CHAPTER 2

Polynomials

EXAM DRILL

SOLUTIONS

1. **(a) :** Since α, β are the zeroes of $2x^2 + 6x - 6$, we have

$\therefore \quad \alpha + \beta = \frac{-6}{2} = -3$ and $\alpha\beta = \frac{-6}{2} = -3.$

Hence, $\alpha + \beta = \alpha\beta$

2. **(a) :** Since the curve cuts the x-axis at 3 points. So, there are 3 zeroes of the given graph.

3. **(b) :** Let $f(x) = px^3 + x^2 - 2x + q$

Since $(x + 1)$ and $(x - 1)$ are factors of

$f(x) = px^3 + x^2 - 2x + q$

$\therefore \quad f(1) = 0$ and $f(-1) = 0$

Now, $f(1) = p + 1 - 2 + q = p + q - 1 = 0$

$\Rightarrow \quad p + q = 1$ ...(i)

$f(-1) = 0 \Rightarrow -p + 1 + 2 + q = 0$

$\Rightarrow \quad -p + q = -3$ ...(ii)

Solving (i) and (ii), we get $p = 2$ and $q = -1$

4. Let $p(x) = (k - 1)x^2 + kx + 1$

Given that, one of the zeroes is –3, then $p(-3) = 0$

$\Rightarrow \quad (k - 1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9(k - 1) - 3k + 1 = 0 \Rightarrow 6k - 8 = 0 \Rightarrow k = 4/3.$

5. Total number of zeroes = 2 + 3 = 5

6. Sum of zeroes $= \alpha + \beta = -\left(\frac{-6}{2}\right) = 3$

Product of zeroes $= \alpha\beta = 7/2$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (3)^2 - 2\left(\frac{7}{2}\right) = 9 - 7 = 2$

7. We have $9x^2 - 5 = (3x)^2 - (\sqrt{5})^2$

$= (3x - \sqrt{5})(3x + \sqrt{5}).$

Therefore, the zeroes of $9x^2 - 5$ are

$x = \frac{\sqrt{5}}{3}$ and $x = \frac{-\sqrt{5}}{3}$

8. Given, α and β are the zeroes of the polynomial, $f(x) = x^2 - 19x + k$

$\therefore \quad \alpha + \beta = 19$ ...(i)

and $\alpha\beta = k$ ...(ii)

Also, $\alpha - \beta = 7$ (Given) ...(iii)

Adding (i) and (iii), we get $2\alpha = 26 \Rightarrow \alpha = 13$

From (i), $\beta = 6$

Now, $\alpha\beta = k \Rightarrow 13 \times 6 = k \Rightarrow k = 78$

9. Given that, sum of zeroes $(S) = -\frac{3}{2\sqrt{5}}$

and product of zeroes $(P) = -\frac{1}{2}$

$\therefore$ Required quadratic polynomial is

$$p(x) = x^2 - Sx + P = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2} = 2\sqrt{5}x^2 + 3x - \sqrt{5}$$

Using factorisation method,

$$p(x) = 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$$

$$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = (2x + \sqrt{5})(\sqrt{5}x - 1)$$

Hence, the zeroes of $p(x)$ are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}}$.

10. We have, $f(x) = x^2 - 10x + 25 = (x - 5)^2$

Putting $f(x) = 0$, we get $(x - 5)^2 = 0 \therefore x = 5,5$

$\therefore$ The required zeroe of $f(x)$ is 5.

11. We have, $g(x) = -2x^2 + 3x - 2$ and $h(x) = x - 3$

$\therefore \quad f(x) = (-2x^2 + 3x - 2)(x - 3) + 4$

$= -2x^3 + 3x^2 - 2x + 6x^2 - 9x + 6 + 4$

$= -2x^3 + 9x^2 - 11x + 10$

12. **(i)** Graph of a quadratic polynomial is a parabolic in shape.

(ii) Since the graph of the polynomial cuts the x-axis at (–6, 0) and (6, 0). So, the zeroes of polynomial are –6 and 6.

$\therefore$ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

(iii) We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

(iv) Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{(2)}{1} = -2$$

(v) The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \quad \frac{-5}{a} = \frac{3a}{a} \quad \Rightarrow \quad a = \frac{-5}{3}$$

13. **(i)** **(b) :** The shape of the path of the soccer ball is a parabola.

(ii) **(c) :** The axis of symmetry of the given curve is a line parallel to y-axis.

(iii) **(a) :** The zeroes of the polynomial, represented in the given graph, are –2 and 7, since the curve cuts the x-axis at these points.

(iv) **(d) :** A polynomial having zeroes –2 and –3 is

$p(x) = x^2 - (-2 - 3)x + (-2)(-3) = x^2 + 5x + 6$

(v) (c) : We have, $f(x) = (x - 3)^2 + 9$
Now, $9 = (x - 3)^2 + 9$
$\Rightarrow (x - 3)^2 = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

14. (i) Since, the graph intersects the x-axis at two points, namely $x = 8, -2$.
So, 8, –2 are the zeroes of the given polynomial.

(ii) The expression of the polynomial given in diagram is $-x^2 + 6x + 16$.

(iii) Let $p(x) = -x^2 + 6x + 16$
When $x = 4$, $p(4) = -4^2 + 6 \times 4 + 16 = 24$

(iv) Let $f(x) = -x^2 + 3x - 2$
Now, consider $f(x) = 0 \Rightarrow -x^2 + 3x - 2 = 0$
$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 2)(x - 1) = 0$
$\Rightarrow x = 1, 2$ are its zeroes.

(v) Let α and β are the zeroes of the required polynomial.
Given, $\alpha + \beta = -3$
If $\alpha = 4$, then $\beta = -7$
$\therefore$ Representation of tunnel is $-x^2 - 3x + 28$.

15. Let $p(x) = x^2 - 12x + 35$
For zeroes, put $p(x) = 0$
$\Rightarrow x^2 - 12x + 35 = 0 \Rightarrow x^2 - 5x - 7x + 35 = 0$
$\Rightarrow x(x - 5) - 7(x - 5) = 0 \Rightarrow (x - 5)(x - 7) = 0$
$\Rightarrow x - 5 = 0$ or $x - 7 = 0 \Rightarrow x = 5$ or $x = 7$
Zeroes of $p(x)$ are 5 and 7.

$$\text{Sum of zeroes} = 5 + 7 = 12 = \frac{-(12)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 5 \times 7 = 35 = \frac{35}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

16. Given quadratic polynomial is $x^2 - 8x + 7$.
Let α and β be its roots.
$\therefore \alpha + \beta = 8$ ...(i)
$\alpha\beta = 7$ ...(ii)

$$\therefore \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(8)^2 - 4 \times 7}$$ [Using (i) and (ii)]

$$= \sqrt{64 - 28} = \sqrt{36} = 6$$

$\Rightarrow \alpha - \beta = 6$ ...(iii)
Adding (i) and (iii), we get $2\alpha = 14 \Rightarrow \alpha = 7$
From (i), we get $\beta = 8 - \alpha = 8 - 7 = 1$

OR

Given polynomial, $f(x) = 25P^2 - 15P + 2$
Since, α and β are roots of $f(x)$

$$\therefore \alpha + \beta = \frac{15}{25} = \frac{3}{5} \text{ and } \alpha\beta = \frac{2}{25}$$

Sum of zeroes of the required polynomial

$$= \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{\alpha+\beta}{2\alpha\beta} = \frac{\left(\frac{3}{5}\right)}{2\left(\frac{2}{25}\right)} = \frac{\frac{3}{5}}{\frac{4}{25}} = \frac{3}{5} \times \frac{25}{4} = \frac{15}{4}$$

and product of zeroes of the required polynomial

$$= \left(\frac{1}{2\alpha}\right)\left(\frac{1}{2\beta}\right) = \frac{1}{4\alpha\beta} = \frac{1}{4\left(\frac{2}{25}\right)} = \frac{1}{\frac{8}{25}} = \frac{25}{8}$$

Hence, the required quadratic polynomial is

$$x^2 - \frac{15}{4}x + \frac{25}{8} \text{ or } \frac{1}{8}(8x^2 - 30x + 25).$$

17. Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$. Then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Let S and P denotes respectively the sum and product of the zeroes of a polynomial, whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

$$\text{Then, } S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$$

$$\text{and } P = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$$

Hence, the required polynomial $g(x)$ is given by

$g(x) = k(x^2 - Sx + P) = k\left(x^2 + \frac{bx}{c} + \frac{a}{c}\right)$, where k is any non-zero constant.

18. We have, $6x^2 - 13x + 6 = 6x^2 - 4x - 9x + 6$
$= 2x(3x - 2) - 3(3x - 2) = (2x - 3)(3x - 2)$
So, the value of $6x^2 - 13x + 6$ is 0, when
$(3x - 2) = 0$ or $(2x - 3) = 0$
i.e., when $x = 2/3$ or $x = 3/2$
Therefore, the zeroes of $6x^2 - 13x + 6$ are 2/3 and 3/2.
Sum of zeroes

$$= \frac{2}{3} + \frac{3}{2} = \frac{13}{6} = \frac{-(-13)}{6} = \frac{(-\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

OR

Given, α and β be the roots of the given polynomial.
$\therefore \alpha + \beta = 5, \alpha\beta = 2$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} - 3\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 3\alpha\beta$$

$$= \frac{5}{2} - 3 \times 2 = \frac{5}{2} - 6 = \frac{5 - 12}{2} = \frac{-7}{2}$$

19. Let α, β be the zeroes of $8x^2 - 18x - m$, where $\alpha = 5/2$

$$\text{Now, } \alpha + \beta = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\frac{5}{2} + \beta = \frac{-(-18)}{8} = \frac{9}{4} \Rightarrow \beta = \frac{9}{4} - \frac{5}{2} = \frac{-1}{4}$$

$$\text{Also, } \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-m}{8}$$

$$\frac{5}{2} \times \left(\frac{-1}{4}\right) = \frac{-m}{8} \Rightarrow \frac{-5}{8} = \frac{-m}{8} \Rightarrow m = 5$$

20. Given polynomial is $23x^2 - 26x + 161$.
Product of zeroes = 161/23 = 7
Now, 2 × product of zeroes = $14p$ [Given]

$$\Rightarrow 2 \times 7 = 14p \Rightarrow p = \frac{14}{14} \Rightarrow p = 1.$$

21. Since, α and β are the zeroes of the polynomial $x^2 - 9x + k$.

$\therefore \quad \alpha+\beta=-\frac{(-9)}{1}=9$ ...(i)

and $\alpha\beta=\frac{k}{1}=k$ ...(ii)

Now, $\alpha - \beta = 1$ [Given]

$\Rightarrow \quad (\alpha-\beta)^2 = 1 \Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = 1$

$\Rightarrow \quad 9^2 - 4k = 1$ [Using (i) and (ii)]

$\Rightarrow \quad 81 - 4k = 1 \Rightarrow -4k = -80 \Rightarrow k=\frac{-80}{-4}=20$

22. We have, $f(x) = abx^2 + (b^2 - ac)x - bc$
$= abx^2 + b^2x - acx - bc = bx(ax + b) - c(ax + b)$
$= (ax + b)(bx - c)$
The zeroes of $f(x)$ are given by $f(x) = 0$
$\Rightarrow \quad (ax + b)(bx - c) = 0$
$\Rightarrow \quad ax + b = 0$ or $bx - c = 0$
$\Rightarrow \quad x=-\frac{b}{a}$ or $x=\frac{c}{b}$

Thus, the zeroes of $f(x)$ are : $\alpha=-\frac{b}{a}$ and $\beta=\frac{c}{b}$

Now, $\alpha+\beta=-\frac{b}{a}+\frac{c}{b}=\frac{ac-b^2}{ab}$ and $\alpha\beta=-\frac{b}{a}\times\frac{c}{b}=-\frac{c}{a}$

Also, $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=-\left(\frac{b^2-ac}{ab}\right)=\frac{ac-b^2}{ab}$

and, $\frac{\text{Constant term}}{\text{Coefficient of } x^2}=-\frac{bc}{ab}=-\frac{c}{a}$

Hence, sum of the zeroes $=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and, product of the zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

23. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.
(i) Here, $\alpha + \beta = 1/4$ and $\alpha \cdot \beta = 1/2$
Thus the required polynomial is
x^2 – (sum of zeroes)x + product of zeroes
$= x^2-\left(\frac{1}{4}\right)x+\frac{1}{2}$ *i.e.*, $4x^2-x+2$

(ii) Here, $\alpha+\beta=2$ and $\alpha\beta=\frac{1}{3}$
Thus the required polynomial is
x^2 – (sum of zeroes) x + product of zeroes
$= x^2-(2)x+\frac{1}{3}$ *i.e.*, $3x^2-6x+1$.

24. Since, α and β are the zeroes of the quadratic polynomial, $f(x) = kx^2 + 4x + 4$

$\therefore \quad \alpha+\beta=-\frac{4}{k}$ and $\alpha\beta=\frac{4}{k}$

Now, $\alpha^2 + \beta^2 = 24 \Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 24$

$\Rightarrow \quad \left(-\frac{4}{k}\right)^2-2\times\frac{4}{k}=24 \Rightarrow \frac{16}{k^2}-\frac{8}{k}=24$

$\Rightarrow \quad 16 - 8k = 24k^2 \Rightarrow 3k^2 + k - 2 = 0$
$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0 \Rightarrow 3k(k + 1) - 2(k + 1)$
$\Rightarrow \quad (k + 1)(3k - 2) = 0 \Rightarrow k + 1 = 0$ or $3k - 2 = 0$
$\Rightarrow \quad k = -1$ or $k = 2/3$
Hence, $k = -1$ or $k = 2/3$

OR

Let α and β be the zeroes of required quadratic polynomial.

Then, $\alpha+\beta=\frac{5+\sqrt{2}}{5-\sqrt{2}}+\frac{5-\sqrt{2}}{5+\sqrt{2}}=\frac{(5+\sqrt{2})^2+(5-\sqrt{2})^2}{(5-\sqrt{2})(5+\sqrt{2})}$

$=\frac{25+2+10\sqrt{2}+25+2-10\sqrt{2}}{5^2-(\sqrt{2})^2}=\frac{54}{23}$

Also, $\alpha\beta=\frac{5+\sqrt{2}}{5-\sqrt{2}}\times\frac{5-\sqrt{2}}{5+\sqrt{2}}=1$

So, required polynomial is given by

x^2 – (sum of zeroes)x + product of zeroes $= x^2-\frac{54}{23}x+1$

Since, zeroes of $x^2-\frac{54}{23}x+1$ is same as $23x^2 - 54x + 23$

$\therefore \quad$ Required polynomial is $23x^2 - 54x + 23$.

25. It is given that α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$.

$\therefore \quad \alpha+\beta=-\left(-\frac{5}{2}\right)=\frac{5}{2}$ and $\alpha\beta=\frac{7}{2}$

Let S and P denotes respectively the sum and product of zeroes of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5\times\frac{5}{2}=\frac{25}{2}$

and, $P = (2\alpha + 3\beta)(3\alpha + 2\beta) = 6(\alpha^2 + \beta^2) + 13\alpha\beta$
$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta = 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta$

$= 6(\alpha+\beta)^2+\alpha\beta = 6\left(\frac{5}{2}\right)^2+\frac{7}{2}=\frac{75}{2}+\frac{7}{2}=41$

Hence, the required polynomial $g(x)$ is given by

$g(x) = k(x^2 - Sx + P) = k\left(x^2-\frac{25}{2}x+41\right)$, where k is any non-zero real number.

26. Let $f(x) = kx^2 + 41x + 42$
Given, product of zeroes = 7
$\Rightarrow \quad 42/k = 7 \Rightarrow 42 = 7k$
$\Rightarrow \quad k = 6$
Putting $k = 6$ in polynomial
$p(x) = (k - 4)x^2 + (k + 1)x + 5$, we get
$p(x) = (6 - 4)x^2 + (6 + 1)x + 5$
$\Rightarrow \quad p(x) = 2x^2 + 7x + 5$
For zeroes of $p(x)$, put $2x^2 + 7x + 5 = 0$
$2x^2 + 5x + 2x + 5 = 0$
$\Rightarrow \quad x(2x + 5) + 1(2x + 5) = 0$
$\Rightarrow \quad (x + 1)(2x + 5) = 0$
$\Rightarrow \quad x = -1, x = -5/2$
$\therefore \quad$ zeroes are -1 and $-5/2$.

27. Let α and β be the zeroes of the polynomial

$p(x) = x^2 + 2kx + k$

$\therefore \quad \alpha + \beta = -2k/1 = -2k$ ...(i)

and $\alpha\beta = k/1 = k$ ...(ii)

Also, $\alpha = \beta$ (given)

From (i), $\alpha + \alpha = -2k$

$\Rightarrow \quad 2\alpha = -2k \Rightarrow \alpha = -k$ ...(iii)

From (ii), $\alpha \cdot \alpha = k$

$\Rightarrow \quad \alpha^2 = k \Rightarrow (-k)^2 = k$ [Using (iii)]

$\Rightarrow \quad k^2 - k = 0 \Rightarrow k(k-1) = 0$

$\Rightarrow \quad k = 0$ or $k = 1$

So, the quadratic polynomial $p(x)$ will have equal zeroes at $k = 0$ and $k = 1$.

$\therefore$ $p(x)$ can have equal zeroes for some odd integer $k > 0$.

28. Given, α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$.

$$\therefore \quad \alpha + \beta = -\frac{\text{Coefficient of } s}{\text{Coefficient of } s^2} = \frac{-(-6)}{3} = \frac{6}{3} = 2$$

$$\text{and } \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } s^2} = \frac{4}{3}$$

$$\text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$$

[$\because a^2 + b^2 = (a+b)^2 - 2ab$]

$$= \frac{(2)^2 - 2(4/3)}{4/3} + 2\left(\frac{2}{4/3}\right) + 3 \times \frac{4}{3}$$

[$\because \alpha + \beta = 2$ and $\alpha\beta = 4/3$]

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2 \times 2 \times \frac{3}{4} + 4 = \frac{12-8}{3} \times \frac{3}{4} + 3 + 4$$

$$= \frac{4}{3} \times \frac{3}{4} + 7 = 1 + 7 = 8$$

29. Let the zeroes of the given polynomial $ax^2 + bx + b$ be $m\alpha$ and $n\alpha$.

$$\therefore \quad \text{Sum of zeroes, } m\alpha + n\alpha = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$$

$$\Rightarrow \quad \alpha(m+n) = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{a(m+n)} \quad \text{...(i)}$$

$$\text{and product of zeroes, } m\alpha \times n\alpha = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{b}{a}$$

$$\Rightarrow \quad mn\alpha^2 = \frac{b}{a} \Rightarrow mn\left[\frac{b^2}{a^2(m+n)^2}\right] = \frac{b}{a} \quad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{mnb}{a(m+n)^2} = 1 \Rightarrow \frac{mn}{(m+n)^2} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{mn}{m^2 + 2mn + n^2} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{1}{\frac{m^2}{mn} + \frac{2mn}{mn} + \frac{n^2}{mn}} = \frac{a}{b}$$

[Dividing numerator and denominator of LHS by mn]

$$\Rightarrow \quad \frac{1}{\frac{m}{n} + 2 + \frac{n}{m}} = \frac{a}{b} \Rightarrow \frac{m}{n} + 2 + \frac{n}{m} = \frac{b}{a}$$

$$\Rightarrow \quad \left(\sqrt{\frac{m}{n}}\right)^2 + 2\sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}} + \left(\sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a} \quad \left[\because 1 = \sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}}\right]$$

$$\Rightarrow \quad \left(\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}\right)^2 = \frac{b}{a}$$

$$\therefore \quad \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \sqrt{\frac{b}{a}}$$

[Here, we take a positive square root, because values of $\sqrt{\frac{m}{n}}$ and $\sqrt{\frac{n}{m}}$ are always positive.]

30. Since, α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$. Then, $\alpha + \beta = -4$ and $\alpha\beta = 3$

Sum of the zeroes

$$= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta} = \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

$$\text{Product of the zeroes} = \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$$

$$1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta} = 2 + \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{2\alpha\beta + \alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$

$\therefore$ Required polynomial is given by

x^2 – (sum of the zeroes)x + product of zeroes

$$= x^2 - \frac{16x}{3} + \frac{16}{3} \text{ or } 3x^2 - 16x + 16.$$

31. Let α and β be the zeroes of $3x^2 - 8x - 2k - 1$.

$$\text{Now, } \alpha + \beta = \frac{8}{3} \Rightarrow 7\beta + \beta = \frac{8}{3} \quad [\because \alpha = 7\beta \text{ (Given)}]$$

$$\therefore 8\beta = \frac{8}{3} \Rightarrow \beta = \frac{1}{3}.$$

$$\text{Now, } \alpha = 7\beta = 7 \times \frac{1}{3} = \frac{7}{3}.$$

$$\text{Also, } \alpha\beta = -\frac{(2k+1)}{3} \Rightarrow \frac{7}{3} \times \frac{1}{3} = -\frac{(2k+1)}{3}$$

$\Rightarrow \quad \frac{7}{9} = -\frac{(2k+1)}{3} \quad \Rightarrow 7 = -3(2k+1) \Rightarrow 7 = -6k - 3$

$\Rightarrow \quad 6k = -10 \Rightarrow k = -\frac{10}{6} = -\frac{5}{3}$

32. Since, α and β are zeroes of $p(x) = 2x^2 - 11x + 12$.

$\therefore \quad \alpha + \beta = \frac{11}{2}$ and $\alpha\beta = 6$

Now, we have to form a polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

$\therefore$ Sum of zeroes $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$= 5\alpha + 5\beta = 5(\alpha + \beta) = 5\left(\frac{11}{2}\right) = \frac{55}{2}$

and product of zeroes $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6\alpha^2 + 6\beta^2 + 4\alpha\beta + 9\alpha\beta = 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$

$= 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$

$= 6\left(\frac{11}{2}\right)^2 + 6 = 6 \times \frac{121}{4} + 6 = \frac{363}{2} + 6 = \frac{375}{2}$

$\therefore$ The required polynomial is

$k\left(x^2 - \frac{55}{2}x + \frac{375}{2}\right)$, where $k\ (\neq 0)$ is real number.

Taking $k = 2$, one such required polynomial is

$\frac{2(2x^2 - 55x + 375)}{2} = 2x^2 - 55x + 375$

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