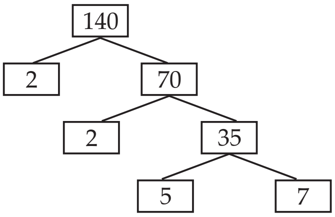
Real Numbers

CHAPTER 1

NCERT FOCUS SOLUTIONS

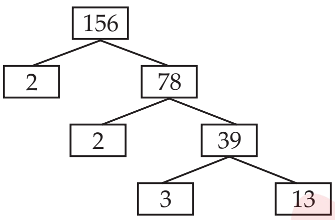
EXERCISE - 1.2

1. (i) Using factor tree method, we have



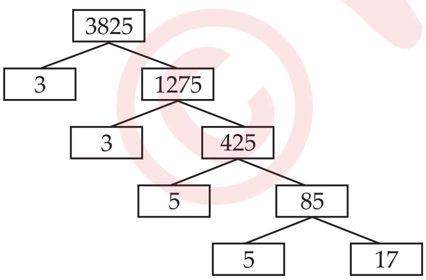
$\therefore$ $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) Using factor tree method, we have



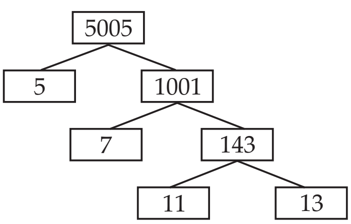
$\therefore$ $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) Using factor tree method, we have



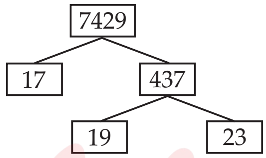
$\therefore$ $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) Using factor tree method, we have



$\therefore$ $5005 = 5 \times 7 \times 11 \times 13$

(v) Using factor tree method, we have



$\therefore$ $7429 = 17 \times 19 \times 23$

2. (i) The prime factorisation of 26 and 91 is,
$26 = 2 \times 13$ and $91 = 7 \times 13$
$\therefore$ LCM (26, 91) $= 2 \times 7 \times 13 = 182$
HCF (26, 91) = 13
Now, LCM × HCF = $182 \times 13 = 2366$ and $26 \times 91 = 2366$
i.e., LCM × HCF = Product of two numbers.

(ii) The prime factorisation of 510 and 92 is,
$510 = 2 \times 3 \times 5 \times 17$ and $92 = 2 \times 2 \times 23$
$\therefore$ LCM (510, 92) $= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
HCF (510, 92) = 2
Now, LCM × HCF = $23460 \times 2 = 46920$
and $510 \times 92 = 46920$
i.e., LCM × HCF = Product of two numbers.

(iii) The prime factorisation of 336 and 54 is,
$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$ and $54 = 2 \times 3 \times 3 \times 3$
$\therefore$ LCM (336, 54) $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$
and HCF(336, 54) $= 2 \times 3 = 6$
Now, LCM × HCF = $3024 \times 6 = 18144$
Also, $336 \times 54 = 18144$
Thus, LCM × HCF = Product of two numbers.

3. (i) The prime factorisation of 12, 15 and 21 is,
$12 = 2 \times 2 \times 3$, $15 = 3 \times 5$ and $21 = 3 \times 7$
$\therefore$ HCF (12, 15, 21) = 3
LCM (12, 15, 21) $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

(ii) We have, $17 = 1 \times 17$, $23 = 1 \times 23$, $29 = 1 \times 29$
$\Rightarrow$ HCF (17, 23, 29) = 1
LCM (17, 23, 29) $= 17 \times 23 \times 29 = 11339$

(iii) The prime factorisation of 8, 9 and 25 is,
$8 = 2 \times 2 \times 2$, $9 = 3 \times 3$ and $25 = 5 \times 5$
$\therefore$ HCF (8, 9, 25) = 1
LCM (8, 9, 25) $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

4. Since, LCM × HCF = Product of the numbers
$\therefore$ LCM $\times 9 = 306 \times 657$

$$\Rightarrow \text{LCM} = \frac{306 \times 657}{9} = 22338$$

Thus, LCM of 306 and 657 is 22338.

5. Here, n is a natural number and let 6^n ends with digit 0.
$\therefore$ 6^n is divisible by 5.

But the prime factors of 6 are 2 and 3. *i.e.*, $6 = 2 \times 3$

$\Rightarrow \quad 6^n = (2 \times 3)^n$

i.e., In the prime factorisation of 6^n, there is no factor 5.

So, by the fundamental theorem of Arithmetic, every composite number can be expressed as a product of primes and this factorisation is unique apart from the order in which the prime factorisation occurs.

$\therefore$ Our assumption that 6^n ends with digit 0, is wrong.

Thus, there does not exist any natural number n for which 6^n ends with zero.

6. We have

$7 \times 11 \times 13 + 13 = 13((7 \times 11) + 1) = 13(78)$, which cannot be a prime number because it has factors 13 and 78.

Also, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5[7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1]$,

which is also not a prime number because it has a factor 5

Thus, $7 \times 11 \times 13 + 13$ and

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

7. Time taken by Sonia to drive one round of the field = 18 minutes

Time taken by Ravi to drive one round of the field = 12 minutes

LCM of 18 and 12 gives the exact number of minutes after which they meet again at the starting point.

Now, $18 = 2 \times 3 \times 3$ and $12 = 2 \times 2 \times 3$

$\therefore$ LCM of 18 and 12 $= 2 \times 2 \times 3 \times 3 = 36$

Thus, they will meet again at the starting point after 36 minutes.

EXERCISE - 1.3

1. Let $\sqrt{5}$ be a rational number.

So, we can find integers a and b ($b \neq 0$ and a, b are co-prime) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad \sqrt{5} \cdot b = a$ ...(i)

Squaring both sides, we get

$5b^2 = a^2$

$\Rightarrow$ 5 divides $a^2 \Rightarrow$ 5 divides a ...(ii)

So, we can write $a = 5m$, where m is an integer.

$\therefore$ Putting $a = 5m$ in (i), we get

$\sqrt{5}b = 5m$

$\Rightarrow \quad 5b^2 = 25m^2$ [Squaring both sides]

$\Rightarrow \quad b^2 = 5m^2$

$\Rightarrow$ 5 divides $b^2 \Rightarrow$ 5 divides b ...(iii)

From (ii) and (iii), we have, a and b have 5 as a common factor which contradicts the fact that a and b are co-prime.

$\therefore$ Our supposition that $\sqrt{5}$ is rational, is wrong.

Hence, $\sqrt{5}$ is irrational.

2. Let $3 + 2\sqrt{5}$ be a rational number.

$\therefore$ We can find two co-prime integers a and b such that

$3 + 2\sqrt{5} = \frac{a}{b}$, where $b \neq 0$

$\therefore \quad \frac{a}{b} - 3 = 2\sqrt{5} \Rightarrow \frac{a - 3b}{b} = 2\sqrt{5} \Rightarrow \frac{a - 3b}{2b} = \sqrt{5}$ ... (i)

$\because$ a and b are integers,

$\therefore \quad \frac{a-3b}{2b}$ is rational

So, $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our supposition is wrong.

Hence, $3 + 2\sqrt{5}$ is irrational.

3. (i) We have $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2} \cdot \sqrt{2}$

Let $\frac{1}{\sqrt{2}}$ be rational,

$\therefore \quad \frac{1}{2}(\sqrt{2})$ is rational

Let $\frac{1}{2}(\sqrt{2}) = \frac{a}{b}$, such that a and b are co-prime integers and $b \neq 0$.

$\therefore \quad \sqrt{2} = \frac{2a}{b}$ ... (i)

Since, the division of two integers is rational.

$\therefore \quad \frac{2a}{b}$ is rational.

From (i), $\sqrt{2}$ is rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our assumption is wrong.

Thus, $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let $7\sqrt{5}$ is rational.

$\therefore$ We can find two co-prime integers a and b such that

$7\sqrt{5} = \frac{a}{b}$, where $b \neq 0$

Now, $7\sqrt{5} = \frac{a}{b} \Rightarrow \sqrt{5} = \frac{a}{7b}$, which is a rational number.

[$\because$ a and b are integers.]

$\Rightarrow \quad \sqrt{5}$ is a rational number.

This contradicts the fact that $\sqrt{5}$ is an irrational number.

$\therefore$ Our assumption is wrong.

Thus, we conclude that $7\sqrt{5}$ is irrational.

(iii) Let $6 + \sqrt{2}$ is rational.

$\therefore$ We can find two co-prime integers a and b such that

$6 + \sqrt{2} = \frac{a}{b}$, where $b \neq 0$

$\therefore \quad \frac{a}{b} - 6 = \sqrt{2} \Rightarrow \sqrt{2} = \frac{a - 6b}{b}$, which is rational

$\Rightarrow \quad \sqrt{2}$ is rational which contradicts the fact that $\sqrt{2}$ is an irrational number.

$\therefore$ Our supposition is wrong.

Hence, $6 + \sqrt{2}$ is an irrational number.