Polynomials

CHAPTER 2

NCERT FOCUS

SOLUTIONS

EXERCISE - 2.1

1. (i) The given graph is parallel to x-axis, it does not intersect the x-axis.

$\therefore$ It has no zero.

(ii) The given graph intersects the x-axis at one point only.

$\therefore$ It has one zero.

(iii) The given graph intersects the x-axis at three points.

$\therefore$ It has three zeroes.

(iv) The given graph intersects the x-axis at two points.

$\therefore$ It has two zeroes.

(v) The given graph intersects the x-axis at four points.

$\therefore$ It has four zeroes.

(vi) The given graph meets the x-axis at three points.

$\therefore$ It has three zeroes.

EXERCISE - 2.2

1. (i) We have, $p(x) = x^2 - 2x - 8$

$= x^2 + 2x - 4x - 8 = x(x + 2) - 4(x + 2) = (x - 4)(x + 2)$

For $p(x) = 0$, we must have $(x - 4)(x + 2) = 0$

Either $x - 4 = 0 \Rightarrow x = 4$ or $x + 2 = 0 \Rightarrow x = -2$

$\therefore$ The zeroes of $x^2 - 2x - 8$ are 4 and -2

Now, sum of the zeroes $= 4+(-2)=2=\dfrac{-(-2)}{1}$

$=\dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 4\times(-2)=\dfrac{-8}{1}=\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $x^2 - 2x - 8$ is verified.

(ii) We have, $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

For $p(s) = 0$, we have, $(2s - 1) = 0 \Rightarrow s = \dfrac{1}{2}$

$\therefore$ The zeroes of $4s^2 - 4s + 1$ are $\dfrac{1}{2}$ and $\dfrac{1}{2}$

Sum of the zeroes $=\dfrac{1}{2}+\dfrac{1}{2}=1=\dfrac{-(-4)}{4}=\dfrac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

and product of zeroes $=\left(\dfrac{1}{2}\right)\times\left(\dfrac{1}{2}\right)=\dfrac{1}{4}=\dfrac{\text{Constant term}}{\text{Coefficient of } s^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $4s^2 - 4s + 1$ is verified.

(iii) We have, $p(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$

$= (3x + 1)(2x - 3)$

For $p(x) = 0$, we have,

Either $(3x + 1) = 0 \Rightarrow x = -\dfrac{1}{3}$

or $(2x - 3) = 0 \Rightarrow x = \dfrac{3}{2}$

Thus, the zeroes of $6x^2 - 3 - 7x$ are $-\dfrac{1}{3}$ and $\dfrac{3}{2}$.

Now, sum of the zeroes $= -\dfrac{1}{3}+\dfrac{3}{2}=\dfrac{7}{6}=\dfrac{-(-7)}{6}$

$=\dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes $=\left(-\dfrac{1}{3}\right)\times\dfrac{3}{2}=\dfrac{-3}{6}$

$=\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $6x^2 - 3 - 7x$ is verified.

(iv) We have, $p(u) = 4u^2 + 8u = 4u(u + 2)$

For $p(u) = 0$, we have

Either $4u = 0 \Rightarrow u = 0$

or $u + 2 = 0 \Rightarrow u = -2$

$\therefore$ The zeroes of $4u^2 + 8u$ are 0 and -2.

Now, $4u^2 + 8u$ can be written as $4u^2 + 8u + 0$.

Sum of the zeroes $= 0+(-2)=-2=\dfrac{-(8)}{4}$

$=\dfrac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

and the product of zeroes $= 0\times(-2)=0=\dfrac{0}{4}$

$=\dfrac{\text{Constant term}}{\text{Coefficient of } u^2}$

Thus, the relationship between the zeroes and the coefficients in the polynomial $4u^2 + 8u$ is verified.

(v) We have, $p(t) = t^2 - 15 = (t^2) - (\sqrt{15})^2$

$= (t+\sqrt{15})(t-\sqrt{15})$ $[\because a^2 - b^2 = (a + b)(a - b)]$

For $p(t) = 0$, we have

Either $(t+\sqrt{15}) = 0 \Rightarrow t = -\sqrt{15}$

or $t-\sqrt{15}=0 \Rightarrow t=\sqrt{15}$

$\therefore$ The zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$

Now, we can write $t^2 - 15$ as $t^2 + 0t - 15$.

$\therefore$ Sum of the zeroes $= -\sqrt{15}+\sqrt{15}=0=\dfrac{-(0)}{1}$

$=\dfrac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes = $(-\sqrt{15})\times(\sqrt{15})=\frac{-(15)}{1}$

$=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$

Thus, the relationship between zeroes and the coefficients in the polynomial $t^2 - 15$ is verified.

(vi) We have, $p(x) = 3x^2 - x - 4$

$= 3x^2 + 3x - 4x - 4 = 3x(x + 1) - 4(x + 1) = (x + 1)(3x - 4)$

For $p(x) = 0$, we have

Either $(x + 1) = 0 \Rightarrow x = -1$

or $3x - 4 = 0 \Rightarrow x = 4/3$

$\therefore$ The zeroes of $3x^2 - x - 4$ are -1 and $4/3$

Now, sum of the zeroes = $(-1)+\frac{4}{3}=\frac{1}{3}=\frac{-(-1)}{3}$

$=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of zeroes = $(-1)\times\frac{4}{3}=\frac{(-4)}{3}$

$=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, the relationship between the zeroes and coefficients in the polynomial $3x^2 - x - 4$ is verified.

2. (i) Since, sum of the zeroes, $(\alpha+\beta)=\frac{1}{4}$

Product of the zeroes, $\alpha\beta = -1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-\left(\frac{1}{4}\right)x+(-1)=x^2-\frac{1}{4}x-1=\frac{1}{4}(4x^2-x-4)$

Since, $\frac{1}{4}(4x^2-x-4)$ and $(4x^2 - x - 4)$ have same zeroes, therefore $(4x^2 - x - 4)$ is the required quadratic polynomial.

(ii) Since, sum of the zeroes, $(\alpha+\beta)=\sqrt{2}$

Product of zeroes, $\alpha\beta=\frac{1}{3}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-(\sqrt{2})x+\left(\frac{1}{3}\right)=\frac{1}{3}(3x^2-3\sqrt{2}x+1)$

Since, $\frac{1}{3}(3x^2-3\sqrt{2}x+1)$ and $(3x^2-3\sqrt{2}x+1)$ have same zeroes, therefore

$(3x^2-3\sqrt{2}x+1)$ is the required quadratic polynomial.

(iii) Since, sum of zeroes, $(\alpha + \beta) = 0$

Product of zeroes, $\alpha\beta=\sqrt{5}$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-(0)x+\sqrt{5}=x^2+\sqrt{5}$

(iv) Since, sum of zeroes, $(\alpha + \beta) = 1$

Product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (1)x + 1 = x^2 - x + 1$

(v) Since, sum of the zeroes, $(\alpha+\beta)=-\frac{1}{4}$

Product of zeroes, $\alpha\beta = 1/4$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta$

$=x^2-\left(-\frac{1}{4}\right)x+\frac{1}{4}=x^2+\frac{x}{4}+\frac{1}{4}=\frac{1}{4}(4x^2+x+1)$

Since, $\frac{1}{4}(4x^2+x+1)$ and $(4x^2+x+1)$ have same zeroes, therefore, the required quadratic polynomial is $(4x^2+x+1)$.

(vi) Since, sum of zeroes, $(\alpha + \beta) = 4$ and product of zeroes, $\alpha\beta = 1$

$\therefore$ The required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 4x + 1.$

EXERCISE - 2.4

1. (i) $\because p(x) = 2x^3 + x^2 - 5x + 2$

$\therefore \quad p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$

$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+\frac{2}{1}=\frac{1+1-10+8}{4}=0$

$\Rightarrow \frac{1}{2}$ is a zero of $p(x)$.

Again, $p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$

$\Rightarrow$ 1 is a zero of $p(x)$.

Also, $p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2 = -16 + 16 = 0$

$\Rightarrow -2$ is a zero of $p(x)$.

Now, $p(x) = 2x^3 + x^2 - 5x + 2$

$\therefore$ Comparing it with $ax^3 + bx^2 + cx + d$, we have $a = 2$, $b = 1$, $c = -5$ and $d = 2$

Also, $\frac{1}{2}$, 1 and -2 are the zeroes of $p(x)$.

Let $\alpha=\frac{1}{2}, \beta=1$ and $\gamma=-2$

$\therefore \quad \alpha+\beta+\gamma=\frac{1}{2}+1+(-2)=-\frac{1}{2}=\frac{-b}{a}$

Again, $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1}{2}(1)+1(-2)+(-2)\left(\frac{1}{2}\right)$

$=\frac{1}{2}-2-1=\frac{-5}{2}=\frac{c}{a}$

and product of zeroes = $\alpha\beta\gamma$

$=\frac{1}{2}\times1\times(-2)=-\frac{2}{2}=\frac{-d}{a}$

Thus, the relationship between the coefficients and the zeroes of $p(x)$ is verified.

(ii) Here, $p(x) = x^3 - 4x^2 + 5x - 2$

$\therefore \quad p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 18 - 18 = 0$

$\Rightarrow$ 2 is a zero of $p(x)$.

Again, $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 6 - 6 = 0$

$\Rightarrow$ 1 is a zero of $p(x)$.

Now, comparing $p(x) = x^3 - 4x^2 + 5x - 2$ with $ax^3 + bx^2 + cx + d = 0$, we have

$a = 1, b = -4, c = 5$ and $d = -2$

Also, 2, 1 and 1 are the zeroes of $p(x)$.

Let $\alpha = 2, \beta = 1, \gamma = 1$

Now, sum of zeroes $= \alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -b/a$

Again, $\alpha\beta + \beta\gamma + \gamma\alpha = 2(1) + 1(1) + 1(2)$

$= 2 + 1 + 2 = 5 = \frac{c}{a}$

and product of zeroes $= \alpha\beta\gamma = (2)(1)(1) = 2 = -d/a$

Thus, the relationship between the zeroes and the coefficients of $p(x)$ is verified.

2. Let the required cubic polynomial be $ax^3 + bx^2 + cx + d = 0$ and its zeroes be α, β and γ.

$$\therefore \quad \alpha+\beta+\gamma = 2 = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = -7 = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$$

$$\alpha\beta\gamma = -14 = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a}$$

If $a = 1$, then $\frac{-b}{a} = \frac{-b}{1} = 2 \Rightarrow b = -2$,

$\frac{c}{a} = \frac{c}{1} = -7 \Rightarrow c = -7$

and $\frac{-d}{a} = -\frac{d}{1} = -14 \Rightarrow d = 14$

$\therefore$ The required cubic polynomial is

$1x^3 + (-2)x^2 + (-7)x + 14 = 0$

$= x^3 - 2x^2 - 7x + 14 = 0$

3. We have, $p(x) = x^3 - 3x^2 + x + 1$.

Comparing it with $Ax^3 + Bx^2 + Cx + D$,

We have $A = 1, B = -3, C = 1$ and $D = 1$

$\because$ It is given that $(a - b)$, a and $(a + b)$ are the zeroes of the polynomial.

$\therefore$ Let $\alpha = (a - b)$, $\beta = a$ and $\gamma = (a + b)$

$\therefore \quad \alpha+\beta+\gamma = -\frac{B}{A} = -\frac{(-3)}{1} = 3$

$\Rightarrow (a - b) + a + (a + b) = 3 \Rightarrow 3a = 3 \Rightarrow a = 1$

Again, $\alpha\beta\gamma = \frac{-D}{A} = -1$

$\Rightarrow (a - b) \times a \times (a + b) = -1$

$\Rightarrow (1 - b) \times 1 \times (1 + b) = -1 \Rightarrow 1 - b^2 = -1$

$\Rightarrow b^2 = 1 + 1 = 2 \Rightarrow b = \pm\sqrt{2}$

Thus, $a = 1$ and $b = \pm\sqrt{2}$