Real Numbers

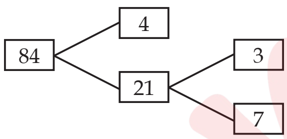
SOLUTIONS

1. **(c) :** Let HCF be x, then LCM = $14x$
We have, $14x + x = 600 \Rightarrow x = 40$
Let other number be y.
As we know that, product of two numbers = product of their LCM & HCF
$\Rightarrow y \times 80 = 40 \times 14 \times 40 \Rightarrow y = 280$

2. **(a) :** The prime factorization of 280 and 674 are
$280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$
and $674 = 2 \times 337$
$\therefore$ HCF (280, 674) = 2.

3. **(c) :** If the sum of 3 prime numbers is even, then one of the numbers must be 2.
Let the second number be x.
$x + (x + 36) + 2 = 100$
$\Rightarrow 2x = 62 \Rightarrow x = 31$
So, the numbers are 2, 31 and 67.
Hence, largest number is 67.

4. **(b) :** Using factor tree method, we have



$\therefore x = 21;\ y = 84$

5. **(d) :** $12 - 7 = 5$, $15 - 10 = 5$ and $16 - 11 = 5$
Hence, the desired number is 5 less than the LCM of 12, 15, 16.
LCM of 12, 15 and 16 is 240.
Hence, the least number = 240 – 5 = 235

6. **(c) :** Since LCM $(a, b) = \dfrac{a \times b}{\text{HCF }(a, b)} = \dfrac{1800}{12} = 150$

7. Given that, $a = x^5y^3 = x \times x \times x \times x \times x \times y \times y \times y$
and $b = x^3y^4 = x \times x \times x \times y \times y \times y \times y$
$\therefore$ HCF of a and b = HCF (x^5y^3, x^3y^4)
$= x \times x \times x \times y \times y \times y = x^3y^3$

8. Clearly, 2 is neither a factor of p nor that of q
$\therefore$ p and q are both odd.
So, $(p + q)$ must be an even number, which is divisible by 2. Hence, the least prime factor of $(p + q)$ is 2.

9. Since, p is prime, then p and $p + 1$ has no common factor other than 1.
$\therefore$ HCF of p and $(p + 1)$ is 1 and LCM of p and $(p + 1)$ is $p(p + 1)$.

10. The factors common to given numbers are 2^2, 5 and 7^2.
$\therefore$ HCF = $2^2 \times 5 \times 7^2 = 980$.

11. HCF = $\dfrac{\text{Product of two numbers}}{\text{LCM}}$

12. **(i)** Here $80 = 2^4 \times 5$, $85 = 17 \times 5$
and $90 = 2 \times 3^2 \times 5$
L.C.M of 80, 85 and 90 = $2^4 \times 3 \times 3 \times 5 \times 17 = 12240$
Hence, the minimum distance each should walk when they at first time is 12240 cm.

(ii) Here $594 = 2 \times 3^3 \times 11$ and $189 = 3^3 \times 7$
HCF of 594 and 189 = $3^3 = 27$
Hence, the maximum number of columns in which they can march is 27.

(iii) Here $768 = 2^8 \times 3$ and $420 = 2^2 \times 3 \times 5 \times 7$
HCF of 768 and 420 = $2^2 \times 3 = 12$
So, the container which can measure fuel of either tanker exactly must be of 12 litres.

(iv) Here, Length = 825 cm, Breadth = 675 cm and Height = 450 cm
Also, $825 = 5 \times 5 \times 3 \times 11$, $675 = 5 \times 5 \times 3 \times 3 \times 3$ and $450 = 2 \times 3 \times 3 \times 5 \times 5$
HCF = $5 \times 5 \times 3 = 75$
Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

(v) LCM of 8 and 12 is 24.
$\therefore$ The least number of pack of pens = 24/8 = 3
$\therefore$ The least number of pack of note pads = 24/12 = 2

13. **(i)** **(b) :** Here $\sqrt{8} = 2\sqrt{2}$ = product of rational and irrational numbers = irrational number

(ii) **(c) :** Here, $\sqrt{9} = 3$
So, $2 + 2\sqrt{9} = 2 + 6 = 8$, which is not irrational.

(iii) **(b) :** Here $\sqrt{15}$ and $\sqrt{10}$ are both irrational and difference of two irrational numbers is also irrational.

(iv) **(c) :** As $\sqrt{5}$ is irrational, so its reciprocal is also irrational.

(v) **(d) :** We know that $\sqrt{6}$ is irrational.
So, $15 + 3\sqrt{6}$ is irrational.
Similarly, $\sqrt{24} - 9 = 2\sqrt{6} - 9$ is irrational.
And $5\sqrt{150} = 5 \times 5\sqrt{6} = 25\sqrt{6}$ is irrational.

14. (i) (b) : LCM of x and $y = p^3q^3$ and HCF of x and $y = p^2q$
Also, LCM = pq^2 × HCF.

(ii) (d) : Number of marbles = $5m + 2$ or $6n + 2$.
Thus, number of marbles, p = (multiple of 5 × 6) + 2
= $30k + 2 = 2(15k + 1)$
= which is an even number but not prime

(iii) (d) : Here, required numbers
= HCF (398 – 7, 436 – 11, 542 –15)
= HCF (391, 425, 527) = 17

(iv) (b) : LCM of 126 and 600 = 2 × 3 × 21 × 100 = 12600
The least positive integer which on adding 1 is exactly divisible by 126 and 600 = 12600 – 1 = 12599

(v) (a) : Here $85C - 340A = 109$ and $425A + 85B = 146$
On adding them, we get
$85A + 85B + 85C = 255 \Rightarrow A + B + C = 3$, which is divisible by 3.

15. The greatest possible speed of the bird is the HCF of 45 and 336.
Now, 45 = 3 × 3 × 5
336 = 2 × 2 × 2 × 2 × 3 × 7
∴ HCF (45, 336) = 3
∴ Required speed is 3 km/h.

16. (i)

3	429
11	143
13	13
	1

∴ Prime factorisation of 429 = 3 × 11 × 13

(ii)

5	7325
5	1465
293	293
	1

∴ Prime factorsiation of 7325 = 5 × 5 × 293

17. We have, 3 × 5 × 13 × 46 + 23
= 3 × 5 × 13 × 2 × 23 + 23 = 23 (3 × 5 × 13 × 2 + 1)
= 23 × 391, which is a product of two numbers.
So, the given number is composite.

18. LCM of 252, 308 and 198 = 2772.
So, A, B and C will again meet at the starting point in 2772 seconds *i.e.*, 46 minutes 12 seconds.

OR

Let the required numbers be x, $2x$ and $3x$. Then their HCF = x. So, $x = 12$
∴ The numbers are 12, 24 and 36.
∴ Required number = $\sqrt{36} = 6$

19. Divisors of 99 are 1, 3, 9, 11, 33 and 99
Divisors of 101 are 1 and 101
Divisors of 176 are 1, 2, 4, 8, 16, 11, 22, 44, 88 and 176
Divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182
Hence, 176 has the most number of divisors.

20. Prime factorisation of 144, 112, 418 are
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$
$112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$
$418 = 2 \times 11 \times 19$
∴ LCM (144, 112, 418) = $2^4 \times 3^2 \times 7 \times 11 \times 19 = 210672$

21. 99 = 1 × 3 × 3 × 11; 101 = 1 × 101;
176 = 1 × 2 × 2 × 2 × 2 × 11;
182 = 1 × 2 × 7 × 13
So, divisors of 99 are 1, 3, 9, 11, 33 and 99;
divisors of 101 are 1 and 101;
divisors of 176 are 1, 2, 4, 8, 16, 11, 22, 44, 88 and 176;
divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182
Hence, 176 has the most number of divisors.

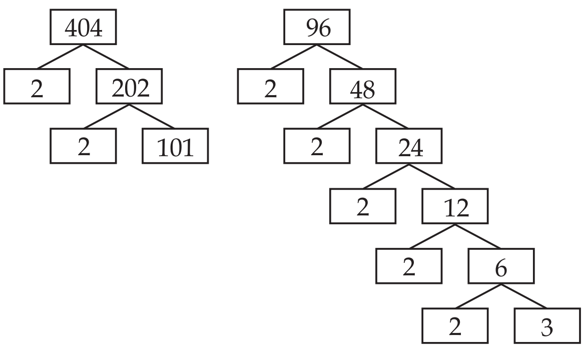
22. Let $x = 1.3\overline{7} = 1.37777....$
Multiplying both sides by 10, we get
$10x = 13.7777...$ ...(i)
Multiplying (i) by 10, we get
$100x = 137.777...$ ...(ii)
Subtracting (i) from (ii), we get
$90x = 124$

$$\Rightarrow \quad x = \frac{124}{90} = \frac{62}{45} = \frac{62}{3^2 \times 5} = \frac{p}{q}$$

Hence, $q = 45$ has prime factors 3 and 5 (or is not of the form $2^m \times 5^n$).

23. LCM of n and p is 21879
$21879 = 3^2 \times 11 \times 13 \times 17$
Since p is a prime, LCM of n, p is np and HCF $(n, p) = 1$
If $p = 13$, $n = 9 \times 11 \times 17 = 1683 \Rightarrow n + p \neq 2000$
If $p = 17$, $n = 9 \times 11 \times 13 = 1287 \Rightarrow n + p \neq 2000$
If $p = 11$, $n = 9 \times 13 \times 17 = 1989 \Rightarrow n + p = 2000$
∴ $p = 11$, $n = 1989$.

24. Using the factor tree method, we have



⇒ 404 = 2 × 2 × 101 and 96 = 2 × 2 × 2 × 2 × 2 × 3
∴ HCF of 404 and 96 = 2 × 2 = 4
LCM of 404 and 96 = 2 × 2 × 2 × 2 × 2 × 3 × 101 = 9696
Also, 404 × 96 = 38784
LCM × HCF = 9696 × 4 = 38784
Thus, HCF × LCM = Product of two numbers.

25. The number of rooms will be minimum if each room accommodates maximum number of participants. Since in each room the same number of participants are to be seated and all of them must be of the same subject. Therefore the number of participants in each room must be the HCF of 60, 84 and 108.

We have, $60 = 2^2 \times 3 \times 5$, $84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$

$\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3 = 12$

Therefore, in each room 12 participants can be seated.

$\therefore$ Number of rooms required

$$= \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12} = \frac{252}{12} = 21$$

26. If possible, let there be a positive integer n for which $\sqrt{n+1}+\sqrt{n-1}$ is rational and equal to $\frac{a}{b}$ (say), where a, b are positive integers. Then,

$$\frac{a}{b} = \sqrt{n+1}+\sqrt{n-1} \quad \text{...(i)}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{n+1}+\sqrt{n-1}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{\{\sqrt{n+1}+\sqrt{n-1}\}\{\sqrt{n+1}-\sqrt{n-1}\}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{(n+1)-(n-1)} = \frac{\sqrt{n+1}-\sqrt{n-1}}{2}$$

$$\Rightarrow \frac{2b}{a} = \sqrt{n+1}-\sqrt{n-1} \quad \text{...(ii)}$$

Adding (i) and (ii) and subtracting (ii) from (i), we get

$$2\sqrt{n+1} = \frac{a}{b}+\frac{2b}{a} \text{ and } 2\sqrt{n-1} = \frac{a}{b}-\frac{2b}{a}$$

$$\Rightarrow \sqrt{n+1} = \frac{a^2+2b^2}{2ab} \text{ and } \sqrt{n-1} = \frac{a^2-2b^2}{2ab}$$

$\Rightarrow$ $\sqrt{n+1}$ and $\sqrt{n-1}$ are rationals.

$\left[\because a \text{ and } b \text{ are integers } \therefore \frac{a^2+2b^2}{2ab} \text{ and } \frac{a^2-2b^2}{2ab} \text{ are rationals.}\right]$

$\Rightarrow$ $(n + 1)$ and $(n - 1)$ are perfect squares of positive integers.

This is not possible as any two perfect squares differ at least by 3.

Hence, there is no positive integer n for which $(\sqrt{n-1}+\sqrt{n+1})$ is rational.