EXAM DRILL

CHAPTER 14

Statistics

SOLUTIONS

1. **(a)**

2. **(d)**

3. **(b) :** We know, mean $= a + \dfrac{\sum f_i d_i}{\sum f_i}$

Here, $a = 55.5$, $n = \Sigma f_i = 100$, $\Sigma f_i d_i = 60$

$\therefore$ Mean $= 55.5 + \dfrac{60}{100} = 55.5 + 0.6 = 56.1$

4. **(a) :** Given, Mode – Median = 24

Also, we know, Mode = 3 Median – 2 Mean

$\Rightarrow$ 24 + Median = 3 Median – 2 Mean

$\Rightarrow$ 2(Median – Mean) = 24

$\Rightarrow$ Median – Mean = 12

5. Since, sum of n natural numbers $= \dfrac{n(n+1)}{2}$ ...(i)

Now, mean of n natural numbers $= \dfrac{5n}{9}$ [Given]

$\Rightarrow \dfrac{n(n+1)}{2n} = \dfrac{5n}{9}$ [From (i)]

$\Rightarrow \dfrac{n^2+n}{2n} = \dfrac{5n}{9} \Rightarrow 9n^2 + 9n = 10n^2 \Rightarrow n(n-9) = 0$

$\Rightarrow$ Either $n = 0$ or $n = 9$ $\Rightarrow$ $n = 9$ ($\because$ n can't be zero)

6. We know, Mean $= \dfrac{\text{Sum of the quantities}}{\text{Number of the quantities}}$

$\Rightarrow x = \dfrac{7+8+x+11+14}{5}$

$\Rightarrow 5x = 40 + x \Rightarrow 4x = 40 \Rightarrow x = 10$

7. Given, $\Sigma f_i u_i = 20$, $\Sigma f_i = 100$, $u_i = \dfrac{x_i - 25}{10} = \dfrac{x_i - a}{h}$

So, $a = 25$, $h = 10$

$\therefore$ Mean, $\bar{x} = a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) \times h = 25 + \dfrac{20}{100} \times 10 = 27$

8. We have, Mode = 50.5 and Median = 45.5

Now, we know that,

3 Median = Mode + 2 Mean

$\Rightarrow$ 3 × 45.5 = 50.5 + 2 Mean

$\Rightarrow$ Mean $= \dfrac{136.5 - 50.5}{2} = 43$

9. We know, mean $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

$\Rightarrow \dfrac{(x-5y)+(x-3y)+(x-y)+(x+y)+(x+3y)+(x+5y)}{6} = 12$

$\Rightarrow \dfrac{6x}{6} = 12 \Rightarrow x = 12$

10. Given, Mode = 1000, Median = 1250

Now, Mode = 3 Median – 2 Mean

$\Rightarrow$ 1000 = 3(1250) – 2 Mean

$\Rightarrow$ 2 Mean = 2750 $\Rightarrow$ Mean = 1375

11. Let us consider the following table :

Class	Class marks (x_i)	$d_i = x_i - A$	Frequency (f_i)	$f_i d_i$
30-40	35	–20	80	–1600
40-50	45	–10	110	–1100
50-60	55 = A	0	120	0
60-70	65	10	70	700
70-80	75	20	40	800
Total			$\Sigma f_i = 420$	$\Sigma f_i d_i = -1200$

(i) **(d) :** Clearly, the possible values of assumed mean (A) are 35, 45, 55, 65, 75.

(ii) **(c) :** The values of $|d_i|$ are 0, 10, 20

Thus, the minimum value of $|d_i|$ is 0.

(iii) **(b):** Required Mean $= A + \dfrac{\sum f_i d_i}{\sum f_i} = 55 - \dfrac{1200}{420}$

= ₹ 52.14

(iv) **(a):** Mean by direct and assumed mean method are always equal.

(v) **(d):** Average toll tax received by a vehicle = ₹ 52.14

Total number of vehicles = 420

$\therefore$ Average toll tax received in a day = ₹(52.14 × 420)

= ₹ 21898.80

12. Given frequency distribution table can be drawn as :

Class interval	Class marks (x_i)	Frequency (f_i)	$x_i f_i$	c.f.
100-120	110	7	770	7
120-140	130	12	1560	19
140-160	150	18	2700	37
160-180	170	13	2210	50
Total		50	7240	

(i) Clearly, average mileage

$= \dfrac{7240}{50} = 144.8$ km/charge

(ii) Since, highest frequency is 18, therefore, modal class is 140-160.

Here, $l = 140, f_1 = 18, f_0 = 12, f_2 = 13, h = 20$

$\therefore$ Mode $= 140+\frac{18-12}{36-12-13}\times 20 = 140+\frac{6}{11}\times 20$

$= 140+\frac{120}{11} = 140+10.91 = 150.91$

(iii) Here, $\frac{N}{2} = \frac{50}{2} = 25$ and the corresponding class whose cumulative frequency is just greater than 25 is 140-160.

Here, $l = 140$, $c.f. = 19$, $h = 20$ and $f = 18$

$\therefore$ Median $= l+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h$

$= 140+\frac{25-19}{18}\times 20 = 140+\frac{60}{9} = 146.67$

(iv) Assumed mean method is useful in determining the mean.

(v) Since, Mean = 144.8, Mode = 150.91 and Median = 146.67 and minimum of which is 144 approx, therefore manufacturer can claim the mileage for his scooter 144 km/charge.

13. (i) (d) : Required number of persons = 9 + 6 = 15

(ii) (c) : Required number of persons = 6 + 8 + 2 = 16

(iii) (a) : 50-60 is the modal class as the maximum frequency is 9.

(iv) (b) : The cumulative frequency distribution table for the given data can be drawn as :

Salaries received (in percent)	Number of employees (f_i)	Cumulative Frequency ($c.f.$)
50-60	9	9
60-70	6	9 + 6 = 15
70-80	8	15 + 8 = 23
80-90	2	23 + 2 = 25
Total	$\Sigma f_i = 25$	

Here, $\frac{N}{2} = \frac{25}{2} = 12.5$

The cumulative frequency just greater than 12.5 lies in the interval 60-70.

Hence, the median class is 60-70.

(v) (a) : We know, Mode = 3 Median – 2 Mean

$\therefore$ 3 Median = Mode + 2 Mean

14. (i) We know that,

Class mark $= \frac{\text{Lower limit + Upper limit}}{2}$

$\Rightarrow m = \frac{\text{Lower limit} + b}{2} \Rightarrow$ Lower limit $= 2m - b$

(ii)

Lifetime (in hours)	Class mark (x_i)	f_i	$d_i = x_i - A$	$f_i d_i$
150-200	175	14	–150	–2100
200-250	225	56	–100	–5600
250-300	275	60	–50	–3000
300-350	325 = A	86	0	0
350-400	375	74	50	3700
400-450	425	62	100	6200
450-500	475	48	150	7200
Total		400		6400

$\therefore$ Average lifetime of a packet

$= A+\frac{\sum f_i d_i}{\sum f_i} = 325+\frac{6400}{400} = 341$ hrs

(iii) Here, $N = 400 \Rightarrow \frac{N}{2} = 200$

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400

$\therefore$ $c.f.$ just greater than 200 is 216, which is corresponding to the interval 300-350.

$l = 300$, $f = 86$, $c.f. = 130$, $h = 50$

$\therefore$ Median $= l+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h = 300+\left(\frac{200-130}{86}\right)\times 50$

= 300 + 40.697 = 340.697 ≈ 340 hrs (approx.)

(iv) We know that Mode = 3 Median – 2 Mean

= 3(340.697) –2(341)

= 1022.091 – 682 = 340.091 ≈ 340 hrs

(v) Since, minimum of mean, median and mode is approximately 340 hrs. So, manufacturer should claim that lifetime of a packet is 340 hrs.

15. Let us construct the cumulative frequency distribution table :

Marks obtained	Frequency	Cumulative frequency
0 - 10	8	8
10 - 20	10	10 + 8 = 18
20 - 30	12	18 + 12 = 30
30 - 40	22	30 + 22 = 52
40 - 50	30	52 + 30 = 82
50 - 60	18	82 + 18 = 100

Here, $n = 100 \Rightarrow n/2 = 50$

Cumulative frequency just greater than 50 is 52, which lies in the interval 30-40. Therefore, 30-40 is the median class.

16. We have, $m = \frac{1+3+4+5+7}{5} = \frac{20}{5} \Rightarrow m = 4$

Also, $m - 1 = \frac{3+2+2+4+3+3+p}{7}$

$\Rightarrow$ $7m - 7 = 17 + p \Rightarrow 7 \times 4 - 7 = 17 + p$ $(\because m = 4)$

$\Rightarrow$ $28 - 7 = 17 + p \Rightarrow p = 21 - 17 = 4$

$\therefore$ The numbers are 3, 2, 2, 4, 3, 3, 4

Here, $n = 7$, which is odd.

$\therefore$ Median, $q = \left(\frac{n+1}{2}\right)^{\text{th}}$ observation $= \left(\frac{7+1}{2}\right)^{\text{th}}$ observation

$= 4^{\text{th}}$ observation $= 4$ $\therefore$ $p + q = 4 + 4 = 8$

17. Let us construct the following table from the given data:

Class-interval	Frequency (f_i)	Class-mark (x_i)	$\Sigma f_i x_i$
0 - 6	7	3	21
6 - 12	5	9	45
12 - 18	10	15	150
18 - 24	12	21	252
24 - 30	6	27	162
Total	$\Sigma f_i = 40$		$\Sigma f_i x_i = 630$

$\therefore$ Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{630}{40} = 15.75$

18. Let the ten numbers be $x_1, x_2, ..., x_5, x_6, x_7, ..., x_{10}$.

Mean of the first six numbers = 15

$\therefore$ $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 90$ ...(i)

Mean of the last five numbers = 10

$\therefore$ $x_6 + x_7 + x_8 + x_9 + x_{10} = 50$ ...(ii)

Mean of 10 numbers $= \frac{x_1 + x_2 + ... + x_9 + x_{10}}{10}$

$$\Rightarrow \quad 12.5 = \frac{(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) + (x_6 + x_7 + x_8 + x_9 + x_{10}) - x_6}{10}$$

$\Rightarrow$ $12.5 \times 10 = 90 + 50 - x_6$ [Using (i) and (ii)]

$\Rightarrow$ $125 = 140 - x_6$ $\Rightarrow x_6 = 140 - 125 = 15$

Hence, the sixth number is 15.

19. Given, mean = 5.5

x_i	f_i	$f_i x_i$
2	3	6
4	5	20
6	6	36
8	y	$8y$
	$\Sigma f_i = 14 + y$	$\Sigma f_i x_i = 62 + 8y$

Mean $= \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 5.5 = \frac{62 + 8y}{14 + y}$

$\Rightarrow$ $5.5(14 + y) = 62 + 8y \Rightarrow 77 + 5.5y = 62 + 8y$

$\Rightarrow$ $8y - 5.5y = 77 - 62 \Rightarrow 2.5y = 15 \Rightarrow y = 6$

20. We have, $\frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \bar{x}$

$\Rightarrow$ $x_1 + x_2 + ... + x_n = n\bar{x}$ ...(i)

Also, $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + ... + x_n \Rightarrow \sum_{i=1}^{n} x_i = n\bar{x}$ [Using (i)]

$\Rightarrow$ $\sum_{i=1}^{n} x_i - n\bar{x} = 0$

21. Let us construct the following table from the given data:

Class-interval	Frequency (f_i)	Class-mark (x_i)	$f_i x_i$
0 - 20	17	10	170
20 - 40	p	30	$30p$
40 - 60	32	50	1600
60 - 80	24	70	1680
80 - 100	19	90	1710
Total	$\Sigma f_i = 92 + p$		$\Sigma f_i x_i = 5160 + 30p$

Now, Mean $= \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 50 = \frac{5160 + 30p}{92 + p}$

$\Rightarrow$ $4600 + 50p = 5160 + 30p \Rightarrow 20p = 560 \Rightarrow p = 28$

22. It is given that mode = 8, which lies in the interval 7 – 10. Therefore, 7 – 10 is the modal class.

So, $l = 7, f_1 = 35, f_0 = 25, f_2 = x$ and $h = 3$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow$ $8 = 7 + \left(\frac{35 - 25}{2 \times 35 - 25 - x}\right) \times 3 \Rightarrow 1 = \left(\frac{10}{70 - 25 - x}\right) \times 3$

$\Rightarrow$ $45 - x = 30 \Rightarrow x = 45 - 30 \Rightarrow x = 15$

Hence, the missing frequency (x) is 15.

OR

23. Let us construct the cumulative frequency distribution table for the given data :

Family size	Number of families	Cumulative frequency
1 - 3	4	4
3 - 5	6	10
5 - 7	2	12
7 - 9	2	14
9 - 11	2	16
Total	$\Sigma f_i = 16$	

Here, $n = 16 \Rightarrow \frac{n}{2} = 8$

The cumulative frequency just greater than 8 is 10 and corresponding interval is 3-5. $\therefore$ Median class is 3-5.

So, $l = 3$, $c.f. = 4$, $f = 6$ and $h = 2$

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h = 3 + \left(\frac{8 - 4}{6}\right) \times 2$

$= 3 + \frac{4}{6} \times 2 = 3 + 1.33 = 4.33$

Hence, median family size is 4.33.

24. Here, $h = 2$

Let us construct the following frequency distribution table:

Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	f_iu_i
11 - 13	3	12	–3	–9
13 - 15	6	14	–2	–12
15 - 17	9	16	–1	–9
17 - 19	13	18 = a (let)	0	0
19 - 21	k	20	1	k
21 - 23	5	22	2	10
23 - 25	4	24	3	12
Total	$\Sigma f_i = 40 + k$			$\Sigma f_iu_i = k - 8$

$$\text{Mean } (\bar{x}) = a + \left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right) \times h \Rightarrow 18 = 18 + \left(\frac{k-8}{40+k}\right) \times 2$$

$$\Rightarrow 0 = \frac{2(k-8)}{40+k} \Rightarrow k - 8 = 0 \Rightarrow k = 8$$

OR

We prepare the following cumulative frequency distribution table:

Salary (in thousand ₹)	Number of persons	Cumulative frequency
5 - 10	49	49
10 - 15	133	182
15 - 20	63	245
20 - 25	15	260
25 - 30	6	266
30 - 35	7	273
35 - 40	4	277
40 - 45	2	279
45 - 50	1	280
Total	$\Sigma f_i = 280$	

We have, $n = 280 \Rightarrow \frac{n}{2} = \frac{280}{2} = 140$

Cumulative frequency just greater than 140 is 182 and corresponding interval is 10–15. Therefore, 10–15 is the median class.

So, $l = 10$, $f = 133$, $c.f. = 49$ and $h = 5$.

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 10 + \left(\frac{140-49}{133}\right) \times 5 = 10 + \frac{455}{133} = 10 + 3.42 = 13.42$$

Hence, median salary is ₹ 13.42 (in thousand).

24. Here, $h = 10$

Let us construct the following table :

Class-interval	Frequency (f_i)	Class-mark (x_i)	$u_i = \frac{x_i - 55}{10}$	f_iu_i
10 - 20	8	15	–4	–32
20 - 30	7	25	–3	–21
30 - 40	12	35	–2	–24
40 - 50	23	45	–1	–23
50 - 60	11	55 = a (let)	0	0
60 - 70	13	65	1	13
70 - 80	8	75	2	16
80 - 90	6	85	3	18
90 - 100	12	95	4	48
Total	$\Sigma f_i = 100$			$\Sigma f_iu_i = -5$

$$\text{Now, Mean } (\bar{x}) = a + \left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right) \times h$$

$$= 55 + \left(\frac{-5}{100}\right) \times 10 = 55 - \frac{50}{100} = 55 - 0.5 = 54.5$$

Hence, mean is 54.5.

25. Let us construct the following cumulative frequency distribution table :

Class-interval	Frequency	Cumulative frequency
0 - 100	2	2
100 - 200	5	7
200 - 300	x	7 + x
300 - 400	12	19 + x
400 - 500	17	36 + x
500 - 600	20	56 + x
600 - 700	15	71 + x
700 - 800	9	80 + x
800 - 900	7	87 + x
900 - 1000	4	91 + x

Given, median = 525, which lies in the interval 500 - 600.

$\therefore$ Median class is 500 - 600.

So, $l = 500$, $f = 20$, $c.f. = 36 + x$ and $h = 100$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$\Rightarrow \quad 525 = 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$$

$$\Rightarrow \quad 25 = 70 - 5x \Rightarrow 5x = 45 \Rightarrow x = \frac{45}{5} \Rightarrow x = 9$$

Thus, the missing frequency is 9.

26. Let us construct the following frequency distribution table:

Daily savings (in ₹)	Number of children (f_i)	Class mark (x_i)	f_ix_i
1 - 3	7	2	14
3 - 5	6	4	24
5 - 7	x	6	$6x$

7 - 9	13	8	104
9 - 11	y	10	$10y$
11 - 13	5	12	60
13 - 15	4	14	56
Total	$\Sigma f_i = 35 + x + y$		$\Sigma f_i x_i = 258 + 6x + 10y$

Given, sum of frequencies = 64

$\Rightarrow \quad 35 + x + y = 64 \Rightarrow x + y = 29 \Rightarrow y = 29 - x$ ...(i)

Now, Mean = $\dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 8 = \dfrac{258 + 6x + 10y}{64}$ ($\because$ Given, mean = 8)

$\Rightarrow \quad 8 \times 64 = 258 + 6x + 10y$

$\Rightarrow \quad 512 = 258 + 6x + 10(29 - x)$ [Using (i)]

$\Rightarrow \quad 512 = 258 + 6x + 290 - 10x$

$\Rightarrow \quad 512 = 548 - 4x \Rightarrow 4x = 548 - 512$

$\Rightarrow \quad 4x = 36 \Rightarrow x = 9$ ...(ii)

From (i) and (ii), we obtain $y = 29 - 9 = 20$

Hence, the missing frequencies are $x = 9$ and $y = 20$.

OR

Let us construct the following cumulative frequency distribution table :

Class-interval	Frequency	Cumulative frequency
0 - 5	12	12
5 - 10	a	$12 + a$
10 - 15	12	$24 + a$
15 - 20	15	$39 + a$
20 - 25	b	$39 + a + b$
25 - 30	6	$45 + a + b$
30 - 35	6	$51 + a + b$
35 - 40	4	$55 + a + b$

It is given that total frequency = 70

$\Rightarrow \quad 55 + a + b = 70$ ...(i)

$\Rightarrow \quad a + b = 70 - 55 = 15$

Given, median is 16 which lies in the class interval 15 - 20.

So, $l = 15$, $f = 15$, $c.f. = 24 + a$, $\dfrac{n}{2} = \dfrac{70}{2} = 35$ and $h = 5$

Now, Median = $l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow \quad 16 = 15 + \left(\dfrac{35 - (24 + a)}{15}\right) \times 5 = 15 + \left(\dfrac{11 - a}{3}\right)$

$\Rightarrow \quad 1 = \dfrac{11 - a}{3} \Rightarrow 3 = 11 - a \Rightarrow a = 11 - 3 = 8$

Now, from (i), we get $8 + b = 15 \Rightarrow b = 15 - 8 = 7$

Hence, the missing frequencies are 8 and 7.

27. Let us construct the following frequency distribution table:

Marks	Number of students (f_i)	Class-mark (x_i)	$f_i x_i$
0 - 10	1	5	5
10 - 20	3	15	45
20 - 30	7	25	175
30 - 40	10	35	350
40 - 50	15	45	675
50 - 60	x	55	$55x$
60 - 70	9	65	585
70 - 80	27	75	2025
80 - 90	18	85	1530
90 - 100	y	95	$95y$
Total	$\Sigma f_i = 90 + x + y$		$\Sigma f_i x_i = 5390 + 55x + 95y$

Total number of students = 120 [Given]

$\therefore \quad 90 + x + y = 120$

$\Rightarrow \quad x + y = 120 - 90 = 30 \Rightarrow y = 30 - x$ ...(i)

Now, mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 59 = \dfrac{5390 + 55x + 95y}{120}$

$\Rightarrow \quad 59 \times 120 = 5390 + 55x + 95y$

$\Rightarrow \quad 7080 = 5390 + 55x + 95(30 - x)$ [Using (i)]

$\Rightarrow \quad 7080 - 5390 = 55x + 2850 - 95x \Rightarrow 1690 = 2850 - 40x$

$\Rightarrow \quad 40x = 2850 - 1690 \Rightarrow 40x = 1160 \Rightarrow x = \dfrac{1160}{40} = 29$

Substituting $x = 29$ in (i), we get $y = 30 - 29 = 1$

Hence, the missing frequencies are $x = 29$ and $y = 1$.

28. Let us construct the cumulative frequency distribution table :

Ages (in years)	Number of persons (f_i)	Cumulative frequency (c.f.)	Class-mark (x_i)	$u_i = \dfrac{x_i - 35}{10}$	$f_i u_i$
0 - 10	50	50	5	–3	–150
10 - 20	400	450	15	–2	–800
20 - 30	108	558	25	–1	–108
30 - 40	530	1088	35 = a (let)	0	0
40 - 50	47	1135	45	1	47
50 - 60	10	1145	55	2	20
60 - 70	5	1150	65	3	15
Total	$\Sigma f_i = 1150$				$\Sigma f_i u_i = -976$

Mean, $\bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right) h$

$= 35 + \dfrac{(-976)}{1150} \times 10 = 35 - \dfrac{976}{115} = 35 - 8.49 = 26.51$

From the table, $n = 1150 \Rightarrow \frac{n}{2} = \frac{1150}{2} = 575$

Cumulative frequency just greater than 575 is 1088 and corresponding interval is 30 - 40.

Therefore, 30 - 40 is the median class. So, $l = 30, f = 530$, $c.f. = 558$ and $h = 10$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 30 + \left(\frac{575 - 558}{530}\right) \times 10 = 30 + \frac{17}{53} = 30 + 0.32 = 30.32$$

Hence, the mean and median of the given distribution is 26.51 years and 30.32 years.

29. We have, $h = 2$. Let us construct the following frequency distribution table :

Age (in years)	Number of students (f_i)	Class-mark (x_i)	$u_i = \frac{x_i - 12}{2}$	$f_i u_i$
5 - 7	67	6	–3	–201
7 - 9	33	8	–2	–66
9 - 11	41	10	–1	–41
11 - 13	95	12 = a (let)	0	0
13 - 15	36	14	1	36
15 - 17	13	16	2	26
17 - 19	15	18	3	45
Total	$\Sigma f_i = 300$			$\Sigma f_i u_i = -201$

$$\therefore \quad \text{Mean, } (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h = 12 + \frac{(-201)}{300} \times 2$$

$$= 12 - \frac{67 \times 2}{100} = 12 - \frac{134}{100} = 12 - 1.34 = 10.66$$

Since, the class 11-13 has the maximum frequency 95, therefore, 11-13 is the modal class.

So, $l = 11, f_1 = 95, f_0 = 41, f_2 = 36$ and $h = 2$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 11 + \left(\frac{95 - 41}{2 \times 95 - 41 - 36}\right) \times 2 = 11 + \left(\frac{54}{190 - 77}\right) \times 2$$

$$= 11 + \frac{108}{113} = 11 + 0.96 = 11.96$$

Hence, mean = 10.66 and mode = 11.96 of the given data.

OR

Since, the maximum frequency is 29 which corresponds to the class 60-80. Therefore, 60-80 is the modal class.

So, $l = 60, f_1 = 29, f_0 = 21, f_2 = 17$ and $h = 20$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{29 - 21}{2 \times 29 - 21 - 17}\right) \times 20 = 60 + \frac{8}{20} \times 20 = 60 + 8 = 68$$

So, mode of the distribution = 68

Given, mean = 53

Empirical relationship between the three measures of central tendency is

3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median = 68 + 2 × 53 = 68 + 106 = 174

$\Rightarrow$ Median $= \frac{174}{3} = 58$

30. Let us construct the following cumulative frequency distribution table:

Class	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	$5 + x$
20 - 30	6	$11 + x$
30 - 40	y	$11 + x + y$
40 - 50	6	$17 + x + y$
50 - 60	5	$22 + x + y$

It is given that $\Sigma f_i = 40$

$\therefore \quad 22 + x + y = 40$

$\Rightarrow \quad x + y = 40 - 22 = 18 \Rightarrow y = 18 - x$ ...(i)

The median is 31, which lies in the class-interval 30–40

So, $l = 30,\ f = y,\ c.f. = 11 + x,\ \frac{n}{2} = \frac{40}{2} = 20$ and $h = 10$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$\Rightarrow \quad 31 = 30 + \left(\frac{20 - (11 + x)}{y}\right) \times 10$$

$$\Rightarrow 31 - 30 = \frac{9 - x}{y} \times 10 \Rightarrow 1 = \frac{9 - x}{18 - x} \times 10 \quad \text{[Using (i)]}$$

$\Rightarrow \quad 18 - x = 90 - 10x \quad \Rightarrow \quad 10x - x = 90 - 18$

$\Rightarrow \quad 9x = 72 \quad \Rightarrow \quad x = 8$

Substituting $x = 8$ in (i), we get $y = 18 - 8 = 10$

Hence, the missing frequencies are $x = 8$ and $y = 10$

31. The frequency distribution table from the given data can be drawn as :

Number of members	(x_i)	(f_i)	$f_i x_i$
1-3	2	2	4
3-5	4	8	32
5-7	6	6	36
7-9	8	10	80
9-11	10	5	50
11-13	12	5	60
13-15	14	7	98
15-17	16	4	64
17-19	18	3	54
		$\Sigma f_i = 50$	$\Sigma f_i x_i = 478$

$\therefore \quad \text{Mean} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{478}{50} = 9.56$

Now, maximum frequency lies in the class interval 7-9.

$\therefore$ Modal class is 7-9.

$l = 7, h = 2, f_1 = 10, f_0 = 6, f_2 = 5$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$\text{Mode} = 7 + \left[\frac{10-6}{2\times 10 - 6 - 5}\right] \times 2$$

$$= 7 + \left[\frac{4}{20-11}\right] \times 2 = 7 + \frac{8}{9} = 7 + 0.889 = 7.889$$

32. Let us construct the following cumulative frequency distribution table :

Class-interval	**Frequency (f_i)**	**Cumulative frequency (*c.f.*)**
0 - 20	6	6
20 - 40	8	6 + 8 = 14
40 - 60	10	14 + 10 = 24
60 - 80	12	24 + 12 = 36
80 - 100	6	36 + 6 = 42
100 - 120	5	42 + 5 = 47
120 - 140	3	47 + 3 = 50
Total	$\Sigma f_i = 50$	

We have, $n = 50 \Rightarrow \dfrac{n}{2} = 25$

Cumulative frequency just greater than 25 is 36, which lies in the interval 60-80.

$\therefore$ 60-80 is the median class. So, $l = 60, f = 12, c.f. = 24$ and $h = 20$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 60 + \left(\frac{25-24}{12}\right) \times 20 = 60 + \frac{20}{12} = 60 + 1.67 = 61.67$$

Since, the class 60-80 has the maximum frequency 12, therefore, 60-80 is the modal class.

So, $l = 60, f_0 = 10, f_1 = 12, f_2 = 6$ and $h = 20$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 60 + \left(\frac{12-10}{2\times 12 - 10 - 6}\right) \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 60 + 5 = 65$$