Quadratic Equations

CHAPTER 4

TRY YOURSELF SOLUTIONS

1. We have, $(x-3)^2+2=5x-9$

$\Rightarrow\quad x^2+9-6x+2=5x-9 \Rightarrow x^2-11x+20=0$

It is of the form $ax^2+bx+c=0,\ a\neq 0$.

So, the given equation is a quadratic equation.

2. We have, $x(7x-12)=7(x^2-7x+9)$

$\Rightarrow\quad 7x^2-12x=7x^2-49x+63 \Rightarrow 37x-63=0$

It is not of the form $ax^2+bx+c=0,\ a\neq 0$.

So, the given equation is not a quadratic equation.

3. We have, $x(6x+5)=8x^2+6x$

$\Rightarrow\quad 6x^2+5x=8x^2+6x \Rightarrow 2x^2+x=0$

It is of the form $ax^2+bx+c=0,\ a\neq 0$.

So, the given equation is a quadratic equation.

4. We have, $2x-\frac{3}{x}-15=5\left(2x-\frac{3}{x}\right),\ x\neq 0$

$\Rightarrow\quad 2x-\frac{3}{x}-15=10x-\frac{15}{x}$

$\Rightarrow\quad 2x^2-3-15x=10x^2-15 \qquad [\because\ x\neq 0]$

$\Rightarrow\quad 8x^2+15x-12=0$

It is of the form $ax^2+bx+c=0,\ a\neq 0$.

So, the given equation is a quadratic equation.

5. Let two consecutive odd integers be $2x+1$ and $2x+3$.

According to question, $(2x+1)(2x+3)=783$

$\Rightarrow\quad 4x^2+6x+2x+3=783$

$\Rightarrow\quad 4x^2+8x-780=0 \Rightarrow x^2+2x-195=0$

This is the required quadratic equation.

6. Let the present age of Raju be x years.

$\therefore$ His father's present age $=(x+30)$ years

After 5 years, Raju's age $=(x+5)$ years

After 5 years, father's age $=(x+30+5)=(x+35)$ years

According to question, $(x+5)(x+35)=450$

$\Rightarrow\quad x^2+35x+5x+175=450 \Rightarrow x^2+40x-275=0$

This is the required quadratic equation.

7. Given equation is in the form $p(x)=0$,

where $p(x)=5x^2-126x+25 \qquad \ldots$(i)

On putting $x=25$ in (i), we get

$p(25)=5(25)^2-126(25)+25=3125-3150+25=0$

On putting $x=\frac{1}{10}$ in (i), we get

$p\left(\frac{1}{10}\right)=5\left(\frac{1}{10}\right)^2-126\left(\frac{1}{10}\right)+25$

$=\frac{5-1260+2500}{100}=\frac{1245}{100}\neq 0.$

Hence, $x=25$ is a solution but $x=\frac{1}{10}$ is not a solution of the given quadratic equation.

8. Given equation is in the form $p(x)=0$,

where $p(x)=x^2+8x+4 \qquad \ldots$(i)

On putting $x=-2$ in (i), we get

$p(-2)=(-2)^2+8(-2)+4=4-16+4=-8\neq 0$

On putting $x=-4$ in (i), we get

$p(-4)=(-4)^2+8(-4)+4=16-32+4=-12\neq 0$

So, $x=-2$ and -4 both are not the solutions of the given equation.

9. Given equation is in the form $p(x)=0$,

where $p(x)=x^2-4\sqrt{2}\,x+2\sqrt{2} \qquad \ldots$(i)

On putting $x=\sqrt{2}$ in (i), we get

$p(\sqrt{2})=(\sqrt{2})^2-4\sqrt{2}(\sqrt{2})+2\sqrt{2}$

$=2-8+2\sqrt{2}=2\sqrt{2}-6\neq 0$

On putting $x=\sqrt{2}+1$ in (i), we get

$p(\sqrt{2}+1)=(\sqrt{2}+1)^2-4\sqrt{2}(\sqrt{2}+1)+2\sqrt{2}$

$=2+1+2\sqrt{2}-8-4\sqrt{2}+2\sqrt{2}=-5\neq 0$

So, $x=\sqrt{2}$ and $\sqrt{2}+1$ both are not the solutions of the given equation.

10. Given, $x^2+kx-192=0$

Since, $x=12$ is a root of the given equation, so it will satisfy the given equation.

$\therefore\quad (12)^2+k(12)-192=0 \Rightarrow 144+12k-192=0$

$\Rightarrow\quad 12k-48=0 \Rightarrow k=\frac{48}{12}=4$

11. Given, $5x^2-8x+k=0$

Since, $x=-2/5$ is a root of the given equation, so it will satisfy the given equation.

$\therefore\quad 5\left(\frac{-2}{5}\right)^2-8\left(\frac{-2}{5}\right)+k=0$

$\Rightarrow\quad \frac{4}{5}+\frac{16}{5}+k=0 \Rightarrow 4+k=0 \Rightarrow k=-4$

12. Given, $3x^2 - 2ax + 2b = 0$ ...(i)

Since, $x = 2$ and $x = 3$ are the roots of (i), so these will satisfy the given equation.

On putting $x = 2$ in (i), we get

$3(2)^2 - 2a(2) + 2b = 0$

$\Rightarrow$ $12 - 4a + 2b = 0 \Rightarrow 4a - 2b = 12$ ...(ii)

On putting $x = 3$ in (i), we get

$3(3)^2 - 2a(3) + 2b = 0$

$\Rightarrow$ $27 - 6a + 2b = 0 \Rightarrow 6a - 2b = 27$ ...(iii)

Subtracting (ii) from (iii), we get

$2a = 15 \Rightarrow a = \frac{15}{2}$

Substituting the value of a in (ii), we get

$4\left(\frac{15}{2}\right) - 2b = 12 \Rightarrow 30 - 2b = 12 \Rightarrow 2b = 18 \Rightarrow b = 9$

13. We have, $11x^2 - 26x - 21 = 0$

$\Rightarrow$ $11x^2 - 33x + 7x - 21 = 0 \Rightarrow 11x(x - 3) + 7(x - 3) = 0$

$\Rightarrow$ $(x - 3)(11x + 7) = 0 \Rightarrow x - 3 = 0$ or $11x + 7 = 0$

$\Rightarrow$ $x = 3$ or $x = \frac{-7}{11}$

Hence, 3 and $\frac{-7}{11}$ are the two roots of the given equation.

14. We have, $2x^2 - 17x + 21 = 0$

$\Rightarrow$ $2x^2 - 14x - 3x + 21 = 0 \Rightarrow 2x(x - 7) - 3(x - 7) = 0$

$\Rightarrow$ $(x - 7)(2x - 3) = 0 \Rightarrow x - 7 = 0$ or $2x - 3 = 0$

$\Rightarrow$ $x = 7$ or $x = \frac{3}{2}$

Hence, 7 and $\frac{3}{2}$ are the two roots of the given equation.

15. We have, $2ax^2 - (2a - b^2)x - b^2 = 0$

$\Rightarrow$ $2ax^2 - 2ax + b^2x - b^2 = 0 \Rightarrow 2ax(x - 1) + b^2(x - 1) = 0$

$\Rightarrow$ $(x - 1)(2ax + b^2) = 0 \Rightarrow x - 1 = 0$ or $2ax + b^2 = 0$

$\Rightarrow$ $x = 1$ or $x = -\frac{b^2}{2a}$

Hence, 1 and $-\frac{b^2}{2a}$ are the two roots of the given equation.

16. We have, $x^2 + (1 + \sqrt{5})x + \sqrt{5} = 0$

$\Rightarrow$ $x^2 + x + \sqrt{5}x + \sqrt{5} = 0$

$\Rightarrow$ $x(x + 1) + \sqrt{5}(x + 1) = 0 \Rightarrow (x + 1)(x + \sqrt{5}) = 0$

$\Rightarrow$ $x + 1 = 0$ or $x + \sqrt{5} = 0 \Rightarrow x = -1$ or $x = -\sqrt{5}$

Hence, -1 and $-\sqrt{5}$ are the two roots of the given equation.

17. Let original average speed of the train be x km/hr.

According to question,

$\frac{63}{x} + \frac{72}{x+6} = 3 \Rightarrow \frac{7}{x} + \frac{8}{x+6} = \frac{1}{3} \Rightarrow \frac{7(x+6)+8x}{x(x+6)} = \frac{1}{3}$

$\Rightarrow$ $3(7x + 42 + 8x) = x^2 + 6x \Rightarrow 45x + 126 = x^2 + 6x$

$\Rightarrow$ $x^2 - 39x - 126 = 0 \Rightarrow x^2 - 42x + 3x - 126 = 0$

$\Rightarrow$ $(x - 42)(x + 3) = 0 \Rightarrow x - 42 = 0$ or $x + 3 = 0$

$\Rightarrow$ $x = 42$ $(\because x > 0 \text{ so } x \neq -3)$

Hence, the original speed of the train is 42 km/hr.

18. Let the marked price of the book be ₹ x.

Total cost = ₹ 300

$\therefore$ Number of books $= \frac{300}{x}$

If price of the book is ₹ $(x - 5)$, then

Number of books $= \frac{300}{x-5}$

According to question,

$$\frac{300}{x-5} - \frac{300}{x} = 5 \Rightarrow \frac{300x - 300(x-5)}{(x-5)x} = 5$$

$\Rightarrow$ $1500 = 5(x^2 - 5x) \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow$ $x^2 - 20x + 15x - 300 = 0 \Rightarrow x(x - 20) + 15(x - 20) = 0$

$\Rightarrow$ $(x - 20)(x + 15) = 0 \Rightarrow x = 20$ or $x = -15$

Since, x has to be a positive integer, so $x = -15$ is rejected.

$\therefore$ $x = 20$

Hence, original marked price of the book is ₹ 20.

19. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

$\therefore$ $b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$

$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0$

So, the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(p^2 - q^2) \pm (p^2 + q^2)}{2p^2}$$

$$\Rightarrow \quad x = \frac{-p^2 + q^2 + p^2 + q^2}{2p^2} = \frac{q^2}{p^2}$$

$$\text{or} \quad x = \frac{-p^2 + q^2 - p^2 - q^2}{2p^2} = -1$$

Hence, the roots are $\frac{q^2}{p^2}$ and -1.

20. We have, $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Comparing this equation with $Ax^2 + Bx + C = 0$, we have

$A = 9$, $B = -9(a + b)$ and $C = 2a^2 + 5ab + 2b^2$

$\therefore$ $B^2 - 4AC = 81(a + b)^2 - 36(2a^2 + 5ab + 2b^2)$

$= 9a^2 + 9b^2 - 18ab = 9(a - b)^2 \geq 0$

So, the roots of the given equation are real and are given by

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$\Rightarrow \quad x = \frac{9(a+b) + 3(a-b)}{18} = \frac{12a + 6b}{18} = \frac{2a + b}{3}$$

or $x = \frac{9(a+b)-3(a-b)}{18} = \frac{6a+12b}{18} = \frac{a+2b}{3}$

Hence, the roots are $\frac{2a+b}{3}$ and $\frac{a+2b}{3}$.

21. We have, $abx^2 + (b^2 - ac)x - bc = 0$

Comparing this equation with $Ax^2 + Bx + C = 0$, we have $A = ab$, $B = b^2 - ac$ and $C = -bc$

$\therefore\ B^2 - 4AC = (b^2 - ac)^2 - 4(ab)(-bc)$

$= (b^2 - ac)^2 + 4ab^2c = b^4 - 2ab^2c + a^2c^2 + 4ab^2c = (b^2 + ac)^2 \geq 0$

So, the given equation has real roots, which are given by

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow\ x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \text{ or } x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$\Rightarrow\ x = \frac{2ac}{2ab} \text{ or } x = \frac{-2b^2}{2ab} \Rightarrow x = \frac{c}{b} \text{ or } x = \frac{-b}{a}$$

Hence, the roots are $\frac{c}{b}$ and $\frac{-b}{a}$.

22. We have, $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$

$\Rightarrow\ \frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6} \Rightarrow (8)6 = x^2 + 2x - 15$

$\Rightarrow\ x^2 + 2x - 63 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 2$ and $c = -63$

$\therefore\ b^2 - 4ac = (2)^2 - 4(1)(-63) = 4 + 252 = 256 > 0$

So, the given equation has real roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{256}}{2(1)} = \frac{-2 \pm 16}{2}$$

$$\Rightarrow\ x = \frac{-2+16}{2} \text{ or } x = \frac{-2-16}{2} \Rightarrow x = 7 \text{ or } x = -9$$

Hence, the roots are 7 and –9.

23. We have, $x^2 + x + 7 = 0$

Here, $a = 1$, $b = 1$ and $c = 7$.

$\therefore\ D = b^2 - 4ac = (1)^2 - 4(1)(7) = 1 - 28 = -27$

24. We have, $(4x - 3)^2 + 20x = 11$

$\Rightarrow\ 16x^2 + 9 - 24x + 20x = 11$

$\Rightarrow\ 16x^2 - 4x - 2 = 0 \Rightarrow 8x^2 - 2x - 1 = 0$

Here, $a = 8$, $b = -2$ and $c = -1$.

$\therefore\ D = b^2 - 4ac = (-2)^2 - 4(8)(-1) = 4 + 32 = 36$

25. We have, $x^2 - 8x + 16 = 0$

Here, $a = 1$, $b = -8$ and $c = 16$.

$\therefore\ D = b^2 - 4ac = (-8)^2 - 4(1)(16) = 64 - 64 = 0$

Thus, the given equation has real and equal roots.

26. We have, $4x^2 - 2\sqrt{3}\,x + 9 = 0$

Here, $a = 4$, $b = -2\sqrt{3}$ and $c = 9$

$\therefore\ D = b^2 - 4ac = \left(-2\sqrt{3}\right)^2 - 4(4)(9) = 12 - 144 = -132 < 0$

Thus, the given equation has no real roots.

27. Let length of park = x m and breadth of park = y m.

Perimeter of park = $2(x + y) = 80$ (Given)

$\Rightarrow\ x + y = 40 \Rightarrow y = 40 - x$

Area of park = $xy = 300$ (Given)

$\Rightarrow\ x(40 - x) = 300 \Rightarrow 40x - x^2 = 300$

$\Rightarrow\ x^2 - 40x + 300 = 0$ ...(i)

Here, $a = 1$, $b = -40$ and $c = 300$.

$\therefore\ D = b^2 - 4ac = (-40)^2 - 4(1)(300) = 1600 - 1200 = 400 > 0$

$\therefore$ Roots of (i) are real and distinct.

Hence, it is possible to design the given rectangular park.

28. We have, $kx^2 + 2x - 3 = 0$

Here, $a = k$, $b = 2$ and $c = -3$.

$\therefore\ D = b^2 - 4ac = (2)^2 - 4(k)(-3) = 4 + 12k$

Now, the given equation has real and equal roots, so $D = 0$

$\Rightarrow\ 4 + 12k = 0 \Rightarrow k = \frac{-4}{12} = \frac{-1}{3}$

29. We have, $2x^2 - 10x + k = 0$

Here, $a = 2$, $b = -10$ and $c = k$.

$\therefore\ D = b^2 - 4ac = (-10)^2 - 4(2)(k) = 100 - 8k$

Now, the given equation has real and equal roots, so $D = 0$

$\Rightarrow\ 100 - 8k = 0 \Rightarrow k = \frac{100}{8} = \frac{25}{2}$

30. We have, $5x^2 + kx - 4 = 0$

Here, $a = 5$, $b = k$ and $c = -4$.

$\therefore\ D = b^2 - 4ac = k^2 - 4(5)(-4) = k^2 + 80$

Now, the given equation has real and equal roots, if $D = 0$

$\Rightarrow\ k^2 + 80 = 0$

But k^2 is always positive.

So, for no value of k, $D = 0$.

Hence, equation has no real and equal roots.

31. We have, $(n + 3)x^2 - (5 - n)x + 1 = 0$

Here, $a = n + 3$, $b = -(5 - n)$ and $c = 1$.

$\therefore\ D = b^2 - 4ac = (-(5 - n))^2 - 4(n + 3)(1)$

$= 25 + n^2 - 10n - 4n - 12$

$= n^2 - 14n + 13 = n^2 - 13n - n + 13$

$= n(n - 13) - 1(n - 13) = (n - 13)(n - 1)$

Now, the given equation has coincident roots, *i.e.*, equal roots, so $D = 0$

$\Rightarrow\ (n - 13)(n - 1) = 0 \Rightarrow n = 1$ or $n = 13$