$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{ cm}^2 = 154 \text{ cm}^2$$

Total surface area of the remaining solid $= (220 + 154) \text{ cm}^2 = 374 \text{ cm}^2$

EXERCISE - 13.2

Here, r = 1 cm and h = 1 cm.

Volume of the conical part = $\frac{1}{2}\pi r^2 h$

and volume of the hemispherical

$$part = \frac{2}{3}\pi r^3$$

2.1 m

:. Volume of the solid shape

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 [h + 2r]$$

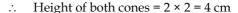
$$= \frac{1}{3}\pi(1)^2[1+2(1)] \text{ cm}^3 = \frac{1}{3}\pi \times 1 \times 3 \text{ cm}^3 = \pi \text{ cm}^3$$

2. Here, diameter = 3 cm

$$\Rightarrow$$
 Radius $(r) = \frac{3}{2}$ cm

Total height = 12 cm

Height of a cone (h) = 2 cm



Height of the cylinder $(h_1) = (12 - 4)$ cm = 8 cm Now, volume of the cylindrical part = $\pi r^2 h$

Volume of both conical parts = $2\left|\frac{1}{3}\pi r^2 h\right|$

:. Volume of the whole model

$$=\pi r^2 h_1 + \frac{2}{3}\pi r^2 h = \pi r^2 \left[h_1 + \frac{2}{3}h\right]$$

$$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \left[8 + \frac{2}{3}(2)\right] = \frac{22}{7} \times \frac{9}{4} \times \left(\frac{24+4}{3}\right)$$

$$=\frac{22}{7}\times\frac{9}{4}\times\frac{28}{3}$$
cm³ = 66 cm³.

Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the gulab jamun = 5 cm.

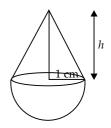
Diameter = $2.8 \text{ cm} \Rightarrow \text{Radius}(r) = 1.4 \text{ cm}$

∴ Length (height) of the cylindrical part (h)

= 5 cm - (1.4 + 1.4) cm = 5 cm - 2.8 cm = 2.2 cm

Now, volume of the cylindrical part = $\pi r^2 h$

and volume of both the hemispherical ends



12 cm

60 cm

220 cm

24!cm-

$$=2\left(\frac{2}{3}\pi r^{3}\right)=\frac{4}{3}\pi r^{3}$$

:. Volume of a gulab jamun

$$= \pi r^2 h + \frac{4}{3} \pi r^3 = \pi r^2 \left[h + \frac{4}{3} r \right]$$

$$= \frac{22}{7} \times (1.4)^2 \left[2.2 + \frac{4}{3} (1.4) \right]$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10} \left[\frac{22}{10} + \frac{56}{30} \right]$$

$$= \frac{22 \times 2 \times 14}{10 \times 10} \left[\frac{66 + 56}{30} \right] = \frac{44 \times 14}{100} \times \frac{122}{30} \text{ cm}^3$$

Volume of 45 gulab jamur

$$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30} \right] = \frac{15 \times 44 \times 14 \times 122}{1000} \text{ cm}^3$$

Since, the quantity of syrup in gulab jamuns

= 30% of [volume] = 30% of
$$\left[\frac{15 \times 44 \times 14 \times 122}{1000} \right]$$

$$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} = 338.184 \text{ cm}^3$$

$$= 338 \text{ cm}^2 \text{ (approx.)}$$

Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.

$$\therefore \text{ Volume of the cuboid} = 15 \times 10 \times \frac{35}{10} = 525 \text{ cm}^3$$

Since each depression is conical in shape with base radius (r) = 0.5 cm and depth (h) = 1.4 cm.

Volume of each depression

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10} = \frac{11}{30} \text{ cm}^3$$

Since there are 4 depressions.

$$\therefore$$
 Total volume of 4 depressions = $4 \times \frac{11}{30} = \frac{44}{30} \text{ cm}^3$

Now, volume of the wood in entire stand

= [Volume of the wooden cuboid]

[Volume of 4 depressions]

$$= 525 - \frac{44}{30} = \frac{15750 - 44}{30} = \frac{15706}{30} = 523.53 \text{ cm}^3$$

Height of the conical vessel (h) = 8 cm

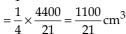
Base radius (r) = 5 cm

Volume of water in conical vessel

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 = \frac{4400}{21} \text{cm}^3$$

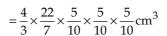
Now, total volume of lead shots

 $=\frac{1}{4}$ of [Volume of water in the cone]



Since, radius of spherical lead shot (r) = 0.5 cm

$$\therefore \quad \text{Volume of 1 lead shot } = \frac{4}{3}\pi r^3$$



1.4 cm

1.4 cm

∴ Number of lead shots =
$$\frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}}$$

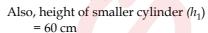
$$= = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]} = 100$$

Thus, the required number of lead shots = 100

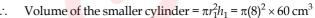
Height of the big cylinder (h) = 220 cm

Base radius (r) =
$$\frac{24}{2}$$
 cm = 12 cm

Volume of the big cylinder $=\pi r^2 h = \pi (12)^2 \times 220 \text{ cm}^3$



Base radius $(r_1) = 8$ cm



Volume of iron pole = [Volume of big cylinder] + [Volume of the smaller cylinder]

$$= (\pi \times 220 \times 12^2 + \pi \times 60 \times 8^2) \text{ cm}^3$$

= 3.14[220 × 12 × 12 + 60 × 8 × 8] cm³

$$=\frac{314}{100}[220\times144+60\times64]$$
 cm³

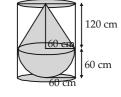
$$= \frac{314}{100} [31680 + 3840] \text{ cm}^3 = \frac{314}{100} \times 35520 \text{ cm}^3$$

Mass of pole =
$$\frac{8 \times 314 \times 35520}{100}$$
g = $\frac{89226240}{100}$ g = $\frac{8922624}{10000}$ kg = 892.2624 kg = 892.26 kg.

Height of the conical part (h) = 120 cm.

Base radius of the conical part (r)

:. Volume of the conical part $=\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120 \text{ cm}^3$



Radius of the hemispherical part (r) = 60 cm

Volume of the hemispherical part

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (60)^3 \text{ cm}^3$$

:. Volume of the solid = [Volume of conical part] + [Volume of hemispherical part]

$$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120 \right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3 \right]$$

$$= \frac{2}{3} \times \frac{22}{7} \times 60^2 [60 + 60]$$

$$=\frac{2}{3}\times\frac{22}{7}\times60\times60\times120=\frac{6336000}{7}$$
 cm³

Radius of cylinder $(r_1) = 60 \text{ cm}$ and, Height of cylinder $(h_1) = 180$ cm Volume of the cylinder = $\pi r_1^2 h$

$$= \frac{22}{7} \times 60^2 \times 180 = \frac{14256000}{7} \text{ cm}^3$$

Volume of water in the cylinder = $\frac{14256000}{7}$ cm³

2 cm

- 8.5 cm -

Volume of the water left in the cylinder

$$= \left[\frac{14256000}{7} - \frac{6336000}{7} \right] = \frac{7920000}{7}$$

- = 1131428.571 cm³ = $\frac{1131428.571}{1000000}$ m³
- $= 1.131428571 \text{ m}^3 = 1.131 \text{ m}^3 \text{ (approx)}.$
- Volume of the cylindrical part = $\pi r^2 h$

$$= 3.14 \times 1^2 \times 8 = \frac{314}{100} \times 8 \text{ cm}^3$$

[: Radius(r) = $\frac{2}{3}$ = 1 cm, height(h) = 8 cm]

Radius of spherical part $(r_1) = \frac{8.5}{2}$ cm

Volume of the spherical part = $\frac{4}{3}\pi r_1^3$

$$=\frac{4}{3}\times\frac{314}{100}\times\frac{85}{20}\times\frac{85}{20}\times\frac{85}{20}cm^3$$

Total volume of the glass-vessel

$$= \left[\frac{314}{100} \times 8 \right] + \left[\frac{4}{3} \times \frac{314}{100} \times \frac{85 \times 85 \times 85}{8000} \right]$$

$$=\frac{314}{100}\left[8+\frac{4\times85\times85\times85}{24000}\right]=\frac{314}{100}\left[8+\frac{614125}{6000}\right]$$

$$=\frac{314}{100}\bigg[\frac{48000+614125}{6000}\bigg]=\frac{314}{100}\bigg[\frac{662125}{6000}\bigg]$$

- $= 346.51 \text{ cm}^3 \text{ (approx.)}$
- \Rightarrow Volume of water in the vessel = 346.51 cm³ Since, the child finds the volume as 345 cm³
- ... The child's answer is not correct

The correct answer is 346.51 cm³.

EXERCISE - 13.5

- 1. Since, diameter of the cylinder = 10 cm
- Radius of the cylinder (r) = 10/2 cm = 5 cm
- Length of wire in one round = $2\pi r$ $= 2 \times 3.14 \times 5 \text{ cm} = 31.4 \text{ cm}$
- Diameter of wire = 3 mm = 3/10 cm
- The thickness of cylinder covered in one round = 3/10 cm
- Number of rounds (turns) of the wire to cover $12 \text{ cm} = \frac{12}{3/10} = 12 \times \frac{10}{3} = 40$
- Length of wire required to cover the whole surface = Length of wire required to complete 40 rounds $l = 40 \times 31.4 \text{ cm} = 1256 \text{ cm}$

Now, radius of the wire $=\frac{3}{2}$ mm $=\frac{3}{20}$ cm

- Volume of wire = $\pi r^2 l = 3.14 \times \frac{3}{20} \times \frac{3}{20} \times 1256 \text{ cm}^3$
- Density of wire = 8.88 g/cm^3
- Weight of the wire = [Volume of the wire] \times density

$$= \left[3.14 \times \frac{3}{20} \times \frac{3}{20} \times 1256 \right] \times 8.88 \text{ g}$$

$$= 3.14 \times \frac{3}{20} \times \frac{3}{20} \times 1256 \times \frac{888}{100} \text{ g}$$

$$= 787.97 \text{ g} = 788 \text{ g (approx.)}$$

- Let us consider the right $\triangle BAC$, right angled at Asuch that AB = 3 cm and AC = 4 cm.
- Hypotenuse $BC = \sqrt{3^2 + 4^2} = 5 \text{ cm}$

Obviously, we have obtained two cones on the same base AA' such that radius = DA or DA'

Now,
$$\frac{AD}{CA} = \frac{AB}{CB}$$
 [: $ADB \sim \Delta CAB$]

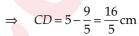
$$\Rightarrow \frac{AD}{A} = \frac{3}{5} \Rightarrow AD = \frac{3}{5} \times 4 = \frac{12}{5} \text{ cm}$$

4 cm

Also,
$$\frac{DB}{AB} = \frac{AB}{CB} \Rightarrow \frac{DB}{3} = \frac{3}{5}$$

$$\Rightarrow DB = \frac{3 \times 3}{5} = \frac{9}{5} \text{cm}$$

Since, CD = BC - DB



Now, volume of the double cone

$$= \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \times \frac{9}{5} + \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \times \frac{16}{5}$$

$$= \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \left[\frac{9}{5} + \frac{16}{5}\right] = \frac{1}{3} \times \frac{314}{100} \times \frac{144}{25} \times 5 = 30.14 \text{ cm}^3$$

$$= \left(\pi \times \frac{12}{5} \times 3\right) + \left(\pi \times \frac{12}{5} \times 4\right) = \pi \times \frac{12}{5} [3+4]$$
$$= \frac{314}{100} \times \frac{12}{5} \times 7 = 52.75 \text{ cm}^2$$

- : Dimensions of the cistern are 150 cm, 120 cm and 110 cm.
- Volume of the cistern = $150 \times 120 \times 110$ $= 1980000 \text{ cm}^3$

Volume of water contained in the cistern = 129600 cm³

Free space (volume) which is not filled with water $= 1980000 - 129600 = 1850400 \text{ cm}^3$

Now, volume of one brick = $22.5 \times 7.5 \times 6.5 = 1096.875 \text{ cm}^3$

Volume of water absorbed by one brick

$$=\frac{1}{17}\times1096.875\,\mathrm{cm}^3$$

Let n bricks can be put in the cistern.

 \therefore Volume of water absorbed by n bricks $=\frac{n}{17}\times1096.875$ cm³

 \therefore Volume occupied by n bricks = Free space in the cistern + Volume of water absorbed by n-bricks

$$\Rightarrow n \times (1096.875) = 1850400 + \frac{n}{17}(1096.875)$$

$$\Rightarrow$$
 1096.875 $n - \frac{n}{17}$ (1096.875) = 1850400

$$\Rightarrow \left(n - \frac{n}{17}\right) \times 1096.875 = 1850400$$

$$\Rightarrow \frac{16}{17}n = \frac{1850400}{1096.875} \Rightarrow n = \frac{1850400}{1096.875} \times \frac{17}{16}$$

$$\Rightarrow$$
 $n = 1792.4102 \approx 1792$

Thus, 1792 bricks can be put in the cistern.

4. Volume of three rivers

= 3 {(Surface area of a river) × Depth}

$$= 3 \left\{ \left(1072 \text{ km} \times \frac{75}{1000} \text{ km} \right) \times \frac{3}{1000} \text{ km} \right\}$$

$$=3\left\{\frac{241200}{1000000}\text{km}^3\right\}=0.7236\text{ km}^3$$

Volume of rainfall

= (Surface area of valley) × (Height of rainfall)

$$= 97280 \times \frac{10}{100 \times 1000} \qquad \left[\because 10 \text{ cm} = \frac{10}{100 \times 1000} \text{ km} \right]$$

$$=\frac{9728}{1000}km^3=9.728\,km^3$$

Thus, amount of rainfall in 1 fortnight *i.e.*, 14 days is 9.728 km^3 .

- \therefore Amount of rainfall in 1 day = 9.728/14 = 0.6949 km³ Since, 0.6949 km³ \approx 0.7236 km³
- :. The additional water in the three rivers is equivalent to the total rainfall.

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