Arithmetic Progressions

CHAPTER 5

TRY YOURSELF — SOLUTIONS

1. (i) Given, list of numbers is 0.8, 1.3, 1.8, 2.3, 2.8, ...
Here, we have, $a_2 - a_1 = 1.3 - 0.8 = 0.5$,
$a_3 - a_2 = 1.8 - 1.3 = 0.5$,
$a_4 - a_3 = 2.3 - 1.8 = 0.5$,
$a_5 - a_4 = 2.8 - 2.3 = 0.5$, ...
Since, $a_{k+1} - a_k$ is same for different values of k, so the given list of numbers forms an A.P.

(ii) Given list of numbers is 15, 1, –13, –27, –41, ...
Here, we have, $a_2 - a_1 = 1 - 15 = -14$,
$a_3 - a_2 = -13 - 1 = -14$,
$a_4 - a_3 = -27 - (-13) = -14$,
$a_5 - a_4 = -41 - (-27) = -14, \ldots$
Since, $a_{k+1} - a_k$ is same for different values of k, so the given list of numbers forms an A.P.

(iii) Given list of numbers is $\sqrt{2}, \sqrt{4}, \sqrt{8}, \sqrt{16}$, ...
which can be written as $\sqrt{2}, 2, 2\sqrt{2}, 4$, ...
Here, we have, $a_2 - a_1 = 2 - \sqrt{2}$,
$a_3 - a_2 = 2\sqrt{2} - 2$,
$a_4 - a_3 = 4 - 2\sqrt{2}, \ldots$
Here, $a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$
∴ The given list of numbers does not form an A.P.

(iv) Given list of numbers is $\frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \frac{1}{16}, \ldots$

Here, we have, $a_2 - a_1 = \frac{1}{8} - \frac{1}{4} = \frac{1-2}{8} = -\frac{1}{8}$,

$a_3 - a_2 = \frac{1}{12} - \frac{1}{8} = \frac{2-3}{24} = -\frac{1}{24}$,

$a_4 - a_3 = \frac{1}{16} - \frac{1}{12} = \frac{3-4}{48} = -\frac{1}{48}, \ldots$

Here, $a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$
∴ The given list of numbers does not form an A.P.

2. (i) Here, $a = 17$, $d = 2.5$
∴ First term, $a = a_1 = 17$,
Second term, $a_2 = 17 + 2.5 = 19.5$,
Third term, $a_3 = 19.5 + 2.5 = 22$,
Fourth term, $a_4 = 22 + 2.5 = 24.5$ and
Fifth term, $a_5 = 24.5 + 2.5 = 27$

(ii) Here, $a = \frac{1}{2}$, $d = -\frac{1}{4}$

∴ First term, $a = a_1 = \frac{1}{2}$

Second term, $a_2 = \frac{1}{2} + \left(-\frac{1}{4}\right) = \frac{2-1}{4} = \frac{1}{4}$,

Third term, $a_3 = \frac{1}{4} + \left(-\frac{1}{4}\right) = 0$,

Fourth term, $a_4 = 0 - \frac{1}{4} = -\frac{1}{4}$ and

Fifth term, $a_5 = -\frac{1}{4} + \left(-\frac{1}{4}\right) = -\frac{1}{2}$

(iii) Here, $a = -3$, $d = 0$
Since $d = 0$
∴ Each term of given A.P. will be same as the first term of A.P.

(iv) Here, $a = -5$, $d = 6$
∴ First term, $a = a_1 = -5$
Second term, $a_2 = -5 + 6 = 1$,
Third term, $a_3 = 1 + 6 = 7$,
Fourth term, $a_4 = 7 + 6 = 13$ and
Fifth term, $a_5 = 13 + 6 = 19$

3. Since $3k + 7$, $k + 19$ and $2k + 1$ are three consecutive terms of an A.P.
∴ $(k + 19) - (3k + 7) = (2k + 1) - (k + 19)$
⇒ $-2k + 12 = k - 18$ ⇒ $3k = 30$ ⇒ $k = 10$

4. (i) Given list of numbers is 9, 7, 5, ...
First term, $a = 9$
Common difference, $d = a_2 - a_1 = 7 - 9 = -2$
Fourth term, $a_4 = 5 + (-2) = 3$
and fifth term, $a_5 = 3 + (-2) = 1$

(ii) Given list of numbers is 18, 14, 10, 6, ...
First term, $a_1 = 18$
Common difference, $d = a_2 - a_1 = 14 - 18 = -4$
Fifth term, $a_5 = 6 + (-4) = 2$
and sixth term, $a_6 = 2 + (-4) = -2$

(iii) Given list of numbers is $\frac{1}{4}, \frac{7}{12}, \frac{11}{12}, \frac{15}{12}$, ...

First term, $a_1 = \frac{1}{4}$

Common difference, $d = a_2 - a_1 = \frac{7}{12} - \frac{1}{4} = \frac{7-3}{12} = \frac{4}{12} = \frac{1}{3}$

Fifth term, $a_5 = \frac{15}{12} + \frac{1}{3} = \frac{19}{12}$

Sixth term, $a_6 = \frac{19}{12} + \frac{1}{3} = \frac{23}{12}$

(iv) Given list of numbers is $(a - b)^2, (a^2 + b^2), (a + b)^2, \ldots$
First term, $a_1 = (a - b)^2$
Common difference, $d = a_2 - a_1 = (a^2 + b^2) - (a - b)^2$
$= a^2 + b^2 - (a^2 + b^2 - 2ab) = 2ab$
Fourth term, $a_4 = (a + b)^2 + (2ab)$
$= a^2 + b^2 + 2ab + 2ab = a^2 + b^2 + 4ab$
Fifth term, $a_5 = (a - b)^2 + 4(2ab) = a^2 + b^2 + 6ab$

5. (i) Let the first term be a and d be the common difference of the given A.P.

Given, $a_2 = 179 \Rightarrow a + d = 179$ ...(i)

Also, $a_3 = 176 \Rightarrow a + 2d = 176$ ...(ii)

Subtracting (i) from (ii), we get $d = -3$

From (i), $a = 179 + 3 = 182$

Also, $a_4 = a + 3d = 182 + 3(-3) = 182 - 9 = 173$

$\therefore$ Missing terms are 182 and 173.

(ii) Given, $a_1 = 8, a_2 = 15$

First term, $a = 8$,

common difference, $d = a_2 - a_1 = 15 - 8 = 7$

$\therefore$ $a_3 = a + 2d = 8 + 2(7) = 22$ and

$a_4 = a + 3d = 8 + 3(7) = 29$

$\therefore$ Missing terms are 22 and 29.

(iii) Let a be the first term and d be the common difference.

Given, $a_3 = 0.78 \Rightarrow a + 2d = 0.78$ ...(i)

Also, $a_4 = 1.01 \Rightarrow a + 3d = 1.01$ ...(ii)

Subtracting (i) from (ii), we get $d = 0.23$

From (i), $a + 2(0.23) = 0.78$

$\Rightarrow a = 0.78 - 0.46 = 0.32$

$\therefore$ $a_2 = a + d = 0.32 + 0.23 = 0.55$

$\therefore$ Missing terms are 0.32 and 0.55.

(iv) Let a be the first term and d be the common difference.

Given, $a_2 = -8 \Rightarrow a + d = -8$ ... (i)

Also, $a_4 = -28 \Rightarrow a + 3d = -28$ ... (ii)

Subtracting (i) from (ii), we get

$2d = -28 + 8 \Rightarrow 2d = -20 \Rightarrow d = -10$

From (i), $a + (-10) = -8 \Rightarrow a = 2$

Now, $a_3 = a + 2d = 2 + 2(-10) = 2 - 20 = -18$

$\therefore$ Missing terms are 2 and -18.

6. Given, first term, $a = 3$

Common difference, $d = 5$

Now, $a_{17} = a + 16d$ [$\because a_n = a + (n-1)d$]

$= 3 + 16(5) = 83$

And $a_{25} = a + 24d$

$= 3 + 24(5) = 3 + 120 = 123$

7. Given A.P. is 8, 6.5, 5, 3.5, ..., -55

Here $a = 8$, $d = 6.5 - 8 = -1.5$

Let the number of terms be n.

$\therefore$ $a_n = -55$ (last term) $\Rightarrow a + (n-1)d = -55$

$\Rightarrow 8 + (n-1)(-1.5) = -55 \Rightarrow (n-1)(-1.5) = -55 - 8$

$\Rightarrow (n-1)(-1.5) = -63$

$\Rightarrow (n-1) = \frac{63}{1.5} = 42 \Rightarrow n = 42 + 1 = 43$

$\therefore$ The number of terms in the given A.P. is 43.

8. All natural numbers between 100 and 500, which are divisible by 8 are

104, 112, 120, 128, ..., 496, which is an A.P.

Here, first term $a = 104$,

common difference, $d = 112 - 104 = 8$

Now, $a_n = a + (n-1)d$

$\Rightarrow 496 = 104 + (n-1)8$

$\Rightarrow 496 - 104 = (n-1)8 \Rightarrow 392 = (n-1)8$

$\Rightarrow (n-1) = \frac{392}{8} \Rightarrow n = 49 + 1 = 50$

$\therefore$ Total numbers are 50.

9. Given, $a_{21} = 46$

$\Rightarrow a + (21-1)d = 46$ [$\because a_n = a + (n-1)d$]

$\Rightarrow a + 20d = 46$ ... (i)

Also, $a_{36} = 70 \Rightarrow a + (36-1)d = 70$

$\Rightarrow a + 35d = 70$ .. (ii)

Subtracting (i) from (ii), we get

$$15d = 24 \Rightarrow d = \frac{24}{15} = \frac{8}{5}$$

From (i), $a = 46 - 20\left(\frac{8}{5}\right) = 14$

$\therefore$ $a_{28} = a + (28-1)d = 14 + 27\left(\frac{8}{5}\right) = \frac{70+216}{5} = \frac{286}{5}$

10. Given, first A.P. is 5, 8, 11, ...

Here, first term $a = 5$

common difference, $d = 8 - 5 = 3$

Now, tenth term, $a_{10} = a + (10-1)d = 5 + 9(3) = 32$ ... (i)

Also, second A.P. is 2, 8, 14, ...

Here, first term $a = 2$, common difference, $d = 8 - 2 = 6$

So, tenth term, $b_{10} = 2 + 9(6) = 56$ ... (ii)

$\therefore$ $\frac{a_{10}}{b_{10}} = \frac{32}{56} = \frac{4}{7}$ [Using (i) and (ii)]

11. Given, $\frac{a_{18}}{a_{11}} = \frac{3}{2} \Rightarrow \frac{a+17d}{a+10d} = \frac{3}{2}$

$\Rightarrow 2a + 34d = 3a + 30d \Rightarrow a = 4d$ ... (i)

Now, $\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$ [Using (i)]

$= \frac{24d}{8d} = \frac{3}{1}$

$\therefore$ Required ratio = 3 : 1

12. Income (in ₹) of Rajat for some years is 100000, 105000, 110000..., 150000, which forms an A.P.

Let there be n terms in the A.P.

Here, $a = 100000$, $d = 105000 - 100000 = 5000$

and $a_n = 150000$

We know that, $a_n = a + (n-1)d$

$\therefore$ $150000 = 100000 + (n-1)\,5000$

$\Rightarrow 5000\,(n-1) = 150000 - 100000 \Rightarrow 5000(n-1) = 50000$

$\Rightarrow n - 1 = \frac{50000}{5000} = 10 \Rightarrow n = 10 + 1 = 11$

Hence, in 11^{th} year his income will reach ₹150000.

13. Given A.P. is 7, 10.5, 14, ..., 213.5

Here, last term, $l = 213.5$

Common difference, $d = 10.5 - 7 = 3.5$

$\therefore$ 19^{th} term from the end $= l - 18d$

[$\because n^{th}$ term from the end $= l - (n-1)d$]

$= 213.5 - 18(3.5) = 213.5 - 63 = 150.5$

14. Given, A.P. is 17, 14, 11, ..., –40
On reversing the given A.P., new A.P. is
–40, ..., 11, 14, 17
Here, first term, $a = -40$
Common difference, $d = 3$
Now, 6th term of new A.P. $= a_6 = a + 5d$
$= -40 + 5(3) = -40 + 15 = -25$
Hence, 6th term from the end of the given A.P. is –25.

15. Given, $a = 10$, $d = 5$, $n = 100$
$\therefore \quad a_{100} = a + (100 - 1)d \qquad [\because a_n = a + (n-1)d]$
$= 10 + 99(5) = 505$
$\therefore \quad l = 505$
Also, 50th term from the end $= l - (n - 1)d$
$= 505 - (50 - 1)5 = 505 - (49)5$
$= 505 - 245 = 260$

16. Given A.P. is 771, 777, ..., 915
Here, $a = 771$, $d = 777 - 771 = 6$
Let there be n terms in the given A.P.
Then, $a_n = 915 \Rightarrow a + (n - 1)d = 915$
$\Rightarrow \quad 771 + (n - 1)6 = 915$
$\Rightarrow \quad (n - 1)6 = 144 \Rightarrow (n - 1) = 24 \Rightarrow n = 25$

Here, n is odd, so $\left(\frac{n+1}{2}\right)^{th}$ *i.e.*, $\left(\frac{25+1}{2}\right)^{th} = 13^{th}$ term
is the middle term and is given by
$a_{13} = a + 12d = 771 + 12(6) = 843$

17. Given, A.P. is 4, 9, 14, ..., 254
Here, $a = 4$, $d = 9 - 4 = 5$
Let there be n terms in the given A.P.
Then, $a_n = 254 \Rightarrow a + (n - 1)d = 254$
$\Rightarrow \quad 4 + (n - 1)5 = 254 \Rightarrow (n - 1)5 = 250$
$\Rightarrow \quad (n - 1) = 50 \Rightarrow n = 51$, which is odd.

So, $\left(\frac{n+1}{2}\right)^{th}$ *i.e.*, $\left(\frac{51+1}{2}\right)^{th} = 26^{th}$ term
is the middle term and is given by
$a_{26} = a + 25d = 4 + 25(5) = 129$

18. Given, first term, $a = 5$, common difference, $d = 3$ and last term, $l = 80$
Let there be n terms, then $a_n = l = 80$
$\Rightarrow \quad a + (n - 1)d = 80 \Rightarrow 5 + (n - 1)3 = 80$
$\Rightarrow \quad (n - 1)3 = 75 \Rightarrow (n - 1) = 25 \Rightarrow n = 26$

Clearly, n is even, so $\left(\frac{n}{2}\right)^{th}$ *i.e.*, 13th and $\left(\frac{n}{2}+1\right)^{th}$
i.e., 14th terms are middle terms and are given by
$a_{13} = a + 12d = 5 + 12(3) = 41$
$a_{14} = a + 13d = 5 + 13(3) = 44$

19. The natural numbers which leave remainder 2 when divided by 5 lying between 100 and 200 are 102, 107, 112, 117, 122, ..., 197.
Which is an A.P.
Here, first term, $a = 102$ and common difference, $d = 107 - 102 = 5$
Let n be the number of terms of the A.P.
$\therefore \quad a_n = 197 \Rightarrow a + (n - 1)d = 197$
$\Rightarrow \quad 102 + (n - 1)5 = 197 \Rightarrow (n - 1)5 = 95$
$\Rightarrow \quad (n - 1) = 19 \Rightarrow n = 20$

Now, $S_{20} = \frac{20}{2}[2(102) + (20 - 1)5]$
$= 10[204 + 95] = 10[299] = 2990$
Thus, the required sum is 2990.

20. Given, $a = 7$ and $S_{20} = -240$
$\Rightarrow \quad \frac{20}{2}(2 \times 7 + 19d) = -240 \qquad \left[\because S_n = \frac{n}{2}(2a + (n-1)d\right]$
$\Rightarrow \quad 10(14 + 19d) = -240$
$\Rightarrow \quad 19d = -24 - 14 \Rightarrow d = -2$
$\therefore \quad a_{24} = a + 23d = 7 + 23(-2) = 7 - 46 = -39$

21. Multiples of 9 between 400 and 800 are 405, 414, 423,..., 792
Clearly, it forms an A.P. with $a = 405$, $d = 9$ and last term, $l = 792$
$\Rightarrow \quad a + (n - 1)d = 792 \Rightarrow 405 + 9n - 9 = 792$
$\Rightarrow \quad 9n = 792 - 396 = 396 \Rightarrow n = 44$

Thus, $S_{44} = \frac{44}{2}(405 + 792) \qquad \left[\because S_n = \frac{n}{2}(a + l)\right]$
$= 22 \times 1197 = 26334.$

22. Let a be the first term and d be the common difference of the required A.P.
$\because \quad S_{10} = \frac{10}{2}[2a + 9d] \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d\right]$
$\Rightarrow \quad 725 = 5(2a + 9d) \Rightarrow 145 = 2a + 9d$ ... (i)
Now, sum of next 10 terms $= S_{20} - S_{10}$
$\Rightarrow \quad 1225 = S_{20} - S_{10}$
$\Rightarrow \quad 1225 = \left[\frac{20}{2}(2a + 19d)\right] - 725$
$\Rightarrow \quad 1950 = 10(2a + 19d) \Rightarrow 2a + 19d = 195$ ... (ii)
Subtracting (i) from (ii), we get
$10d = 50 \Rightarrow d = 5$
From (i), $145 = 2a + 9(5)$
$\Rightarrow \quad 100 = 2a \Rightarrow a = 50$
$\therefore$ The A.P. is 50, 55, 60, ...

23. Given A.P. is 7, 4, 1, –2, ...
Here, $a = 7$, $d = 4 - 7 = -3$
Let there be n terms.

Since, $S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow \quad -333 = \frac{n}{2}[2(7) + (n - 1)(-3)]$ [Given, $S_n = -333$]
$\Rightarrow \quad -666 = n(14 - 3n + 3) \Rightarrow -666 = n(17 - 3n)$
$\Rightarrow \quad 3n^2 - 17n - 666 = 0$

$$\Rightarrow \quad n = \frac{17 \pm \sqrt{(17)^2 - 4(3)(-666)}}{2(3)}$$

$$\Rightarrow \quad n = \frac{17 \pm \sqrt{289 + 7992}}{6} = \frac{17 \pm \sqrt{8281}}{6} = \frac{17 \pm 91}{6}$$

$$\Rightarrow \quad n = 18, \frac{-74}{6}$$

As, n can't be negative.
$\therefore$ Required number of terms is 18.

24. Let the three terms of an A.P. be $(a-d)$, a and $(a+d)$.

$\because$ Sum of these terms is 36.

$\Rightarrow$ $3a = 36 \Rightarrow a = 12$

Also, product of these three terms is 960.

$\Rightarrow$ $(a+d)\,a\,(a-d) = 960 \Rightarrow (12+d)\,12(12-d) = 960$

$\Rightarrow$ $(12+d)(12-d) = 80$

$\Rightarrow$ $144 - d^2 = 80 \Rightarrow d^2 = 64 \Rightarrow d = \pm 8$

Taking $d \pm 8$, we get the terms as 4, 12 and 20.

25. Let the four parts be
$(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

The sum of these four parts is 124.

$\Rightarrow$ $4a = 124 \Rightarrow a = 31$

Also, $(a-3d)(a+3d) = (a-d)(a+d) - 128$ (Given)

$\Rightarrow$ $a^2 - 9d^2 = a^2 - d^2 - 128$

$\Rightarrow$ $8d^2 = 128 \Rightarrow d = \pm 4$

As, $a = 31$, taking $d = 4$, the four parts are 19, 27, 35 and 43.

Note : If d is taken as –4, then the same four numbers are obtained, but in decreasing order.