Triangles

CHAPTER 6

TRY YOURSELF — SOLUTIONS

1. (i) True :
Since, a regular quadrilateral is always a square.
Hence, in any two squares, all the angles are equal (each 90°) and sides would be proportional.
∴ Two regular quadrilaterals are always similar.
(ii) True :
Since, two right angled triangles can also be similar.
∴ If two triangles are similar, then they are not necessarily equilateral.
(iii) False :
Since, two congruent figures are always similar but two similar figures need not be congruent always.
(iv) True :
Since, all the sides are equal in equilateral triangle.
∴ The corresponding sides of equilateral triangles are always proportional.

2. In $ABCDE$, $\angle E = 540° - (\angle A + \angle B + \angle C + \angle D)$
$= 540° - (80° + 130° + 70° + 140°) = 540° - 420° = 120°$
Similarly, $\angle T = 120°$
Now, in pentagons, $ABCDE$ and $PQRST$, we have
(i) $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R, \angle D = \angle S$ and $\angle E = \angle T$
(ii) $\frac{AB}{PQ} = \frac{2}{1}, \frac{BC}{QR} = \frac{5}{2.5} = \frac{2}{1}, \frac{CD}{RS} = \frac{3}{1.5} = \frac{2}{1},$
$\frac{DE}{ST} = \frac{2}{1}, \frac{EA}{TP} = \frac{3.6}{1.8} = \frac{2}{1}$
Hence, the two figures are similar as their corresponding angles are equal and their corresponding sides are in the same ratio *i.e.* 2/1.

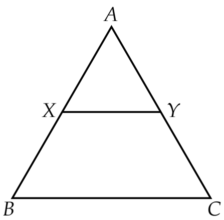
3. Given, $\Delta ABC \sim \Delta DFE$
$\Rightarrow \angle A = \angle D, \angle B = \angle F$ and $\angle C = \angle E$ ...(i)
Now, in ΔABC, $\angle A = 30°, \angle C = 50°$
∴ $\angle B = 180° - (\angle A + \angle C)$
$= 180° - (30° + 50°) = 180° - 80° = 100°$
Hence, using (i), $\angle F = 100°$
Also, $\frac{AB}{DF} = \frac{AC}{DE}$ $(\because \Delta ABC \sim \Delta DFE)$
$\Rightarrow \frac{5}{7.5} = \frac{8}{DE} \Rightarrow DE = \frac{8 \times 7.5}{5} = 12$ cm

4. Given, $\Delta ABC \sim \Delta PQR$, $AC = 4\sqrt{3}$ cm, $BC = 8$ cm, $PQ = 3$ cm, $QR = 6$ cm
Now, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \Rightarrow \frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$
From last two terms, we get
$\frac{8}{6} = \frac{4\sqrt{3}}{y} \Rightarrow y = \frac{6 \times 4\sqrt{3}}{8} = 3\sqrt{3}$ cm ...(i)
From first two terms, we get
$\frac{z}{3} = \frac{8}{6} \Rightarrow z = \frac{24}{6} \Rightarrow z = 4$ cm ...(ii)
From (i) and (ii), we get $y + z = (3\sqrt{3} + 4)$ cm

5. Given, $\Delta PQR \sim \Delta TSM$
∴ $\angle P = \angle T, \angle Q = \angle S$ and $\angle R = \angle M$ ...(i)
and $\frac{PQ}{TS} = \frac{QR}{SM} = \frac{RP}{MT}$ ...(ii)
Given, $\angle P = 55°, \angle S = 25°$ ...(iii)
and $PQ = 7$ cm, $QR = 9$ cm, $TS = 21$ cm, $MT = 24$ cm ...(iv)
Now, using (iv) in (ii), we have
$\frac{7}{21} = \frac{9}{SM} = \frac{RP}{24}$
Using first two terms, $\frac{7}{21} = \frac{9}{SM}$
$\Rightarrow SM = \frac{9 \times 21}{7} \Rightarrow SM = 27$ cm
Using first and last term, $\frac{7}{21} = \frac{RP}{24}$
$\Rightarrow RP = \frac{7 \times 24}{21} = 8$ cm
∴ Difference of remaining two sides
$= SM - RP = 27 - 8 = 19$ cm
Using (iii) in (i), we have
$\angle P = 55° = \angle T$
$\angle Q = \angle S = 25°$
$\angle R = \angle M = 180° - (55° + 25°) = 180° - 80° = 100°$

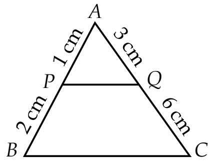
6. Given, $PX = 3$ cm, $PY = 7.5$ cm and $XQ = 2$ cm
In ΔXYZ, $PQ \parallel YZ$
∴ By basic proportionality theorem, we have
$\frac{XP}{PY} = \frac{XQ}{QZ}$
$\Rightarrow \frac{3}{7.5} = \frac{2}{QZ} \Rightarrow QZ = \frac{2 \times 7.5}{3} = 5$ cm
Now, $XZ = XQ + QZ = (2 + 5)$ cm $= 7$ cm

7. **Given :** A ΔABC, X and Y are points on AB and AC respectively such that $XY \parallel BC$ and $BX = CY$



To prove : ABC is an isosceles triangle.
Proof : Since $XY \parallel BC$
∴ $\frac{AX}{BX} = \frac{AY}{CY}$
[By basic proportionality theorem]
$\Rightarrow \frac{AX}{BX} = \frac{AY}{BX}$ $[\because BX = CY]$
$\Rightarrow AX = AY$ ...(i)
Also, $BX = CY$ ...(ii)
Adding (i) and (ii), we get
$AX + BX = AY + CY$
$\Rightarrow AB = AC \Rightarrow ABC$ is an isosceles triangle.

8. In ΔABC, we have $\frac{AP}{PB} = \frac{1}{2}$



and $\frac{AQ}{QC} = \frac{3}{6} = \frac{1}{2}$

Hence, $\frac{AP}{PB} = \frac{AQ}{QC}$

$\therefore$ $PQ \parallel BC$

[By converse of basic proportionality theorem]

In ΔAPQ and ΔABC

$\angle APQ = \angle ABC$ and $\angle AQP = \angle ACB$

[$\because$ Corresponding angles as $PQ \parallel BC$]

$\angle PAQ = \angle BAQ$ [Common]

$\therefore$ $\Delta APQ \sim \Delta ABC$ [By AAA similarity criterion]

$\Rightarrow$ $\frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

Now, $\frac{AP}{AB} = \frac{PQ}{BC} \Rightarrow \frac{AP}{AP+BP} = \frac{PQ}{BC}$

$\Rightarrow$ $\frac{1}{1+2} = \frac{PQ}{BC} \Rightarrow BC = 3PQ$

9. Let AB be the lamp post of height 3.9 m.

Height of Rama, CD = 120 cm = 1.2 m

Distance covered by Rama in 3 seconds = 1.5 × 3 = 4.5 m

Let DE be the shadow of Rama after 3 seconds.

In ΔABE and ΔCDE

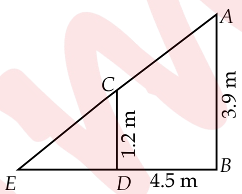
$\angle ABE = \angle CDE = 90°$

$\angle AEB = \angle CED$ [Common]

$\therefore$ $\Delta ABE \sim \Delta CDE$ [By AA similarity criterion]

$\therefore$ $\frac{AB}{CD} = \frac{BE}{DE}$

$\Rightarrow$ $\frac{3.9}{1.2} = \frac{4.5+DE}{DE}$



$\Rightarrow$ $3.9DE = 5.4 + 1.2\,DE$

$\Rightarrow$ $(3.9 - 1.2)DE = 5.4$

$\Rightarrow$ $DE = \frac{5.4}{2.7} = 2$ m

Hence, the shadow of Rama is 2 m.

10. In ΔPRQ and ΔSTQ,

$\angle PRQ = \angle STQ = 90°$

$\angle PQR = \angle SQT$ [Common]

So, $\Delta PRQ \sim \Delta STQ$ [By AA similarity criterion]

$\Rightarrow$ $\frac{QR}{QT} = \frac{QP}{QS}$

$\Rightarrow$ $QR \times QS = QP \times QT$

11. In ΔABC and ΔBDC,

$\angle ABC = \angle BDC = 90°$

$\angle ACB = \angle BCD$ [Common]

$\therefore$ $\Delta ABC \sim \Delta BDC$ [By AA similarity criterion]

$\Rightarrow$ $\angle BAC = \angle DBC$ ...(i)

Now, in ΔADB and ΔBDC,

$\angle BDA = \angle CDB = 90°$

$\angle BAD = \angle CBD$ [From (i)]

$\therefore$ $\Delta ADB \sim \Delta BDC$ [By AA similarity criterion]

$\Rightarrow$ $\frac{BD}{CD} = \frac{AD}{BD} \Rightarrow CD = \frac{BD^2}{AD}$

$\Rightarrow$ $CD = \frac{8^2}{4} = \frac{64}{4} = 16$ cm

12. (i) In ΔABO and ΔDCO, $\frac{AO}{DO} = \frac{16}{9}$ But $\frac{BO}{CO} = \frac{9}{5}$.

Hence, $\frac{AO}{DO} \neq \frac{BO}{CO}$

Thus, ΔABO and ΔDCO are not similar.

(ii) In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow$ $\angle R = 180° - 45° - 78° = 57°$

In ΔLMN, $\angle L + \angle M + \angle N = 180°$

$\Rightarrow$ $\angle N = 180° - 57° - 45° = 78°$

So, $\angle P = \angle M$, $\angle Q = \angle N$, $\angle R = \angle L$

$\therefore$ $\Delta PQR \sim \Delta MNL$ (By AAA similarity criterion)

13. In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \frac{BC}{EF} = \frac{8}{16} = \frac{1}{2}, \frac{AC}{DF} = \frac{9}{18} = \frac{1}{2}$

$\Rightarrow$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\therefore$ $\Delta ABC \sim \Delta DEF$ [By SSS similarity criterion]

$\Rightarrow$ $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

$\Rightarrow$ $x = 87°, y = 58°, z = 35°$

14. In ΔABC and ΔDFE,

$\frac{AB}{DF} = \frac{3.8}{11.4} = \frac{1}{3}, \frac{BC}{FE} = \frac{6}{18} = \frac{1}{3}, \frac{AC}{DE} = \frac{3\sqrt{3}}{9\sqrt{3}} = \frac{1}{3}$

$\Rightarrow$ $\frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}$

$\therefore$ $\Delta ABC \sim \Delta DFE$ (By SSS similarity criterion)

$\Rightarrow$ $\angle A = \angle D, \angle B = \angle F, \angle C = \angle E$ ...(i)

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$

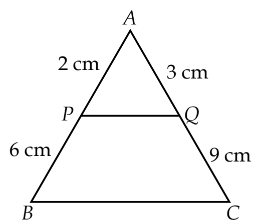
$\Rightarrow$ $\angle C = 180° - 75° - 65° = 40°$

$\therefore$ $\angle E = 40°$ [From (i)]

15. In ΔAPQ and ΔABC,

$\frac{AP}{AB} = \frac{2}{2+6} = \frac{2}{8} = \frac{1}{4}$

$\frac{AQ}{AC} = \frac{3}{3+9} = \frac{3}{12} = \frac{1}{4}$



$\Rightarrow$ $\frac{AP}{AB} = \frac{AQ}{AC}$

and $\angle PAQ = \angle BAC$ [Common]

$\therefore$ $\Delta APQ \sim \Delta ABC$ [By SAS similarity criterion]

$\Rightarrow$ $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} = \frac{1}{4}$

$\Rightarrow$ $BC = 4PQ$.

16. In ΔABC and ΔFED

$\frac{AB}{FE} = \frac{6}{4.5} = \frac{4}{3}$ and $\frac{BC}{ED} = \frac{4}{3}$

Also, $\angle ABC = \angle FED = 85°$

$\therefore$ $\Delta ABC \sim \Delta FED$ [By SAS similarity criterion]

In ΔABC and ΔQPR

$\frac{AB}{PQ} = \frac{6}{12} = \frac{1}{2}$ and $\frac{BC}{PR} = \frac{4}{9} \Rightarrow \frac{AB}{PQ} \neq \frac{BC}{PR}$

$\Rightarrow$ ΔABC and ΔQPR are not similar.

Similarly, ΔDEF and ΔRPQ are not similar.