Pair of Linear Equations in Two Variables

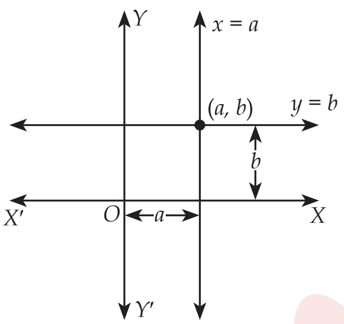
CHAPTER 3

EXAM DRILL

SOLUTIONS

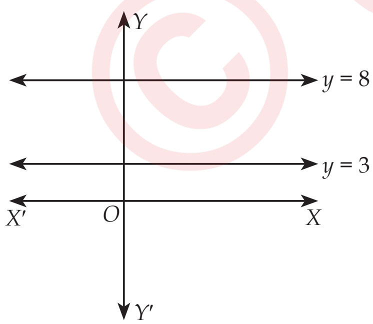
1. (a) : Let x be the number of boys and y be the number of girls in the class.
According to the question,
$12x + 6y = 900$ ...(i)
and $10x + 5y = 900$ ...(ii)
$\therefore$ (i) and (ii) are the required algebraic representation of given situation.

2. (d) : $x = a$ and $y = b$ represents lines parallel to y axis and x axis respectively.



$\therefore$ Graphically, $x = a$ and $y = b$ represents lines intersecting each other at (a, b).

3. (d) : Clearly, $y = 3$ and $y = 8$ represents two parallel lines.



$\therefore$ Given pair of equations has no solution.

4. (a) : For coincident lines,

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

5. (d) : Since, $x = 2$ and $y = 3$ is a solution of $2x - 3y + a = 0$ and $2x + 3y - b + 2 = 0$
$\therefore$ $2(2) - 3(3) + a = 0 \Rightarrow a = 5$ ...(i)
and $2(2) + 3(3) - b + 2 = 0 \Rightarrow b = 15$ ...(ii)
From (i) and (ii), we get $3a = b$

6. (a) : Lines are parallel when $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
$\therefore$ Another linear equation in two variables can be $6x + 8y + k = 0$, where k is constant not equal to –16.
$\therefore$ Another linear equation can be $6x + 8y - 12 = 0$

7. Given equations are $x + 2y - 8 = 0$ and $2x + 4y - 16 = 0$
Here, $a_1 = 1, b_1 = 2, c_1 = -8$ and $a_2 = 2, b_2 = 4, c_2 = -16$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

Since, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$
$\therefore$ System of equations has infinitely many solutions.

8. Condition for lines to be parallel is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{2}{k} = \frac{-3}{-9} \neq \frac{-9}{-18}$$

Now, $\dfrac{2}{k} = \dfrac{-3}{-9} = \dfrac{1}{3} \Rightarrow k = 6$

9. Condition for lines to be inconsistent is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{3}{k} \neq \frac{-4}{12}$$

Now, $\dfrac{1}{3} = \dfrac{3}{k} \Rightarrow k = 9.$

10. For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} \neq -\frac{(k-2)}{-k}$$

Now, $\dfrac{k}{12} = \dfrac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$
$k = \pm 6$ also satisfies the last two terms.

11. For coincident lines, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{5}{15} = \frac{7}{21} = \frac{3}{k}$$

Now, $\dfrac{7}{21} = \dfrac{3}{k} \Rightarrow k = 9.$

12. For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-k} \Rightarrow k \neq 10$$

So, the system of equations has no solution for every real value of k except when $k = 10$.

13. For infinitely many solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

$\Rightarrow \frac{2}{k+2} = \frac{1}{2} \Rightarrow 4 = k + 2 \Rightarrow k = 2$

$k = 2$ also satisfies the last two terms.

14. For unique solution, we have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{p}{2} \Rightarrow p \neq 4$

$\therefore$ p can have any real value except $p = 4$.

15. Given, $x + 2y = 9$ ...(i)

$x - y = 6$ ..(ii)

Multiplying (ii) by 2, we have

$2x - 2y = 12$ ...(iii)

Adding (i) and (iii), we have $3x = 21 \Rightarrow x = 7$

Put $x = 7$ in (ii), we get $y = 1$.

16. For infinitely many solutions, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k-1}{k+1} = \frac{-1}{1-k} = \frac{-5}{-3k-1}$

$\Rightarrow \frac{-1}{1-k} = \frac{5}{3k+1} \Rightarrow -3k - 1 = 5 - 5k \Rightarrow 2k = 6 \Rightarrow k = 3$

17. (i) (a) : For Anu:

Fixed charge + cost of food for 25 days = ₹ 4500

i.e., $x + 25y = 4500$

For Bindu:

Fixed charges + cost of food for 30 days = ₹ 5200

i.e., $x + 30y = 5200$

(ii) (b) : From above, we have $a_1 = 1, b_1 = 25,$ $c_1 = -4500$ and $a_2 = 1, b_2 = 30, c_2 = -5200$

$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{25}{30} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-4500}{-5200} = \frac{45}{52}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, system of linear equations has unique solution.

(iii) (c) : We have, $x + 25y = 4500$ ...(i)

and $x + 30y = 5200$ ...(ii)

Subtracting (i) from (ii), we get

$5y = 700 \Rightarrow y = 140$

$\therefore$ Cost of food per day is ₹ 140

(iv) (c) : We have, $x + 25y = 4500$

$\Rightarrow x = 4500 - 25 \times 140$

$\Rightarrow x = 4500 - 3500 = 1000$

$\therefore$ Fixed charges per month for the hostel is ₹ 1000

(v) (d) : We have, $x = 1000$, $y = 140$ and Bindu takes food for 20 days.

$\therefore$ Amount that Bindu has to pay

= ₹ (1000 + 20 × 140) = ₹ 3800

18. (i) 1st situation can be represented as

$x + 7y = 650$ ...(i)

and 2nd situation can be represented as

$x + 11y = 970$ ...(ii)

(ii) Subtracting equations (i) from (ii), we get

$4y = 320 \Rightarrow y = 80$

$\therefore$ Proportional expense for each person is ₹ 80.

(iii) Putting $y = 80$ in equation (i), we get

$x + 7 \times 80 = 650 \Rightarrow x = 650 - 560 = 90$

$\therefore$ Fixed expense for the party is ₹ 90

(iv) If there will be 15 guests, then amount that Mr Jindal has to pay = ₹ (90 + 15 × 80) = ₹ 1290

(v) We have $a_1 = 1, b_1 = 7, c_1 = -650$ and $a_2 = 1, b_2 = 11, c_2 = -970$

$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{7}{11}, \frac{c_1}{c_2} = \frac{-650}{-970} = \frac{65}{97}$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, system of linear equations has unique solution.

19. (i) (b) : Algebraic representation of situation of day-I is $2x + y = 1600$.

(ii) (a) : Algebraic representation of situation of day-II is $4x + 2y = 3000 \Rightarrow 2x + y = 1500$.

(iii) (c) : At x-axis, $y = 0$

$\therefore$ At $y = 0$, $2x + y = 1600$ becomes $2x = 1600$

$\Rightarrow x = 800$

$\therefore$ Linear equation represented by day-I intersect the x-axis at (800, 0).

(iv) (d) : At y-axis, $x = 0$

$\therefore$ $2x + y = 1500 \Rightarrow y = 1500$

$\therefore$ Linear equation represented by day-II intersect the y-axis at (0, 1500).

(v) (b) : We have, $2x + y = 1600$

and $2x + y = 1500$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ *i.e.*, $\frac{1}{1} = \frac{1}{1} \neq \frac{16}{15}$

$\therefore$ System of equations have no solution.

$\therefore$ Lines are parallel.

20. (i)(a) : At x-axis, $y = 0$

$\therefore$ $2x + 4y = 8 \Rightarrow x = 4$

At y-axis, $x = 0$

$\therefore$ $2x + 4y = 8 \Rightarrow y = 2$

$\therefore$ Required coordinates are (4, 0), (0, 2).

(ii) (c) : At x-axis, $y = 0$

$\therefore$ $3x + 6y = 18 \Rightarrow 3x = 18 \Rightarrow x = 6$

At y-axis, $x = 0$

$\therefore$ $3x + 6y = 18 \Rightarrow 6y = 18 \Rightarrow y = 3$

$\therefore$ Required coordinates are (6, 0), (0, 3).

(iii) (d) : Since, lines are parallel.

So, point of intersection of these lines does not exist.

(iv) (a)

(v) **(a) :** Since the lines are parallel.

$\therefore$ These equations have no solution *i.e.*, the given system of linear equations is inconsistent.

21. Let the original length and breadth of the lawn are x m and y m respectively.

Then, perimeter of lawn = $2(x + y)$

According to the question,

$2(x + y) = 54 \Rightarrow x + y - 27 = 0$ ...(i)

Also, $2\left[\frac{3}{5}x + \frac{4}{5}y\right] = 36$

$\Rightarrow 3x + 4y = 18 \times 5 = 90 \Rightarrow 3x + 4y - 90 = 0$ ...(ii)

$\therefore$ (i) and (ii) is the required algebraic representation of the given situation.

22. Given pair of linear equations are:

$2x - 3y + 15 = 0$ ...(i)

$3x - 5 = 0$ ...(ii)

From (ii), we have, $x = 5/3$

Substituting $x = \frac{5}{3}$ in (i), we get

$$2\left(\frac{5}{3}\right) - 3y + 15 = 0 \Rightarrow \frac{10}{3} - 3y + 15 = 0$$

$$\Rightarrow -3y = -15 - \frac{10}{3} \Rightarrow -3y = \frac{-45-10}{3} \Rightarrow y = \frac{55}{9}$$

$$\therefore \quad x = \frac{5}{3},\ y = \frac{55}{9}$$

23. Let the numbers be x and y.

According to the question,

$\frac{x}{y} = \frac{3}{4} \Rightarrow x = \frac{3y}{4}$ ...(i)

and $\frac{x+6}{y+6} = \frac{7}{8} \Rightarrow 8x + 48 = 7y + 42$

$\Rightarrow 8x - 7y = -6$ ...(ii)

Using (i) in (ii), we get

$$8\left(\frac{3y}{4}\right) - 7y = -6 \Rightarrow 6y - 7y = -6 \Rightarrow y = 6$$

Putting $y = 6$ in (i), we have $x = \frac{3y}{4} = \frac{3}{4} \times 6 = 4.5$

Hence, the numbers are 4.5 and 6.

24. Given pair of linear equations are

$3x - y = 5$ ...(i) and $5x - y = 11$ ...(ii)

Subtracting (ii) from (i), we get

$3x - y - (5x - y) = 5 - 11$

$\Rightarrow -2x = -6 \Rightarrow x = 3$

Substituting the value of x in (i), we get

$3 \times 3 - y = 5 \Rightarrow -y = 5 - 9 \Rightarrow y = 4$

Hence, $x = 3$ and $y = 4$ is the required solution.

25. Let the tens digit of a number be x and ones digit be y.

$\therefore$ The number be $10x + y$.

According to the question,

$$\frac{10x + y}{x + y} = 7 \Rightarrow 10x + y = 7x + 7y$$

$\Rightarrow 3x - 6y = 0 \Rightarrow x - 2y = 0$ ...(i)

and $10x + y - 27 = 10y + x$

$\Rightarrow 9x - 9y = 27 \Rightarrow x - y = 3$ ...(ii)

Subtracting (i) from (ii), we get $y = 3$

From (ii), $x - 3 = 3 \Rightarrow x = 6$

$\therefore$ Required number is 63.

26. We know that for a pair of linear equations

$a_1x + b_1y + c_1 = 0\ ;\ a_2x + b_2y + c_2 = 0$

has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

For no solution, we have

$$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5} \Rightarrow \frac{3k+1}{k^2+1} = \frac{3}{k-2} \text{ and } \frac{3}{k-2} \neq \frac{2}{5}$$

$\Rightarrow (k-2)(3k+1) = 3(k^2+1) \Rightarrow 3k^2 + k - 6k - 2 = 3k^2 + 3$

$\Rightarrow -5k = 5 \Rightarrow k = -1$

Also, $k = -1$ satisfy last two terms.

27. Let the cost of a chair be ₹ x and the cost of a table be ₹ y.

Then, according to the question,

$2x + 3y = 5650$ ...(i)

$3x + 2y = 7100$ ...(ii)

Multiply (i) by 2 and (ii) by 3, we get

$4x + 6y = 11300$ ...(iii)

$9x + 6y = 21300$ ...(iv)

Subtracting (iii) from (iv), we get

$5x = 10000 \Rightarrow x = 2000$

Putting the value of x in (i), we get

$2 \times 2000 + 3y = 5650$

$\Rightarrow 3y = 5650 - 4000 \Rightarrow 3y = 1650$

$\Rightarrow y = \frac{1650}{3} = 550$

Hence, the cost of a chair is ₹ 2000 and cost of a table is ₹ 550.

28. Let the number of rows be x and number of students in each row be y. Then, total number of students in the class = xy.

According to question $(y + 3)(x - 1) = xy$

$\therefore xy + 3x - y - 3 = xy$ or $3x - y = 3$ ...(i)

Also, $(y - 3)(x + 2) = xy$

$\therefore xy - 3x + 2y - 6 = xy$ or $-3x + 2y = 6$ ...(ii)

On adding (i) and (ii), we get $y = 9$

Put $y = 9$ in (i), we get

$3x - 9 = 3 \Rightarrow 3x = 12 \Rightarrow x = 4$

$\therefore$ Number of students in class = $xy = 4 \times 9 = 36$

29. Since $BC \parallel DE$ and $BE \parallel CD$ with $BC \perp CD$ and $BCDE$ is a rectangle

$\therefore$ Opposite sides are equal

$\Rightarrow BE = CD \Rightarrow x + y = 5$ ...(i)

Since perimeter of $ABCDE$ is 21. [Given]

$\Rightarrow AB + BC + CD + DE + EA = 21$

$\Rightarrow 3 + (x - y) + (x + y) + (x - y) + 3 = 21$

$\Rightarrow 6 + 3x - y = 21 \Rightarrow 3x - y = 15$ ...(ii)

On adding (i) and (ii), we get $4x = 20 \Rightarrow x = 5$
Putting $x = 5$ in (i), we get $y = 0$.

30. Let the constant expenditure be ₹ x and consumption of wheat be y quintals.
Then total expenditure = $x + y$ × Rate per quintal
According to the question,
$x + 250y = 1000$ ...(i) and $x + 240y = 980$ ...(ii)
Subtracting (ii) from (i), we get $10y = 20 \Rightarrow y = 2$
Now, substituting $y = 2$ in (i), we get
$x + 250(2) = 1000$
$\Rightarrow x = 1000 - 500 = 500$
∴ Total expenses when the price of wheat is ₹ 350 per quintal = $x + 350y = 500 + 350 \times 2 = 500 + 700 =$ ₹ 1200

OR

Let fare from A to B be ₹ x and fare from A to C be ₹ y.
Then, according to the question,
$2x + 3y = 795$ ...(i)
$3x + 5y = 1300$ ...(ii)
Multiplying (i) by 3 and (ii) by 2, we get
$6x + 9y = 2385$ ...(iii)
$6x + 10y = 2600$ ...(iv)
Subtracting (iii) from (iv), we get $y = 215$
Putting $y = 215$ in (i), we get
$2x + 3(215) = 795$
$\Rightarrow 2x = 150 \Rightarrow x = 75$
Hence, fare from A to B is ₹ 75 and fare from A to C is ₹ 215.

31. Let the digits at ten's and unit place be x and y respectively. Then, required number = $10x + y$.
Also, number obtained by reversing the digit = $10y + x$
According to the question, we have
$x = 2y + 2$ ...(i)
and $10y + x = 3(x + y) + 5$
$\Rightarrow 10y + x = 3x + 3y + 5$ or $-2x + 7y = 5$ ...(ii)
Using (i) in (ii), we get
$-2(2y + 2) + 7y = 5$
$\Rightarrow -4y - 4 + 7y = 5 \Rightarrow 3y = 9 \Rightarrow y = 3$
Putting $y = 3$ in (i), we have $x = 2(3) + 2 = 8$
∴ Required number = 10(8) + 3 = 83

32. Let man's starting salary and his fixed annual increment be ₹ x and ₹ y respectively.
According to the question, $x + 4y = 15000$ ...(i)
and $x + 10y = 18000$ ...(ii)
So, (i) and (ii) represents a pair of linear equations in two variables of given situation.
Subtracting (i) from (ii), we get $6y = 3000 \Rightarrow y = 500$
Putting $y = 500$ in (i), we get
$x + 4(500) = 15000 \Rightarrow x + 2000 = 15000 \Rightarrow x = 13000$
Hence, man's starting salary is ₹ 13000 and his fixed annual increment is ₹ 500.

33. Given equations are
$a^2x - b^2y = a^2 - 2b^2$ ...(i)
and $b^2x + a^2y = b^2 + 2a^2$ ...(ii)
From (i) we get
$a^2x = a^2 - 2b^2 + b^2y \Rightarrow x = \dfrac{a^2 - 2b^2 + b^2y}{a^2}$ ...(iii)
Substituting the value of x in (ii), we get
$$b^2\left(\frac{a^2 - 2b^2 + b^2y}{a^2}\right) + a^2y = b^2 + 2a^2$$
$$\Rightarrow \frac{b^2a^2 - 2b^4 + b^4y}{a^2} + a^2y = b^2 + 2a^2$$
$$\Rightarrow \frac{b^2a^2 - 2b^4 + b^4y + a^4y}{a^2} = b^2 + 2a^2$$
$$\Rightarrow b^2a^2 - 2b^4 + (b^4 + a^4)y = a^2b^2 + 2a^4$$
$$\Rightarrow (b^4 + a^4)y = 2a^4 + 2b^4 \Rightarrow y = \frac{2(a^4 + b^4)}{b^4 + a^4} \Rightarrow y = 2$$
When $y = 2$, (iii) becomes
$$x = \frac{a^2 - 2b^2 + b^2 \times 2}{a^2} \Rightarrow x = \frac{a^2}{a^2} \Rightarrow x = 1$$
∴ $x = 1, y = 2$

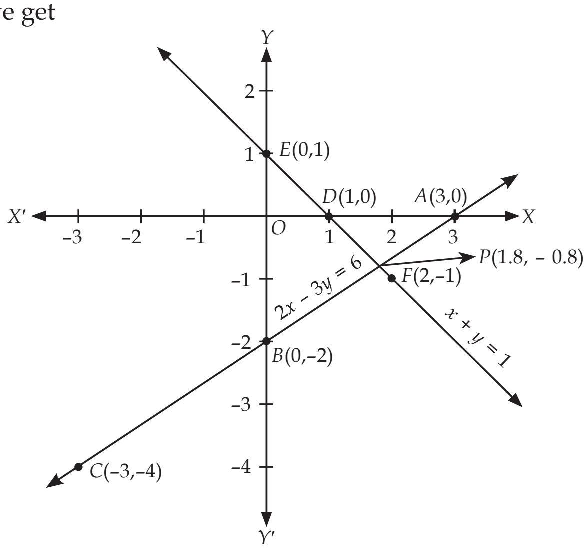
34. The given system of equations is $2x - 3y - 6 = 0$ ...(i)
and $x + y = 1$ ...(ii)
Table of solutions for (i) is :

x	3	0	–3
y	0	–2	–4

Also, table of solutions for (ii) is :

x	1	0	2
y	0	1	–1

Plotting the points on the graph paper and joining them, we get



Clearly from the graph, we see that equations given by (i) and (ii) are intersect each other at point $P(1.8, -0.8)$ and hence, they have a unique solution given by $x = 1.8$, $y = -0.8$.

OR

The given system of equations is $3x - 2y - 1 = 0$ ...(i)

and $2x - 3y + 6 = 0$ ...(ii)

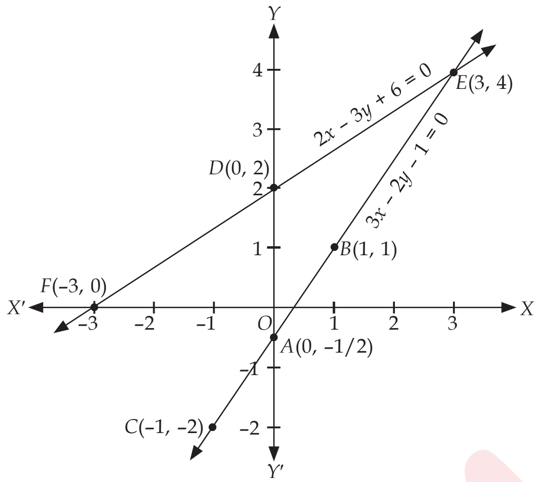
Table of solutions for (i) is :

x	0	1	–1
y	–1/2	1	–2

Table of solutions for (ii) is :

x	0	3	–3
y	2	4	0

Plotting the points on graph paper and joining them, we get



From the graph, we see that the two lines represented by (i) and (ii) intersect each other at point $E(3, 4)$.
Hence, $x = 3$ and $y = 4$ is the required solution.

35. Let Ravi invested ₹x at 8% simple interest and ₹y at 9% simple interest.

According to the first condition, $\frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} = 163$

$\Rightarrow \quad 8x + 9y = 16300$ ...(i)

According to the second condition,

$\frac{x \times 9 \times 1}{100} + \frac{y \times 8 \times 1}{100} = 160$

$\Rightarrow \quad 9x + 8y = 16000$ ...(ii)

Multiplying (i) by 9 and (ii) by 8, we get

$72x + 81y = 146700$ ...(iii)

and $72x + 64y = 128000$ ...(iv)

Subtracting (iv) from (iii), we get

$17y = 18700 \Rightarrow y = 1100$

Putting $y = 1100$ in (i), we get $8x + 9(1100) = 16300$

$\Rightarrow \quad 8x = 16300 - 9900 = 6400 \Rightarrow x = 800$

Hence, he invested ₹800 at 8% simple interest and ₹1100 at 9% simple interest.

36. Let x units and y units be the length and breadth of rectangle respectively.

Then, its area = xy sq. units

According to the question,

$(x - 5)(y + 2) = xy - 80$

$\Rightarrow \quad xy + 2x - 5y - 10 = xy - 80 \Rightarrow 2x - 5y = -70$ ...(i)

and $(x + 10)(y - 5) = xy + 50$

$\Rightarrow \quad xy - 5x + 10y - 50 = xy + 50$

$\Rightarrow \quad -5x + 10y = 100$ or $x - 2y = -20$ ...(ii)

From (i), $x = \frac{5y - 70}{2}$ ...(iii)

Substituting the value of x from (iii) in (ii), we get

$\left(\frac{5y - 70}{2}\right) - 2y = -20 \Rightarrow 5y - 70 - 4y = -40 \Rightarrow y = 30$

Substituting the value of $y = 30$ in (iii), we get

$x = \frac{5(30) - 70}{2} = \frac{80}{2} = 40$

Hence, length = 40 units, breadth = 30 units.

$\therefore$ Perimeter of the rectangle = $2(l + b)$

$= 2(40 + 30) = 2(70) = 140$ units.

OR

Comparing the given system of equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 2, b_1 = -3, c_1 = -7$ and

$a_2 = (a + b), b_2 = -(a + b - 3), c_2 = -(4a + b)$

For infinite solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$$

Taking $\frac{2}{a+b} = \frac{3}{(a+b-3)}$

$\Rightarrow \quad 2a + 2b - 6 = 3a + 3b \Rightarrow a + b + 6 = 0$ ...(i)

and $\frac{3}{a+b-3} = \frac{7}{4a+b}$

$\Rightarrow \quad 12a + 3b = 7a + 7b - 21 \Rightarrow 5a - 4b + 21 = 0$ ...(ii)

From (i), $a = -(6 + b)$ ...(iii)

Using (iii) in (ii), we get

$5[-(6 + b)] - 4b + 21 = 0$

$\Rightarrow \quad -30 - 5b - 4b + 21 = 0$

$\Rightarrow \quad -9b = 9 \Rightarrow b = -1$

From (iii), $a = -(6 - 1) = -5$

$\therefore \quad a = -5$ and $b = -1$