CHAPTER 4

Quadratic Equations

EXAM DRILL

SOLUTIONS

1. **(b) :** Given α and β be roots of the equation $kx^2 + bx + c = 0$.

We have, $\alpha = \frac{-b+\sqrt{b^2-12c}}{6}$ and $\beta = \frac{-b-\sqrt{b^2-12c}}{6}$

$\therefore$ $2k = 6 \Rightarrow k = 3$

2. **(b) :** We have, $9x^2 + 3px + 4 = 0$

Here, $a = 9$, $b = 3p$ and $c = 4$.

$\therefore$ $D = b^2 - 4ac = (3p)^2 - 4(9)(4) = 9p^2 - 144$

The equation has real and equal roots, so $D = 0$

$\Rightarrow$ $9p^2 - 144 = 0 \Rightarrow p^2 = \frac{144}{9} \Rightarrow p^2 = 16$

$\Rightarrow$ $p = \pm 4$

3. **(d) :** We have, $m^2x^2 + 2mcx = (a^2 - c^2) - x^2$

$\Rightarrow$ $(m^2 + 1)x^2 + 2mcx - a^2 + c^2 = 0$

Here, $A = m^2 + 1$, $B = 2mc$ and $C = -a^2 + c^2$.

$\therefore$ $D = B^2 - 4AC = 4m^2c^2 - 4(m^2 + 1)(c^2 - a^2)$

$= 4m^2c^2 - 4m^2c^2 + 4a^2m^2 - 4c^2 + 4a^2 = 4(a^2 - c^2 + a^2m^2)$

Since, the equation has equal roots, so $D = 0$

$\Rightarrow$ $4(a^2 - c^2 + a^2m^2) = 0 \Rightarrow c^2 = a^2(1 + m^2)$

4. **(b) :** Let $x = \sqrt{20+\sqrt{20+\sqrt{20+\ldots\infty}}} \Rightarrow x = \sqrt{20+x}$

Squaring on both sides, we get

$x^2 = 20 + x \Rightarrow x^2 - x - 20 = 0$

$\Rightarrow$ $(x - 5)(x + 4) = 0 \Rightarrow x = 5$ or $x = -4$

But x is a positive quantity.

$\therefore$ $x = 5$

5. **(b) :** Given, $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$

$\Rightarrow$ $a^2x^2 - a^2b^2x - x + b^2 = 0 \Rightarrow a^2x(x - b^2) - 1(x - b^2) = 0$

$\Rightarrow$ $(a^2x - 1)(x - b^2) = 0$

$\Rightarrow$ $a^2x - 1 = 0$ or $x - b^2 = 0 \Rightarrow x = 1/a^2$ or $x = b^2$

$\therefore$ $1/a^2, b^2$ are the required roots.

6. **(b) :** We have, $21x^2 - 2x + 1/21 = 0$

$\Rightarrow$ $441x^2 - 42x + 1 = 0$

Here, $a = 441$, $b = -42$ and $c = 1$.

$\therefore$ $D = b^2 - 4ac = (-42)^2 - 4(441)(1) = 1764 - 1764 = 0$

Hence, both roots are real and repeated.

7. We have, $x(x + 2c) = -ab \Rightarrow x^2 + 2cx + ab = 0$ ...(i)

(i) has real and unequal roots, so $D = b^2 - 4ac > 0$

$\Rightarrow$ $4c^2 - 4ab > 0 \Rightarrow c^2 > ab$

Also, we have $x^2 - 2(a + b)x + 2c^2 + a^2 + b^2 = 0$ ...(ii)

Here, $D = 4(a + b)^2 - 4(2c^2 + a^2 + b^2)$

$= 4(a^2 + b^2 + 2ab - 2c^2 - a^2 - b^2) = 8(ab - c^2) < 0$ $[\because c^2 > ab]$

So, (ii) has no real roots.

8. For equal roots, discriminant = 0

$\therefore$ $(k + 1)^2 - 4(k + 4)(1) = 0$

$\Rightarrow$ $k^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow$ $(k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$

9. Given, $x = 1$ is root of the given equation, so it will satisfy the given equation.

$\therefore$ $a(1)^2 - 5(a - 1) \times 1 - 1 = 0$

$\Rightarrow$ $a - 5a + 5 - 1 = 0 \Rightarrow -4a = -4 \Rightarrow a = \frac{-4}{-4} = 1$

10. We have, $p^2q^2x^2 - q^2x - p^2x + 1 = 0$

$\Rightarrow$ $q^2x(p^2x - 1) - 1(p^2x - 1) = 0$

$\Rightarrow$ $(p^2x - 1)(q^2x - 1) = 0 \Rightarrow x = \frac{1}{p^2}$ or $x = \frac{1}{q^2}$

11. Let the numbers be x and $(x + 4)$.

According to the question, $x(x + 4) = 45$

$\Rightarrow$ $x^2 + 4x - 45 = 0 \Rightarrow x^2 + 9x - 5x - 45 = 0$

$\Rightarrow$ $x(x + 9) - 5(x + 9) = 0$

$\Rightarrow$ $(x + 9)(x - 5) = 0 \Rightarrow x + 9 = 0$ or $x - 5 = 0$

$\Rightarrow$ $x = -9$ or $x = 5$

If $x = -9$, numbers are $-9, -9 + 4$ *i.e.*, $-9, -5$

If $x = 5$, numbers are $5, 5 + 4$ *i.e.*, $5, 9$

12. Let the number be x.

According to question, $x + 2x^2 = 21$

$\Rightarrow$ $2x^2 + x - 21 = 0 \Rightarrow 2x^2 - 6x + 7x - 21 = 0$

$\Rightarrow$ $2x(x - 3) + 7(x - 3) = 0$

$\Rightarrow$ $(x - 3)(2x + 7) = 0 \Rightarrow x = 3$ or $x = \frac{-7}{2}$

13. The given quadratic equation is $3x^2 + 7x + k = 0$ ...(i)

Here, $a = 3$, $b = 7$ and $c = k$.

$\therefore$ $D = b^2 - 4ac = (7)^2 - 4(3)(k) = 49 - 12k$

$\because$ Equation (i) has real and equal roots, so $D = 0$.

$\Rightarrow$ $49 - 12k = 0 \Rightarrow 12k = 49 \Rightarrow k = \frac{49}{12}$

14. The given quadratic equation is

$x(x - 4) + p = 0 \Rightarrow x^2 - 4x + p = 0$

Here, $a = 1$, $b = -4$ and $c = p$.

For real and equal roots : $D = b^2 - 4ac = 0$

$\Rightarrow \quad (-4)^2 - 4(1)(p) = 0$

$\Rightarrow \quad 16 - 4p = 0 \Rightarrow 4p = 16 \Rightarrow p = 4$

15. Since 2 is a root of the equation $x^2 + kx + 12 = 0$.

$\therefore \quad (2)^2 + k(2) + 12 = 0 \Rightarrow 4 + 2k + 12 = 0 \Rightarrow 2k + 16 = 0$

$\Rightarrow \quad k = -16/2 \Rightarrow k = -8$

Putting $k = -8$ in the equation $x^2 + kx + q = 0$, we get

$x^2 - 8x + q = 0$ ...(i)

The equation (i) will have equal roots, if discriminant = 0

$\Rightarrow \quad (-8)^2 - 4(1)q = 0$

$\Rightarrow \quad 64 - 4q = 0 \Rightarrow q = 64/4 \Rightarrow q = 16$

16. We have, $x^2 - x + 2 = 0$

Here, $a = 1$, $b = -1$ and $c = 2$

$\therefore \quad D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

$\therefore$ The given quadratic equation does not have real roots.

17. (i) (a) : To have no real roots, discriminant ($D = b^2 - 4ac$) should be < 0.

(a) $D = 7^2 - 4(-4)(-4) = 49 - 64 = -15 < 0$

(b) $D = 7^2 - 4(-4)(-2) = 49 - 32 = 17 > 0$

(c) $D = 5^2 - 4(-2)(-2) = 25 - 16 = 9 > 0$

(d) $D = 6^2 - 4(3)(2) = 36 - 24 = 12 > 0$

(ii) (b) : To have rational roots, discriminant ($D = b^2 - 4ac$) should be > 0 and also a perfect square.

(a) $D = 1^2 - 4(1)(-1) = 1 + 4 = 5$, which is not a perfect square.

(b) $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1$, which is a perfect square.

(c) $D = (-3)^2 - 4(4)(-2) = 9 + 32 = 41$, which is not a perfect square.

(d) $D = (-1)^2 - 4(6)(11) = 1 - 264 = -263$, which is not a perfect square.

(iii) (c) : To have irrational roots, discriminant ($D = b^2 - 4ac$) should be > 0 but not a perfect square.

(a) $D = 2^2 - 4(3)(2) = 4 - 24 = -20 < 0$

(b) $D = (-7)^2 - 4(4)(3) = 49 - 48 = 1 > 0$ and also a perfect square.

(c) $D = (-3)^2 - 4(6)(-5) = 9 + 120 = 129 > 0$ and not a perfect square.

(d) $D = 3^2 - 4(2)(-2) = 9 + 16 = 25 > 0$ and also a perfect square.

(iv) (d) : To have equal roots, discriminant ($D = b^2 - 4ac$) should be = 0.

(a) $D = (-3)^2 - 4(1)(4) = 9 - 16 = -7 < 0$

(b) $D = (-2)^2 - 4(2)(1) = 4 - 8 = -4 < 0$

(c) $D = (-10)^2 - 4(5)(1) = 100 - 20 = 80 > 0$

(d) $D = 6^2 - 4(9)(1) = 36 - 36 = 0$

(v) (a) : To have two distinct real roots, discriminant ($D = b^2 - 4ac$) should be > 0.

(a) $D = 3^2 - 4(1)(1) = 9 - 4 = 5 > 0$

(b) $D = 3^2 - 4(-1)(-3) = 9 - 12 = -3 < 0$

(c) $D = 8^2 - 4(4)(4) = 64 - 64 = 0$

(d) $D = 6^2 - 4(3)(4) = 36 - 48 = -12 < 0$

18. (i) Roots of the quadratic equation are 2 and –3.

$\therefore$ The required quadratic equation is

$(x - 2)(x + 3) = 0 \Rightarrow x^2 + x - 6 = 0$

(ii) We have, $2x^2 + kx + 1 = 0$

Since, –1/2 is the root of the equation, so it will satisfy the given equation.

$$\therefore \quad 2\left(-\frac{1}{2}\right)^2 + k\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow 1 - k + 2 = 0 \Rightarrow k = 3$$

(iii) We have, $16x^2 - 9 = 0$ ...(i)

$$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \frac{\pm 3}{4}$$

$\Rightarrow$ Roots of (i) are $\frac{3}{4}$ and $\frac{-3}{4}$.

(iv) The given equation is $(x - 2)^2 + 19 = 0$

$\Rightarrow \quad x^2 - 4x + 4 + 19 = 0 \Rightarrow x^2 - 4x + 23 = 0$

(v) If one root of a quadratic equation is irrational, then its other root is also irrational and also its conjugate *i.e.*, if one root is $p + \sqrt{q}$, then its other root is $p - \sqrt{q}$.

19. (i) We have, $6x^2 + x - 2 = 0$

$\Rightarrow \quad 6x^2 - 3x + 4x - 2 = 0$

$\Rightarrow \quad (3x + 2)(2x - 1) = 0$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{-2}{3}$$

(ii) $2x^2 + x - 300 = 0$

$\Rightarrow \quad 2x^2 - 24x + 25x - 300 = 0$

$\Rightarrow \quad (x - 12)(2x + 25) = 0$

$$\Rightarrow \quad x = 12, \frac{-25}{2}$$

(iii) $x^2 - 8x + 16 = 0$

$\Rightarrow \quad (x - 4)^2 = 0 \Rightarrow (x - 4)(x - 4) = 0 \Rightarrow x = 4, 4$

(iv) $6x^2 - 13x + 5 = 0$

$\Rightarrow \quad 6x^2 - 3x - 10x + 5 = 0 \Rightarrow (2x - 1)(3x - 5) = 0$

$$\Rightarrow \quad x = \frac{1}{2}, \frac{5}{3}$$

(v) $100x^2 - 20x + 1 = 0$

$$\Rightarrow \quad (10x - 1)^2 = 0 \Rightarrow x = \frac{1}{10}, \frac{1}{10}$$

20. **(i)** **(d)** **(ii)** **(b)**

(iii) **(a) :** $x(x+3)+7=5x-11$

$\Rightarrow x^2+3x+7=5x-11$

$\Rightarrow x^2-2x+18=0$ is a quadratic equation.

(b) $(x-1)^2-9=(x-4)(x+3)$

$\Rightarrow x^2-2x-8=x^2-x-12$

$\Rightarrow x-4=0$ is not a quadratic equation.

(c) $x^2(2x+1)-4=5x^2-10$

$\Rightarrow 2x^3+x^2-4=5x^2-10$

$\Rightarrow 2x^3-4x^2+6=0$ is not a quadratic equation.

(d) $x(x-1)(x+7)=x(6x-9)$

$\Rightarrow x^3+6x^2-7x=6x^2-9x$

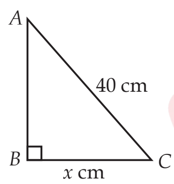
$\Rightarrow x^3+2x=0$ is not a quadratic equation.

(iv) **(d)** **(v)** **(d)**

21. Let ΔABC is the given triangle.

Let base, $BC = x$ cm, then altitude, $AB = (x+8)$ cm

By Pythagoras theorem, we have



$(AB)^2+(BC)^2=(AC)^2$

$\Rightarrow (x+8)^2+x^2=40^2$

$\Rightarrow x^2+64+16x+x^2=1600$

$\Rightarrow 2x^2+16x-1536=0$

$\Rightarrow x^2+8x-768=0$

$\Rightarrow x^2+32x-24x-768=0 \Rightarrow x(x+32)-24(x+32)=0$

$\Rightarrow (x+32)(x-24)=0 \Rightarrow x=-32$ or $x=24$

But side of a triangle can't be negative.

$\therefore\ x=24$

22. Let the first part be x, then the second part will be $12-x$.

According to the given condition,

$x^2+(12-x)^2=74 \Rightarrow x^2+144+x^2-24x-74=0$

$\Rightarrow 2x^2-24x+70=0 \Rightarrow x^2-12x+35=0$

$\Rightarrow x^2-7x-5x+35=0 \Rightarrow x(x-7)-5(x-7)=0$

$\Rightarrow (x-7)(x-5)=0 \Rightarrow x-7=0$ or $x-5=0$

$\Rightarrow x=7$ or $x=5$

$\therefore$ Two parts of 12 are 7 and 5.

23. Let one number be x, then other number will be $x-7$.

According to question, $x(x-7)=408 \Rightarrow x^2-7x-408=0$

$\Rightarrow x^2-24x+17x-408=0 \Rightarrow x(x-24)+17(x-24)=0$

$\Rightarrow (x-24)(x+17)=0 \Rightarrow x=24$ or $x=-17$ (rejected)

Thus, one number is 24 and other number is 17.

Sum of numbers = 24 + 17 = 41

24. Given, $4x^2-2(c+1)x+(c+4)=0$

Here, $A=4$, $B=-2(c+1)$ and $C=c+4$

Now, $D=B^2-4AC$

$=\{-2(c+1)\}^2-4\times4\times(c+4)=4(c^2+2c+1)-16(c+4)$

$=4c^2+8c+4-16c-64=4c^2-8c-60$

For equal roots, $D=0$

$\therefore\ 4c^2-8c-60=0 \Rightarrow c^2-2c-15=0$

$\Rightarrow (c+3)(c-5)=0 \Rightarrow c=-3$ or $c=5$

25. Given, $\dfrac{1}{2a+b+2x}=\dfrac{1}{2a}+\dfrac{1}{b}+\dfrac{1}{2x}$

$\Rightarrow \dfrac{1}{2a+b+2x}-\dfrac{1}{2x}=\dfrac{1}{2a}+\dfrac{1}{b}$

$\Rightarrow \dfrac{2x-2a-b-2x}{2x(2a+b+2x)}=\dfrac{b+2a}{2ab}$

$\Rightarrow \dfrac{-(2a+b)}{2x(2a+b+2x)}=\dfrac{b+2a}{2ab} \Rightarrow \dfrac{-1}{x(2a+b+2x)}=\dfrac{1}{ab}$

$\Rightarrow 2x^2+2ax+bx+ab=0 \Rightarrow 2x(x+a)+b(x+a)=0$

$\Rightarrow (x+a)(2x+b)=0 \Rightarrow x=-a$ or $x=\dfrac{-b}{2}$

26. Given, $\dfrac{6}{x}-\dfrac{2}{x-1}=\dfrac{1}{x-2} \Rightarrow \dfrac{6x-6-2x}{x(x-1)}=\dfrac{1}{x-2}$

$\Rightarrow \dfrac{4x-6}{x^2-x}=\dfrac{1}{x-2} \Rightarrow 4x^2-6x-8x+12=x^2-x$

$\Rightarrow 4x^2-14x+12=x^2-x$

$\Rightarrow 3x^2-13x+12=0 \Rightarrow 3x^2-9x-4x+12=0$

$\Rightarrow 3x(x-3)-4(x-3)=0 \Rightarrow (x-3)(3x-4)=0$

$\Rightarrow x-3=0$ or $3x-4=0$

$\Rightarrow x=3$ or $x=4/3$

OR

Let the number of persons in 1st condition is x and in 2nd condition is $(x+15)$.

Amount to be divided = ₹ 6500

According to the question, $\dfrac{6500}{x}-\dfrac{6500}{x+15}=30$

$\Rightarrow \dfrac{6500x+97500-6500x}{x(x+15)}=\dfrac{30}{1}$

$\Rightarrow 30x^2+450x=97500 \Rightarrow 30x^2+450x-97500=0$

$\Rightarrow x^2+15x-3250=0 \Rightarrow x^2+65x-50x-3250=0$

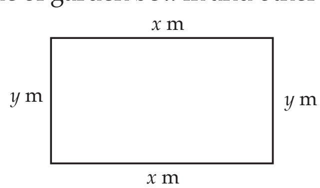
$\Rightarrow x(x+65)-50(x+65)=0 \Rightarrow (x+65)(x-50)=0$

$\Rightarrow x+65=0$ or $x-50=0 \Rightarrow x=-65$ or $x=50$

$\because$ Number of persons cannot be negative

$\therefore$ Original number of persons = 50.

27. Let the length of one side of garden be x m and other side be y m. Then,



$x+y+x=30$

$\Rightarrow y=30-2x$ ...(i)

Given, area of the vegetable garden = 100 m^2

$\Rightarrow xy=100$

$\Rightarrow x(30-2x)=100$ [Using (i)]

$\Rightarrow 30x-2x^2=100 \Rightarrow 15x-x^2=50$

$\Rightarrow x^2-15x+50=0 \Rightarrow x^2-10x-5x+50=0$

$\Rightarrow\quad (x-10)(x-5)=0 \Rightarrow x=5 \text{ or } 10$

When $x=5$, then $y=30-2\times 5=20$ [Using (i)]

When $x=10$, then $y=30-2\times 10=10$ [Using (i)]

Hence, the dimensions of the vegetable garden are 5 m and 20 m or 10 m and 10 m.

OR

Let x be the total number of students of the class.

Number of students opted for visiting an old age home $=\frac{3}{8}x$.

Number of students opted for having a nature walk = 16.

Number of students opted for tree plantation in the school $=\sqrt{x}$.

According to the given condition,

$$\frac{3}{8}x=16+\sqrt{x} \Rightarrow 3x=128+8\sqrt{x}$$

$\Rightarrow\quad 3y^2=128+8y$, where $\sqrt{x}=y$

$\Rightarrow\quad 3y^2-8y-128=0 \Rightarrow 3y^2-24y+16y-128=0$

$\Rightarrow\quad 3y(y-8)+16(y-8)=0 \Rightarrow (y-8)(3y+16)=0$

$\Rightarrow\quad y-8=0 \text{ or } 3y+16=0$

$\Rightarrow\quad y=8 \text{ or } y=-\frac{16}{3} \Rightarrow \sqrt{x}=8 \quad \left[\because \sqrt{x}\neq -\frac{16}{3}\right]$

$\Rightarrow\quad x=64$

Hence, the total number of students of the class is 64.

28. Let the numerator of the fraction $=x$

Then denominator of the fraction $=2x+1$

$\therefore\quad$ Fraction $=\frac{x}{2x+1}$ and its reciprocal $=\frac{2x+1}{x}$

According to given condition, $\frac{x}{2x+1}+\frac{2x+1}{x}=2\frac{16}{21}$

$\Rightarrow\quad \frac{x^2+4x^2+1+4x}{2x^2+x}=\frac{58}{21} \Rightarrow \frac{5x^2+1+4x}{2x^2+x}=\frac{58}{21}$

$\Rightarrow\quad 116x^2+58x=105x^2+84x+21$

$\Rightarrow\quad 116x^2+58x-105x^2-84x-21=0$

$\Rightarrow\quad 11x^2-26x-21=0 \Rightarrow 11x^2-33x+7x-21=0$

$\Rightarrow\quad 11x(x-3)+7(x-3)=0 \Rightarrow (x-3)(11x+7)=0$

$\Rightarrow\quad x-3=0 \text{ or } 11x+7=0 \Rightarrow x=3 \text{ or } x=-7/11$

$\therefore\quad x=3$ (Neglecting negative value)

$\therefore\quad$ Fraction $=\frac{x}{2x+1}=\frac{3}{6+1}=\frac{3}{7}$

29. Let the original price of the toy = ₹ x

Then the reduced price of the toy = ₹ $(x-2)$

According to the question,

$$\frac{360}{x-2}-\frac{360}{x}=2 \quad \left(\because \text{ Number of toys}=\frac{\text{Total amount}}{\text{Price of 1 toy}}\right)$$

$\Rightarrow\quad \frac{360x-360x+720}{x(x-2)}=2 \Rightarrow \frac{720}{x(x-2)}=\frac{2}{1}$

$\Rightarrow\quad x(x-2)=360$

$\Rightarrow\quad x^2-2x-360=0 \Rightarrow x^2-20x+18x-360=0$

$\Rightarrow\quad x(x-20)+18(x-20)=0 \Rightarrow (x-20)(x+18)=0$

$\Rightarrow\quad x-20=0 \text{ or } x+18=0$

$\Rightarrow\quad x=20 \text{ or } x=-18$

$\therefore\quad x=20$ [$\because$ Price cannot be negative]

$\therefore\quad$ Original price of the toy = ₹ 20

30. Given, $(2p+1)x^2-(7p+2)x+(7p-3)=0$ ...(i)

$\because\quad$ Roots are equal. $\therefore\quad D=0$

$\Rightarrow\quad (-(7p+2))^2-4(2p+1)(7p-3)=0$

$\Rightarrow\quad 49p^2+4+28p-4(14p^2+7p-6p-3)=0$

$\Rightarrow\quad 49p^2+28p+4-56p^2-4p+12=0$

$\Rightarrow\quad 7p^2-24p-16=0 \Rightarrow 7p^2+4p-28p-16=0$

$\Rightarrow\quad p(7p+4)-4(7p+4)=0 \Rightarrow (p-4)(7p+4)=0$

$\Rightarrow\quad p=4 \text{ or } p=\frac{-4}{7}$

When $p=4$, (i) becomes $9x^2-30x+25=0$

$\Rightarrow\quad (3x)^2-2(3x)(5)+(5)^2=0$

$\Rightarrow\quad (3x-5)^2=0 \Rightarrow x=\frac{5}{3},\frac{5}{3}$

When $p=\frac{-4}{7}$, (i) becomes

$$\frac{-x^2}{7}+2x-7=0 \Rightarrow x^2-14x+49=0$$

$\Rightarrow\quad (x-7)^2=0 \Rightarrow x=7,7$

Thus, equal roots of given equation are either 5/3 or 7.

31. Let the denominator of the fraction $=x$

$\therefore\quad$ Numerator of the fraction $=x-4$

$\Rightarrow\quad$ Fraction $=\frac{x-4}{x}$

According to question,

$$\frac{x-4}{x+1}=\frac{x-4}{x}-\frac{1}{18} \Rightarrow \frac{x-4}{x}-\frac{x-4}{x+1}=\frac{1}{18}$$

$\Rightarrow\quad (x-4)\left[\frac{1}{x}-\frac{1}{x+1}\right]=\frac{1}{18} \Rightarrow (x-4)\left[\frac{x+1-x}{x(x+1)}\right]=\frac{1}{18}$

$\Rightarrow\quad 18(x-4)=x(x+1) \Rightarrow 18x-72=x^2+x$

$\Rightarrow\quad x^2-17x+72=0 \Rightarrow x^2-9x-8x+72=0$

$\Rightarrow\quad x(x-9)-8(x-9)=0 \Rightarrow (x-8)(x-9)=0$

$\Rightarrow\quad x=8 \text{ or } x=9$

But $x=8$ is not possible $\therefore x=9$

Hence, the fraction $\frac{x-4}{x}$ is $\frac{5}{9}$.

32. Let breadth of rectangular park $=x$ m

Then, length of rectangular park $=(x+3)$m

Now, area of rectangular park $=x(x+3)=(x^2+3x)\text{m}^2$

Given, base of triangular park = Breadth of the rectangular park

$\therefore$ Base of triangular park = x m

and also it is given that altitude of triangular park = 12 m

$\therefore$ Area of triangular park = $\frac{1}{2} \times x \times 12 = 6x$ m^2

According to the question,

Area of rectangular park = 4 + Area of triangular park

$\Rightarrow x^2 + 3x = 4 + 6x \Rightarrow x^2 + 3x - 6x - 4 = 0$

$\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x^2 - 4x + x - 4 = 0$

$\Rightarrow x(x - 4) + 1(x - 4) = 0 \Rightarrow (x - 4)(x + 1) = 0$

$\Rightarrow x - 4 = 0$ or $x + 1 = 0 \Rightarrow x = 4$ or $x = -1$

Since, breadth cannot be negative.

$\therefore x = 4$

Hence, breadth of the rectangular park = 4 m

and length of the rectangular park = $x + 3 = 4 + 3 = 7$ m.

OR

Let the length of piece of cloth = x m

Increased length of piece of cloth = $(x + 5)$m

Total cost of piece of cloth = ₹ 200

According to the question,

$\frac{200}{x} - \frac{200}{x+5} = 2$ $\left[\because \text{Rate per metre} = \frac{\text{Total cost}}{\text{Length}}\right]$

$\Rightarrow \frac{200x + 1000 - 200x}{x(x+5)} = 2$

$\Rightarrow 1000 = 2x^2 + 10x \Rightarrow 2x^2 + 10x - 1000 = 0$

$\Rightarrow x^2 + 5x - 500 = 0 \Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x + 25) - 20(x + 25) = 0 \Rightarrow (x + 25)(x - 20) = 0$

$\Rightarrow x + 25 = 0$ or $x - 20 = 0 \Rightarrow x = -25$ or $x = 20$

But, length can never be negative.

$\therefore$ Length of cloth = 20 m

and rate per metre = ₹ $\frac{200}{20}$ = ₹ 10.

33 Let the number of students in the group in the beginning be x.

Total internet service charges for x students = ₹ 4800

$\therefore$ Internet service charges for each student = ₹ $\frac{4800}{x}$

It is given that 4 more students join the group.

$\therefore$ The number of students in group for internet service $= (x + 4)$

Now, the internet service charges for each student = ₹ $\frac{4800}{x+4}$

According to question, $\frac{4800}{x} - \frac{4800}{x+4} = 200$

$\Rightarrow \frac{4800x + 19200 - 4800x}{x(x+4)} = 200$

$\Rightarrow 19200 = 200(x^2 + 4x) \Rightarrow 96 = x^2 + 4x$

$\Rightarrow x^2 + 4x - 96 = 0 \Rightarrow x^2 + 12x - 8x - 96 = 0$

$\Rightarrow x(x + 12) - 8(x + 12) = 0 \Rightarrow (x - 8)(x + 12) = 0$

$\Rightarrow x - 8 = 0$ or $x + 12 = 0 \Rightarrow x = 8$ or $x = -12$

But number of students cannot be negative

$\therefore x = 8$

Hence, the number of students in the group in the beginning is 8.

OR

Let the speed of the train be x km/hour.

When the speed is 9 km/hour more, then the new speed of the train is $(x + 9)$ km/hour.

Time taken by the train with speed x km/hour for a journey of 180 km = $\frac{180}{x}$ hours

Time taken by the train with new speed $(x + 9)$ km/hour for a journey of 180 km = $\frac{180}{(x+9)}$ hours

According to the question, $\frac{180}{x} - \frac{180}{x+9} = 1$

$\Rightarrow 180\left[\frac{1}{x} - \frac{1}{x+9}\right] = 1 \Rightarrow 180\left[\frac{x+9-x}{x(x+9)}\right] = 1$

$\Rightarrow 180 \times 9 = x(x + 9) \Rightarrow x^2 + 9x - 1620 = 0$

$\Rightarrow x^2 + 45x - 36x - 1620 = 0 \Rightarrow x(x + 45) - 36(x + 45) = 0$

$\Rightarrow (x + 45)(x - 36) = 0 \Rightarrow x + 45 = 0$ or $x - 36 = 0$

$\Rightarrow x = -45$ or $x = 36$

But, speed can't be negative.

$\therefore x = 36$

Hence, the uniform speed of the train is 36 km/hour.

34. Let x and y be the sides of two squares, respectively such that $x > y$, where x is the side of the first square and y is the side of the second square.

$\therefore$ Area of the first square + Area of the second square = 640 m^2

$\Rightarrow x^2 + y^2 = 640$ ...(i)

Again, it is given that the difference of their perimeters = 64 m

$\Rightarrow 4x - 4y = 64 \Rightarrow x = 16 + y$ ...(ii)

From (i) and (ii), we have, $(16 + y)^2 + y^2 = 640$

$\Rightarrow 256 + y^2 + 32y + y^2 = 640 \Rightarrow 2y^2 + 32y - 384 = 0$

$\Rightarrow y^2 + 16y - 192 = 0 \Rightarrow y^2 + 24y - 8y - 192 = 0$

$\Rightarrow y(y + 24) - 8(y + 24) = 0 \Rightarrow (y + 24)(y - 8) = 0$

$\Rightarrow y + 24 = 0$ or $y - 8 = 0 \Rightarrow y = -24$ or $y = 8$

But, side of a square can't be negative. $\therefore y = 8$

When $y = 8$, then from (ii), we get $x = 16 + 8 = 24$.

Hence, the sides of the two squares are 24 m and 8 m respectively.

OR

Let the speed of Deccan Queen = x km/hr

and speed of other train = $(x - 20)$ km/hr

Time taken by Deccan Queen = $\frac{192}{x}$ hr

and time taken by other train = $\frac{192}{(x-20)}$ hr

According to the question, $\frac{192}{(x-20)} - \frac{192}{x} = \frac{48}{60}$ or $\frac{4}{5}$

$\Rightarrow \quad \frac{192x - 192x + 3840}{x(x-20)} = \frac{4}{5}$

$\Rightarrow \quad 5(3840) = 4x(x - 20) \Rightarrow 19200 = 4x^2 - 80x$

$\Rightarrow \quad 4x^2 - 80x - 19200 = 0 \Rightarrow x^2 - 20x - 4800 = 0$

$\Rightarrow \quad x^2 - 80x + 60x - 4800 = 0 \Rightarrow x(x - 80) + 60(x - 80) = 0$

$\Rightarrow \quad (x - 80)(x + 60) = 0 \Rightarrow x - 80 = 0$ or $x + 60 = 0$

$\Rightarrow \quad x = 80$ or $x = -60$

As speed can never be negative. $\therefore x = 80$

$\therefore$ Speed of Deccan Queen = 80 km/hr.

35. Let the usual speed of the plane be x km/hr.

$\therefore$ Time taken to travel 1500 km at x km/hr

$= \frac{1500}{x}$ hour

Increased speed of the plane = $(x + 250)$ km/hr

$\therefore$ Time taken to travel 1500 km at $(x + 250)$ km/hr

$= \frac{1500}{x+250}$ hour

According to question,

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60} \Rightarrow 1500\left(\frac{x+250-x}{x(x+250)}\right) = \frac{1}{2}$$

$\Rightarrow \quad 2 \times 1500 \times 250 = x^2 + 250x$

$\Rightarrow \quad x^2 + 250x - 750000 = 0$

$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow \quad x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow \quad (x + 1000)(x - 750) = 0$

$\Rightarrow \quad x = 750$ or $x = -1000$ (But speed can't be negative)

$\therefore \quad x = 750$

Hence, usual speed of the plane is 750 km/hr.