Triangles

CHAPTER 6

NCERT FOCUS

SOLUTIONS

EXERCISE - 6.1

1. (i) All circles are similar.
(ii) All squares are similar.
(iii) All equilateral triangles are similar.
(iv) Two polygons of the same number of sides are similar, if
(a) their corresponding angles are equal and
(b) their corresponding sides are proportional.

2. (i) (a) Any two circles are similar figures.
(b) Any two squares are similar figures.
(ii) (a) A circle and a triangle are non-similar figures.
(b) An isosceles triangle and a scalene triangle are non-similar figures.

3. On observing the given figures, we find that their corresponding sides are proportional but their corresponding angles are not equal.
$\therefore$ The given figures are not similar.

EXERCISE - 6.2

1. (i) Since $DE \parallel BC$ [Given]
$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Since, $AD = 1.5$ cm, $DB = 3$ cm and $AE = 1$ cm

$$\therefore \quad \frac{1.5 \text{ cm}}{3 \text{ cm}} = \frac{1 \text{ cm}}{EC}$$

$$\Rightarrow \quad EC \times 1.5 = 1 \times 3$$

$$\Rightarrow \quad EC = \frac{1 \times 3}{1.5} = \frac{1 \times 3 \times 10}{15} \Rightarrow EC = 2 \text{ cm}$$

(ii) In ΔABC, $DE \parallel BC$
$\therefore$ Using the basic proportionality theorem, we have

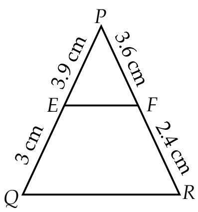
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow AD \times 5.4 = 1.8 \times 7.2$$

$$\Rightarrow \quad AD = \frac{1.8 \times 7.2}{5.4} = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54} = \frac{24}{10} = 2.4$$

$\therefore$ $AD = 2.4$ cm

2. (i) We have, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm



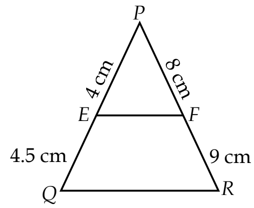
$$\therefore \quad \frac{PE}{EQ} = \frac{3.9 \text{ cm}}{3 \text{ cm}} = \frac{1.3}{1}$$

$$\text{And} \quad \frac{PF}{FR} = \frac{3.6 \text{ cm}}{2.4 \text{ cm}} = \frac{1.5}{1}$$

$$\because \quad \frac{1.3}{1} \neq \frac{1.5}{1} \quad \therefore \quad \frac{PE}{EQ} \neq \frac{PF}{FR}$$

$\Rightarrow$ EF is not parallel to QR.

(ii) We have, $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm



$$\therefore \quad \frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$$

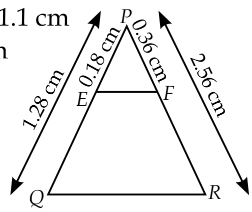
$$\text{And} \quad \frac{PF}{FR} = \frac{8}{9}$$

$$\text{Since,} \quad \frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow$ EF is parallel to QR.

(iii) We have, $PE = 0.18$ cm, $PQ = 1.28$ cm, $PF = 0.36$ cm and $PR = 2.56$ cm

$\therefore$ $EQ = PQ - PE = 1.28 - 0.18 = 1.1$ cm
$FR = PR - PF = 2.56 - 0.36 = 2.2$ cm



$$\therefore \quad \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{18}{110} = \frac{9}{55}$$

$$\text{And} \quad \frac{PF}{FR} = \frac{0.36}{2.2} = \frac{36}{220} = \frac{9}{55}$$

$$\text{Since,} \quad \frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow EF \text{ is parallel to } QR.$$

3. In ΔABC, $LM \parallel CB$ [Given]
$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AM}{MB} = \frac{AL}{LC} \Rightarrow \frac{MB}{AM} + 1 = \frac{LC}{AL} + 1$$

$$\Rightarrow \quad \frac{MB + AM}{AM} = \frac{LC + AL}{AL} \Rightarrow \frac{AB}{AM} = \frac{AC}{AL}$$

$$\Rightarrow \quad \frac{AM}{AB} = \frac{AL}{AC} \quad \text{...(i)}$$

Similarly, in ΔACD, $LN \parallel CD$
$\therefore$ Using the basic proportionality theorem, we have

$$\frac{AL}{AC} = \frac{AN}{AD} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{AM}{AB} = \frac{AL}{AC} = \frac{AN}{AD} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

4. In ΔABC, $DE \parallel AC$ [Given]

$$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC} \quad \text{[By basic proportionality theorem] ...(i)}$$

In ΔABE, $DF \parallel AE$ [Given]

$$\therefore \quad \frac{BD}{DA} = \frac{BF}{FE} \quad \text{[By basic proportionality theorem] ...(ii)}$$

From (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{BF}{FE} = \frac{BE}{EC}$$

5. In ΔPQO, $DE \parallel OQ$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad ...(i)$$

Similarly, in ΔPOR, $DF \parallel OR$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{PD}{DO} = \frac{PF}{FR} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PD}{DO} = \frac{PF}{FR} \Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in ΔPQR, E and F are two distinct points on PQ and PR respectively and $\frac{PE}{EQ} = \frac{PF}{FR}$ *i.e.*, E and F divides the two sides PQ and PR in the same ratio.

$\therefore$ By converse of basic proportionality theorem, $EF \parallel QR$.

6. In ΔPQR, O is a point and OP, OQ and OR are joined. We have points A, B, and C on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$.

Now, in ΔOPQ, $AB \parallel PQ$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad ...(i)$$

Again, in ΔOPR, $AC \parallel PR$ [Given]

$\therefore$ Using the basic proportionality theorem, we have

$$\frac{OA}{AP} = \frac{OC}{CR} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OA}{AP} = \frac{OC}{CR} \Rightarrow \frac{OB}{BQ} = \frac{OC}{CR}$$

Now, in ΔOQR, B is a point on OQ, C is a point on OR and $\frac{OB}{BQ} = \frac{OC}{CR}$

i.e., B and C divide the sides OQ and OR in the same ratio

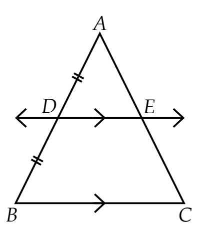
$\therefore$ $BC \parallel QR$

[By converse of basic proportionality theorem]

7. Given, ΔABC, in which D is the mid-point of AB and E is a point on AC such that $DE \parallel BC$.

$\therefore$ Using basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC} \quad ...(i)$$



But D is the mid-point of AB

$\therefore$ $AD = DB$

$$\Rightarrow \frac{AD}{DB} = 1 \quad ...(ii)$$

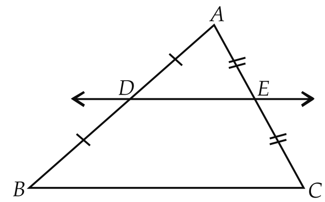
From (i) and (ii), we get

$$1 = \frac{AE}{EC} \Rightarrow EC = AE$$

$\Rightarrow$ E is the mid-point of AC. Hence, it is proved that a line through the mid-point of one side of a triangle parallel to another side bisects the third side.

8. We have ΔABC, in which D and E are the mid-points of sides AB and AC respectively.

$\therefore$ $AD = DB$ and $AE = EC$



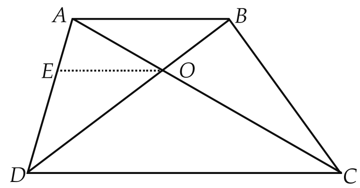
$$\Rightarrow \frac{AD}{DB} = 1 \text{ and } \frac{AE}{EC} = 1$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow$ $DE \parallel BC$

[By converse of basic proportionality theorem]

9. We have, a trapezium $ABCD$ such that $AB \parallel DC$. The diagonals AC and BD intersect each other at O.



Let us draw OE parallel to either AB or DC.

In ΔADC, $OE \parallel DC$ [By construction]

$\therefore$ Using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{AO}{CO} \quad ...(i)$$

In ΔABD, $OE \parallel AB$ [By construction]

$\therefore$ Using basic proportionality theorem, we get

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{BO}{DO} = \frac{AO}{CO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

Note : Remember this as a result.

10. It is given that $\frac{AO}{BO} = \frac{CO}{DO} \Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$ ...(i)

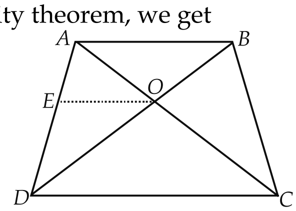
Through O, draw $OE \parallel BA$

In ΔADB, $OE \parallel AB$ [By construction]

$\therefore$ Using basic proportionality theorem, we get

$$\frac{DE}{EA} = \frac{DO}{BO}$$

$$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO} \quad ...(ii)$$



From (i) and (ii), we have

$$\frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$$

i.e., the points O and E on the sides AC and AD (of ΔADC) respectively are in the same ratio.

$\therefore$ Using basic proportionality theorem, we get

$OE \parallel DC$

Also, $OE \parallel AB$ [By construction]

$\Rightarrow$ $AB \parallel DC$

$\Rightarrow$ $ABCD$ is a trapezium.

EXERCISE - 6.3

1. (i) In ΔABC and ΔPQR,

$\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

$\therefore$ The corresponding angles are equal.

$\therefore$ $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

(ii) In ΔABC and ΔQRP,

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2},$$

$$\frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2} \Rightarrow \frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

$\therefore$ $\Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii) In ΔPML and ΔDEF,

$\because$ $\frac{PM}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{ML}{EF} = \frac{2.7}{5} = \frac{27}{50}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow$ $\frac{PM}{DE} \neq \frac{ML}{EF} \neq \frac{LP}{DF}$ $\Rightarrow$ Triangles are not similar.

(iv) In ΔMNL and ΔQPR,

$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}, \frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$ and

$\angle NML = \angle PQR = 70°$

$\therefore$ $\Delta MNL \sim \Delta QPR$ [By SAS similarity criterion]

(v) In ΔABC and ΔFDE,

$\angle A = \angle F = 80°$

Here, $\frac{AB}{AC}$ and $\frac{FD}{DE}$ are unknown.

$\therefore$ The triangles cannot be said similar.

(vi) In ΔDEF and ΔPQR, $\angle D = \angle P = 70°$

[$\because \angle P = 180° - (80° + 30°) = 180° - 110° = 70°$]

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$ [$\because \angle F = 180° - (80° + 70°) = 30°$]

$\therefore$ $\Delta DEF \sim \Delta PQR$ [By AAA similarity criterion]

2. We have, $\angle BOC = 125°$ and $\angle CDO = 70°$

Since, $\angle DOC + \angle BOC = 180°$ [Linear pair]

$\Rightarrow$ $\angle DOC = 180° - 125° = 55°$ ...(i)

Using the angle sum property in ΔODC, we get

$\angle DOC + \angle ODC + \angle DCO = 180°$

$\Rightarrow$ $55° + 70° + \angle DCO = 180°$

$\Rightarrow$ $\angle DCO = 180° - 55° - 70° = 55°$

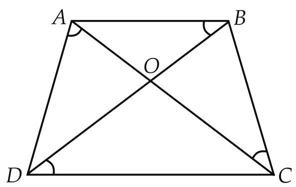
Also, $\angle OAB = \angle DCO = 55°$ ...(ii)

[Corresponding angles of similar triangles]

Thus, from (i) and (ii)

$\angle DOC = 55°, \angle DCO = 55°$ and $\angle OAB = 55°$.

3. We have a trapezium $ABCD$ in which $AB \parallel DC$. The diagonals AC and BD intersect at O.



In ΔOAB and ΔOCD,

$\because$ $AB \parallel DC$ and AC and BD are transversals.

$\therefore$ $\angle OBA = \angle ODC$ [Alternate angles]

and $\angle OAB = \angle OCD$ [Alternate angles]

$\therefore$ $\Delta OAB \sim \Delta OCD$ [By AA similarity criterion]

So, $\frac{OB}{OD} = \frac{OA}{OC}$ [Ratios of corresponding sides of the similar triangles]

4. In ΔPQR, $\angle 1 = \angle 2$ [Given]

$\therefore$ $PR = QP$ ...(i)

[Sides opposite to equal angles are equal]

Also, $\frac{QR}{QS} = \frac{QT}{PR}$ [Given] ...(ii)

From (i) and (ii), we get

$\frac{QR}{QS} = \frac{QT}{QP} \Rightarrow \frac{QS}{QR} = \frac{QP}{QT}$ [By taking reciprocals] ...(iii)

Now, in ΔPQS and ΔTQR,

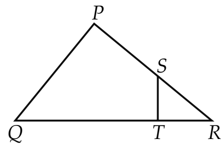
$\frac{QS}{QR} = \frac{QP}{QT}$ [From (iii)]

and $\angle SQP = \angle RQT = \angle 1$

$\therefore$ $\Delta PQS \sim \Delta TQR$ [By SAS similarity criterion]

5. In ΔPQR,

T is a point on QR and S is a point on PR such that $\angle RTS = \angle P$.



Now, in ΔRPQ and ΔRTS,

$\angle RPQ = \angle RTS$ [Given]

$\angle PRQ = \angle TRS$ [Common]

$\therefore$ $\Delta RPQ \sim \Delta RTS$ [By AA similarity criterion]

6. We have, $\Delta ABE \cong \Delta ACD$

$\therefore$ Their corresponding parts are equal,

i.e., $AB = AC$, $AE = AD$

$\Rightarrow$ $\frac{AB}{AC} = 1$ and $\frac{AE}{AD} = 1$ $\therefore$ $\frac{AB}{AC} = \frac{AE}{AD} \Rightarrow \frac{AB}{AE} = \frac{AC}{AD}$

$\Rightarrow$ $\frac{AB}{AD} = \frac{AC}{AE}$ [$\because AE = AD$] .....(i)

Now in ΔADE and ΔABC, $\frac{AB}{AD} = \frac{AC}{AE}$ [From (i)]

and $\angle DAE = \angle BAC$ [common]

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

7. We have a ΔABC in which altitude AD and CE intersect each other at P.

$\Rightarrow$ $\angle D = \angle E = 90°$ ...(1)

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ [From (1)]

$\angle EPA = \angle DPC$ [Vertically opposite angles]

$\therefore$ $\Delta AEP \sim \Delta CDP$ [By AA similarity criterion]

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ [From (1)]

Also, $\angle ABD = \angle CBE$ [Common]

$\therefore$ $\Delta ABD \sim \Delta CBE$ [By AA similarity criterion]

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ [From (1)]

Also, $\angle EAP = \angle DAB$ [Common]

$\therefore$ $\Delta AEP \sim \Delta ADB$ [By AA similarity criterion]

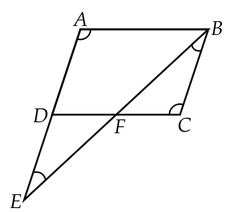
(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ [From (1)]

Also, $\angle DCP = \angle ECB$ [Common]

$\therefore$ $\Delta PDC \sim \Delta BEC$ [By AA similarity criterion]

8. We have a parallelogram $ABCD$ in which AD is produced to E and BE is joined such that BE intersects CD at F.



Now, in ΔABE and ΔCFB,

$\angle BAE = \angle FCB$ [Opposite angles of a parallelogram]

$\angle AEB = \angle CBF$

[Alternate angles, as $AE \parallel BC$ and BE is a transversal.]

$\therefore \quad \Delta ABE \sim \Delta CFB$ [By AA similarity criterion]

9. We have ΔABC, right angled at B and ΔAMP, right angled at M.

$\therefore \quad \angle B = \angle M = 90°$ ...(1)

(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ [From (1)]

and $\angle BAC = \angle MAP$ [Common]

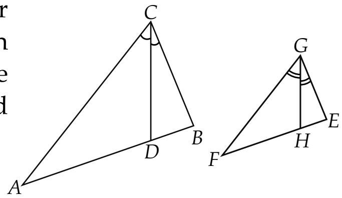
$\therefore \quad \Delta ABC \sim \Delta AMP$ [By AA similarity criterion]

(ii) $\because \quad \Delta ABC \sim \Delta AMP$ [As proved above]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CA}{PA} = \frac{BC}{MP}$

10. We have, two similar ΔABC and ΔFEG such that CD and GH are the bisectors of $\angle ACB$ and $\angle FGE$ respectively.



(i) In ΔACD and ΔFGH,

$\angle A = \angle F$ [$\because \Delta ABC \sim \Delta FEG$] ...(1)

Since $\Delta ABC \sim \Delta FEG$

$\therefore \quad \angle C = \angle G \Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \quad \angle ACD = \angle FGH$ ...(2)

From (1) and (2), we have

$\Delta ACD \sim \Delta FGH$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta FEG$] ...(1)

Again, $\Delta ABC \sim \Delta FEG \Rightarrow \angle ACB = \angle FGE$

$\therefore \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \quad \angle DCB = \angle HGE$ ...(2)

From (1) and (2), we have

$\Delta DCB \sim \Delta HGE$ [By AA similarity criterion]

(iii) In ΔDCA and ΔHGF,

$\because \quad \Delta ABC \sim \Delta FEG \Rightarrow \angle CAB = \angle GFE$

$\Rightarrow \angle CAD = \angle GFH \Rightarrow \angle DAC = \angle HFG$ ...(1)

Also, $\Delta ABC \sim \Delta FEG \Rightarrow \angle ACB = \angle FGE$

$\therefore \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \quad \angle DCA = \angle HGF$ ...(2)

From (1) and (2), we have

$\Delta DCA \sim \Delta HGF$ [By AA similarity criterion]

11. We have an isosceles ΔABC in which $AB = AC$.

In ΔABD and ΔECF,

$\angle ACB = \angle ABC$

$\Rightarrow \quad \angle ECF = \angle ABD$ ...(i)

Again, $AD \perp BC$ and $EF \perp AC$

$\Rightarrow \quad \angle ADB = \angle EFC = 90°$ ...(ii)

From (i) and (ii), we have

$\Delta ABD \sim \Delta ECF$ [By AA similarity criterion]

12. We have ΔABC and ΔPQR in which AD and PM are medians corresponding to sides BC and QR respectively such that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{(1/2)BC}{(1/2)QR} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \quad \Delta ABD \sim \Delta PQM$ [By SSS similarity criterion]

$\therefore$ Their corresponding angles are equal.

$\Rightarrow \quad \angle ABD = \angle PQM \Rightarrow \angle ABC = \angle PQR$

Now, in ΔABC and ΔPQR, $\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

Also, $\angle ABC = \angle PQR$

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

13. We have a ΔABC and a point D on its side BC such that $\angle ADC = \angle BAC$.

In ΔBAC and ΔADC,

$\angle BAC = \angle ADC$ [Given]

and $\angle BCA = \angle ACD$ [Common]

$\therefore \quad \Delta BAC \sim \Delta ADC$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{CA}{CD} = \frac{CB}{CA} \Rightarrow CA \times CA = CB \times CD$

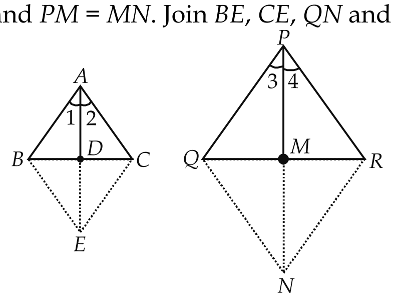
$\Rightarrow \quad CA^2 = CB \times CD$

14. Given : ΔABC and ΔPQR in which AD and PM are medians.

Also, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ ...(i)

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E and PM to N such that $AD = DE$ and $PM = MN$. Join BE, CE, QN and RN.



Proof : Quadrilaterals $ABEC$ and $PQNR$ are parallelograms, since their diagonals bisect each other at point D and M respectively.

$\Rightarrow \quad BE = AC$ and $QN = PR$

$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ [From (i)]

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(ii)

From (i), $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AE}{PN}$ ...(iii)

From (ii) and (iii), we have

$$\frac{AB}{PQ}=\frac{BE}{QN}=\frac{AE}{PN}$$

$\Rightarrow$ $\Delta ABE \sim \Delta PQN$ [By SSS similarity criterion]

$\Rightarrow$ $\angle 1 = \angle 3$ ...(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \Rightarrow \angle 2 = \angle 4$ ...(v)

From (iv) and (v), $\angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow$ $\angle A = \angle P$ ...(vi)

Now, in ΔABC and ΔPQR, we have

$\frac{AB}{PQ}=\frac{AC}{PR}$ [From (i)]

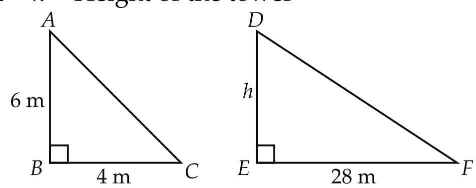
and $\angle A = \angle P$ [From (vi)]

$\therefore$ $\Delta ABC \sim \Delta PQR$ [By SAS similarity criterion]

15. Let $AB = 6$ m be the pole and $BC = 4$ m be its shadow (in right ΔABC), whereas DE and EF denote the tower and its shadow respectively.

EF = Length of the shadow of the tower = 28 m

Let $DE = h$ = Height of the tower



In ΔABC and ΔDEF, we have $\angle B = \angle E = 90°$

$\angle A = \angle D$

[$\because$ Angular elevation of the sun at the same time is equal]

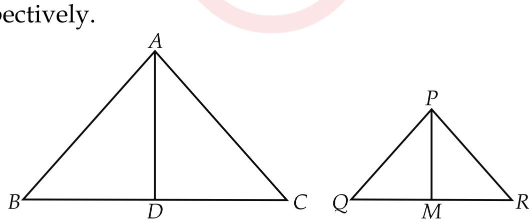
$\therefore$ $\Delta ABC \sim \Delta DEF$ [By AA similarity criterion]

$\therefore$ Their sides are proportional *i.e.*, $\frac{AB}{DE}=\frac{BC}{EF}$

$\Rightarrow$ $\frac{6}{h}=\frac{4}{28} \Rightarrow h = \frac{6\times 28}{4} = 42$ m

Thus, the required height of the tower is 42 m.

16. We have $\Delta ABC \sim \Delta PQR$ such that AD and PM are the medians corresponding to the sides BC and QR respectively.



$\because$ $\Delta ABC \sim \Delta PQR$

$\Rightarrow$ The corresponding sides of similar triangles are proportional.

$\therefore$ $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{CA}{RP}$ ...(i)

$\because$ Corresponding angles are also equal in two similar triangles.

$\therefore$ $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ ...(ii)

Since, AD and PM are medians.

$\therefore$ $BC = 2BD$ and $QR = 2QM$

$\therefore$ From (i), $\frac{AB}{PQ}=\frac{2BD}{2QM}=\frac{BD}{QM}$ ...(iii)

And $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM$ ...(iv)

From (iii) and (iv), we have

$\Delta ABD \sim \Delta PQM$ [By SAS similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow$ $\frac{AB}{PQ}=\frac{AD}{PM}$.

EXERCISE - 6.6

1. We have, ΔPQR in which PS is the bisector of $\angle QPR$.

$\therefore$ $\angle QPS = \angle RPS$

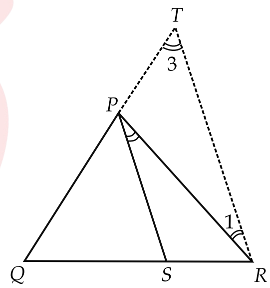
Let us draw $RT \parallel PS$ to meet QP produced at T, such that

$\angle 1 = \angle RPS$ [Alternate angles]

Also, $\angle 3 = \angle QPS$

[Corresponding angles]

But $\angle RPS = \angle QPS$ [Given]



$\therefore$ $\angle 1 = \angle 3$

$\Rightarrow$ $PT = PR$ [Sides opposite to equal angles are equal]

Now, in ΔQRT, $PS \parallel RT$ [By construction]

$\therefore$ Using the basic proportionality theorem, we have

$\frac{QS}{SR}=\frac{PQ}{PT} \Rightarrow \frac{QS}{SR}=\frac{PQ}{PR}$ [$\because PT = PR$]

2. We have AC as the hypotenuse of ΔABC.

Also, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$.

$\Rightarrow$ $BMDN$ is a rectangle.

$\therefore$ $BM = ND$ [Opposite sides of a rectangle]

(i) In ΔBMD and ΔDMC,

$\angle DMB = 90° = \angle DMC$ ...(1)

$\because$ $BD \perp AC$ [Given]

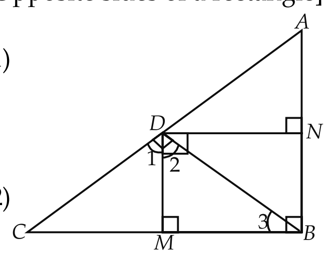
$\therefore$ $\angle 1 + \angle 2 = 90°$

In ΔBDM, $\angle 3 + \angle 2 = 90°$

$\Rightarrow$ $\angle 1 = \angle 3$ ...(2)

$\therefore$ From (1) and (2),

$\Delta BMD \sim \Delta DMC$



[By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow$ $\frac{BM}{DM}=\frac{MD}{MC} \Rightarrow \frac{DN}{DM}=\frac{DM}{MC}$ [$\therefore DN = BM$]

$\Rightarrow$ $DN \times MC = DM \times DM$

$\Rightarrow$ $DN \times MC = DM^2$ or $DM^2 = DN \times MC$

(ii) In ΔBND and ΔDNA, we have

$\angle BND = \angle DNA$ [Each equals 90°]

$\angle DBN = \angle ADN$ [Similarly, as proved in part (i)]

$\therefore$ $\Delta BND \sim \Delta DNA$ [By AA similarity criterion]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow$ $\frac{BN}{DN}=\frac{ND}{NA} \Rightarrow \frac{DM}{DN}=\frac{DN}{NA}$

[∵ BN and DM are opposite sides of a rectangle]

$\Rightarrow \quad DM \times NA = DN \times DN$

$\Rightarrow \quad DM \times AN = DN^2$ or $DN^2 = DM \times AN$

3. We have two chords AB and CD of a circle. AB and CD intersect at P.

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [Vertically opposite angles]

$\angle CAP = \angle BDP$ [Angles in the same segment]

$\therefore \quad \Delta APC \sim \Delta DPB$ [By AA similarity criterion]

(ii) Since, $\Delta APC \sim \Delta DPB$ [As proved above]

$\therefore$ Their corresponding sides are proportional,

$\Rightarrow \quad \frac{AP}{DP} = \frac{CP}{BP} \quad \Rightarrow \quad AP \cdot BP = CP \cdot DP$

4. We have two chords AB and CD, when produced meet outside the circle at P.

(i) Since, in a cyclic quadrilateral, the exterior angle is equal to the interior opposite angle.

$\therefore \quad \angle PAC = \angle PDB$

and $\angle PCA = \angle PBD$

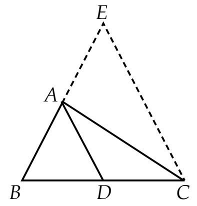
$\therefore \quad \Delta PAC \sim \Delta PDB$ [By AA similarity criterion]

(ii) Since, $\Delta PAC \sim \Delta PDB$ [As proved above]

$\therefore$ Their corresponding sides are proportional.

$\Rightarrow \quad \frac{PA}{PD} = \frac{PC}{PB} \quad \Rightarrow \quad PA \cdot PB = PC \cdot PD$

5. Let us produce BA to E such that $AE = AC$. Join EC.



Since, $\frac{BD}{CD} = \frac{AB}{AC}$ [Given]

But $AC = AE$ [By construction]

$\therefore \quad \frac{BD}{CD} = \frac{AB}{AE}$

$\therefore \quad AD \parallel CE$

[By the converse of the basic proportionality theorem]

$\Rightarrow \quad \angle BAD = \angle AEC$ [Corresponding angles] ...(1)

Also, $\angle CAD = \angle ACE$ [Alternate angles] ...(2)

Since, $AC = AE$

$\Rightarrow \quad \angle AEC = \angle ACE$ ...(3)

[∵ Angles opposite to equal sides are equal]

From (1) and (3), we have

$\angle BAD = \angle ACE$ ...(4)

From (2) and (4), we have

$\angle BAD = \angle CAD$

$\Rightarrow \quad AD$ is bisector of $\angle BAC$.