Polynomials

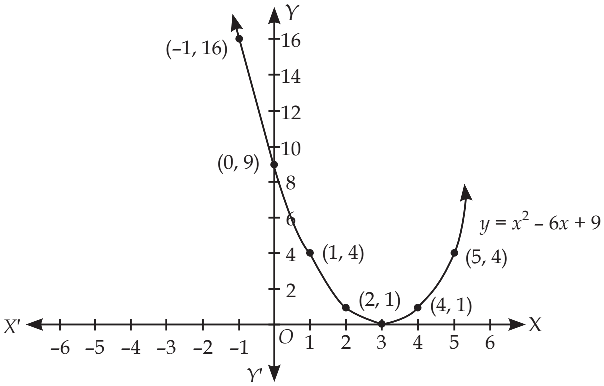
CHAPTER 2

TRY YOURSELF SOLUTIONS

1. Let $y = f(x) = x^2 - 6x + 9 = (x - 3)^2$

Now, we find values of y for different values of x.

x	–1	0	1	2	3	4	5
$y = f(x)$	16	9	4	1	0	1	4



The given curve represents a parabola opening upwards. Also, it intersect x axis at one point only. Hence, $y = f(x)$ has coincident zeroes.

$\therefore$ Number of zeroes will be 1.

2. (i) Since, curve represents a parabola opening upwards.

$\therefore$ $a > 0$.

(ii) Since, curve represents a parabola opening downwards.

$\therefore$ $a < 0$.

3. (i) Let $p(y) = y^3 - 2y^2 - \sqrt{3}y + 1/2$

Yes, it is a polynomial is $p(y)$ is of the form $a_0 + a_1y + a_2y^2 + a_3y^3$, where, $a_0 = \frac{1}{2}$, $a_1 = -\sqrt{3}$, $a_2 = -2$, $a_3 = 1$ (all being real numbers).

Clearly, $p(y)$ is of degree 3.

(ii) Let $p(x) = \sqrt{7}x^4 - \sqrt{x} + 2x - 1/3$

$p(x)$ is not of the form $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

Hence, $p(x)$ is not a polynomial in x.

4. We know, $x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

$\therefore$ Zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$.

Now, sum of zeroes $= (\sqrt{3}) + (-\sqrt{3}) = 0$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

and product of zeroes $= (\sqrt{3})(-\sqrt{3}) = -3 = \frac{(-3)}{1}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

5. Let $f(x) = x^3 - 27x + 54$

Now, $f(-6) = (-6)^3 - 27(-6) + 54$

$= -216 + 162 + 54 = -216 + 216 = 0$

$f(3) = (3)^3 - 27(3) + 54 = 27 - 81 + 54 = 81 - 81 = 0$

Hence, –6, 3, 3 are the zeroes of $f(x)$, where 3 is a repeated zero of $f(x)$.

Let $\alpha = -6$, $\beta = 3$, $\gamma = 3$ be the roots of $f(x)$.

Now, $\alpha + \beta + \gamma = -6 + 3 + 3 = 0 = \dfrac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-6)(3) + (3)(3) + 3(-6)$

$= -18 + 9 - 18 = -27 = \dfrac{(-27)}{1} = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta\gamma = (-6)(3)(3) = -54 = \dfrac{-(54)}{1} = \dfrac{-(\text{Constant term})}{\text{Coefficient of } x^3}$

6. Let $f(x) = x^2 + 3mx + 8m$

$\because$ 2 is a root of $f(x)$ $\therefore$ $f(2) = 0$

Now, $f(2) = (2)^2 + 3m(2) + 8m = 0$

$\Rightarrow$ $4 + 14m = 0 \Rightarrow 14m = -4 \Rightarrow m = \dfrac{-2}{7}$

Let the other zero of $f(x)$ be α

Sum of the roots $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow$ $2 + \alpha = \dfrac{-3m}{1} = -3 \times \dfrac{-2}{7} = \dfrac{6}{7} \Rightarrow \alpha = \dfrac{6}{7} - 2 = \dfrac{6-14}{7} = -\dfrac{8}{7}$

7. Since, α and β are the zeroes of $f(x) = x^2 - x - 4$

$\therefore$ $\alpha\beta = \dfrac{(-4)}{1} = -4$ ...(i) and $\alpha + \beta = -\dfrac{(-1)}{1} = 1$ ...(ii)

Now, $\dfrac{1}{\alpha} + \dfrac{1}{\beta} - \alpha\beta = \dfrac{(\beta+\alpha) - (\alpha\beta)^2}{\alpha\beta} = \dfrac{(\alpha+\beta) - (\alpha\beta)^2}{\alpha\beta}$

$= \dfrac{1-(-4)^2}{-4}$ [Using (i) and (ii)]

$= \dfrac{1-16}{-4} = \dfrac{-15}{-4} = \dfrac{15}{4}$

8. Since, α and β are the zeroes of $f(t) = t^2 - 4t + 3$

$\therefore$ $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } t^2} = \dfrac{3}{1} = 3$ ...(i)

and $\alpha + \beta = \dfrac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2} = \dfrac{-(-4)}{1} = 4$ ...(ii)

Now, $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = (\alpha\beta)^3(\alpha + \beta)$

$= (3)^3(4)$ [Using (i) and (ii)]

$= 27 \times 4 = 108$

9. Since, α and β are the zeroes of $p(z) = 5z^2 - 9z + 4$

$\therefore \quad \alpha + \beta = \dfrac{-(\text{Coefficient of } z)}{\text{Coefficient of } z^2} = -\dfrac{(-9)}{5} = \dfrac{9}{5}$ ...(i)

and $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } z^2} = \dfrac{4}{5}$ ...(ii)

Now, $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \dfrac{4}{5}\left(\dfrac{9}{5}\right) = \dfrac{36}{25}$

10. The quadratic polynomial $f(x)$ whose sum of zeroes, (S) and product of zeroes, (P) is given by $k(x^2 - Sx + P)$

Here, $S = \sqrt{2}$ and $P = \dfrac{1}{3}$

$\therefore \quad f(x) = k\left(x^2 - \sqrt{2}x + \dfrac{1}{3}\right)$, k being any non zero real number

$= k\left(\dfrac{3x^2 - 3\sqrt{2}x + 1}{3}\right) = 3x^2 - 3\sqrt{2}x + 1$ [Taking $k = 3$]

Hence, required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$

11. Here, sum of zeroes $(S) = \sqrt{2}$

Sum of the product of zeroes taken two at a time $(S') = \sqrt{3}$.

and product of zeroes, $P = \dfrac{1}{\sqrt{6}}$

Now, required cubic polynomial is given by

$k(x^3 - Sx^2 + S'x - P)$, k being any non zero real number.

$k\left(x^3 - \sqrt{2}x^2 + \sqrt{3}x - \dfrac{1}{\sqrt{6}}\right)$

$k\left(\dfrac{\sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1}{\sqrt{6}}\right) = \sqrt{6}x^3 - 2\sqrt{3}x^2 + 3\sqrt{2}x - 1$

[Taking $k = \sqrt{6}$]

12. Given, α and β are zeroes of the polynomial, $f(x) = x^2 - 2x + 3$

$\therefore \quad \alpha + \beta = \dfrac{-(-2)}{1} = 2$ and $\alpha\beta = 3$

Let S and P denotes the sum and product of polynomial, whose roots are $\dfrac{\alpha - 1}{\alpha + 1}$ and $\dfrac{\beta - 1}{\beta + 1}$

Now, $S = \dfrac{\alpha - 1}{\alpha + 1} + \dfrac{\beta - 1}{\beta + 1} = \dfrac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)}$

$= \dfrac{(\alpha\beta + \alpha - \beta - 1) + (\alpha\beta - \alpha + \beta - 1)}{\alpha\beta + (\alpha + \beta) + 1}$

$= \dfrac{2\alpha\beta - 2}{\alpha\beta + \alpha + \beta + 1} = \dfrac{2(\alpha\beta - 1)}{\alpha\beta + (\alpha + \beta) + 1} = \dfrac{2(3 - 1)}{3 + 2 + 1} = \dfrac{4}{6} = \dfrac{2}{3}$

$P = \dfrac{(\alpha - 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)} = \dfrac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1} = \dfrac{3 - (2) + 1}{3 + 2 + 1} = \dfrac{2}{6} = \dfrac{1}{3}$

$\therefore$ Required polynomial is $k(x^2 - Sx + P)$, k being any real number.

$= k\left(x^2 - \dfrac{2}{3}x + \dfrac{1}{3}\right) = \dfrac{k(3x^2 - 2x + 1)}{3}$

$= 3x^2 - 2x + 1$ [Taking $k = 3$]

Thus, one of the polynomial is given by $3x^2 - 2x + 1$.