CHAPTER 7

EXAM DRILL

Coordinate Geometry

SOLUTIONS

1. **(b) :** Distance between $A(0, 6)$ and $B(0, -2)$,

$$AB = \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{0+(-8)^2} = \sqrt{8^2} = 8 \text{ units}$$

2. **(a) :** Let $A(4, p)$ and $B(1, 0)$ be the given points.
Given, $AB = 5$ units

$$\Rightarrow \left(\sqrt{(1-4)^2 + (0-p)^2}\right)^2 = (5)^2$$

$$\Rightarrow 9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$$

3. **(a) :** Let $A(4, 7)$ be the given point. Suppose the perpendicular from A meet the y-axis at B.
So, the coordinates of B is $(0, 7)$.

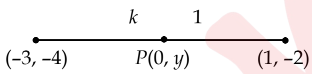
$$\therefore AB = \sqrt{(4-0)^2 + (7-7)^2} = \sqrt{16+0} = 4 \text{ units}$$

4. **(d) :** Let a point on x-axis be $(x_1, 0)$, then its distance from the point $(2, 3) = \sqrt{(x_1 - 2)^2 + 9} = c$

$$\Rightarrow (x_1 - 2)^2 = c^2 - 9 \Rightarrow x_1 - 2 = \sqrt{c^2 - 9}$$

But $c < 3 \Rightarrow c^2 - 9 < 0$ $\therefore$ No real point exist.

5. **(c) :** Let the point $P(0, y)$ on the y-axis divides the line segment joining the points $(-3, -4)$ and $(1, -2)$ in the ratio $k : 1$.



Using section formula, we have $(0, y) = \left(\dfrac{k-3}{k+1}, \dfrac{-2k-4}{k+1}\right)$

$$\Rightarrow \frac{k-3}{k+1} = 0 \Rightarrow k - 3 = 0 \Rightarrow k = 3$$

Hence, the required ratio is 3 : 1.

6. **(c) :** Let $A(3, -5)$, $B(-7, 4)$ and $C(10, -k)$ be the given points.
Given, centroid of $\Delta ABC = (k, -1)$

$$\Rightarrow \left(\frac{3-7+10}{3}, \frac{-5+4-k}{3}\right) = (k, -1)$$

$$\Rightarrow \left(\frac{6}{3}, \frac{-1-k}{3}\right) = (k, -1) \Rightarrow \frac{6}{3} = k \text{ and } \frac{-1-k}{3} = -1$$

$$\Rightarrow k = 2 \text{ and } k = 3 - 1 = 2$$

7. Let $A(0, 0)$, $B(2, 0)$, $C(0, 3)$ and $D(x, y)$ be the vertices of rectangle $ABCD$.
Since, diagonals of rectangle bisect each other.
$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{0+0}{2}, \frac{0+3}{2}\right) = \left(\frac{2+x}{2}, \frac{0+y}{2}\right)$$

$$\Rightarrow \left(0, \frac{3}{2}\right) = \left(\frac{2+x}{2}, \frac{y}{2}\right)$$

$$\Rightarrow \frac{2+x}{2} = 0 \text{ and } \frac{y}{2} = \frac{3}{2}$$

$$\Rightarrow x = -2 \text{ and } y = 3$$

8. Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3}\right) = \left(\frac{-6}{3}, \frac{9}{3}\right) = (-2, 3)$$

Since, G lies on the median, $x - 2y + k = 0$

$$\therefore -2 - 2(3) + k = 0 \Rightarrow -2 - 6 + k = 0 \Rightarrow k = 8$$

9. Let coordinates of P be $(0, y)$ and $A(-5, -2)$, $B(3, 2)$
We have $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (-5-0)^2 + (-2-y)^2 = (3-0)^2 + (2-y)^2$$

$$\Rightarrow 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$$

$$\Rightarrow 8y = -16 \Rightarrow y = -2$$

Coordinates of P are $(0, -2)$

$$PA = \sqrt{25 + 4 + (-2)^2 + (-2)4} = \sqrt{25} = 5 \text{ cm}$$

10. Required distance

$$= \sqrt{[c+a-(b+c)]^2 + [a+b-(c+a)]^2}$$

$$= \sqrt{[c+a-b-c]^2 + [a+b-c-a]^2}$$

$$= \sqrt{(a-b)^2 + (b-c)^2} = \sqrt{a^2 + 2b^2 + c^2 - 2ab - 2bc} \text{ units}$$

11. Here, point A lies on Y-axis, so its abscissa is zero and given its ordinate is 5, therefore, its coordinates are $A(0, 5)$.

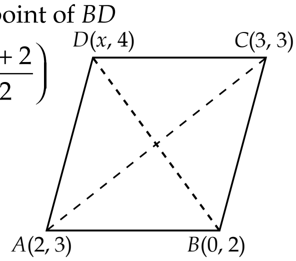
$$\text{Now, } AB = \sqrt{(-5-0)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29} \text{ units}$$

12. Diameter of circle, $d = \sqrt{(2-24)^2 + (23-1)^2}$

$$= \sqrt{(-22)^2 + (22)^2} = \sqrt{(22)^2(1+1)} = 22\sqrt{2} \text{ units}$$

$$\therefore \text{ Radius of a circle, } r = \frac{d}{2} = \frac{22\sqrt{2}}{2} = 11\sqrt{2} \text{ units}$$

13. As the diagonals of parallelogram bisect each other.
$\therefore$ Mid-point of AC = Mid-point of BD



$$\Rightarrow \left(\frac{2+3}{2}, \frac{3+3}{2}\right) = \left(\frac{x+0}{2}, \frac{4+2}{2}\right)$$

$$\Rightarrow \left(\frac{5}{2}, 3\right) = \left(\frac{x}{2}, 3\right)$$

$$\Rightarrow \frac{x}{2} = \frac{5}{2} \Rightarrow x = 5$$

14. Let coordinates of P and Q be $(x, 0)$ and $(0, y)$ respectively.
Let $M(-2, -6)$ be the mid-point of PQ.
$\therefore$ By mid-point formula, we have

$$-2 = \frac{x+0}{2} \text{ and } -6 = \frac{0+y}{2} \Rightarrow -4 = x \text{ and } -12 = y$$

$\therefore$ Points are $P(-4, 0)$ and $Q(0, -12)$.

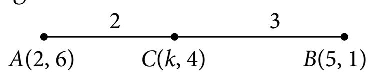
15. Let $A(1, 2)$, $B(4, 3)$ and $C(6, 6)$ and $D(x, y)$ be the vertices of parallelogram.
Since diagonals AC and BD bisect each other.
$\therefore$ Mid-point of BD = Mid-point of AC

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2}\right) = \left(\frac{1+6}{2}, \frac{2+6}{2}\right)$$

$$\Rightarrow \left(\frac{4+x}{2}, \frac{3+y}{2}\right) = \left(\frac{7}{2}, \frac{8}{2}\right)$$

$\Rightarrow$ $4 + x = 7$ and $3 + y = 8$ $\Rightarrow x = 3$ and $y = 5$
$\therefore$ Coordinates of D are $(3, 5)$.

16. We have, $A(2, 6)$, $B(5, 1)$ and $C(k, 4)$ divides the given line segment in the ratio 2 : 3.



Using section formula, we have

$$(k, 4) = \left(\frac{2\times5+3\times2}{2+3}, \frac{2\times1+3\times6}{2+3}\right) = \left(\frac{16}{5}, \frac{20}{5}\right) \Rightarrow k = \frac{16}{5}$$

17. **(i)** **(a) :** We have, $OA = 2\sqrt{2}$ km

$\Rightarrow \sqrt{2^2 + y^2} = 2\sqrt{2}$

$\Rightarrow 4 + y^2 = 8 \Rightarrow y^2 = 4$

$\Rightarrow y = 2$ $(\because y = -2$ is not possible$)$

(ii) **(c) :** We have, $OB = 8\sqrt{2}$

$\Rightarrow \sqrt{x^2 + 8^2} = 8\sqrt{2}$

$\Rightarrow x^2 + 64 = 128 \Rightarrow x^2 = 64$

$\Rightarrow x = 8$ $(\because x = -8$ is not possible$)$

(iii) **(c) :** Coordinates of A and B are $(2, 2)$ and $(8, 8)$ respectively, therefore coordinates of point M are $\left(\frac{2+8}{2}, \frac{2+8}{2}\right)$, *i.e.*, $(5, 5)$

(iv) **(d) :** Let A divides OM in the ratio $k : 1$.

Then, $2 = \frac{5k+0}{k+1} \Rightarrow 2k + 2 = 5k \Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$

$\therefore$ Required ratio = 2 : 3

(v) **(b) :** Since M is the mid-point of A and B therefore $AM = MB$. Hence, he should try his luck moving towards B.

18. Consider the house is at origin $(0, 0)$, then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively $(2, 3)$, $(-4, -6)$, $(6, -8)$ and $(-6, 8)$

(i) **(d) :** Since, grocery store is at $(2, 3)$ and food cart is at $(6, -8)$

$\therefore$ Required distance $= \sqrt{(6-2)^2 + (-8-3)^2}$

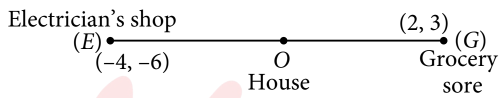
$= \sqrt{4^2 + 11^2} = \sqrt{16+121} = \sqrt{137}$ cm

(ii) **(b) :** Required distance

$= \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$ cm

(iii) **(c) :** Let O divides EG in the ratio $k : 1$, then

$0 = \frac{2k-4}{k+1}$



$\Rightarrow 2k = 4$

$\Rightarrow k = 2$

Thus, O divides EG in the ratio 2 : 1
Hence, required ratio = $OG : OE$ *i.e.*, 1 : 2

(iv) **(c) :** Since, $(0, 0)$ is the mid-point of $(-6, 8)$ and $(6, -8)$, therefore both bus stand and food cart are at equal distances from the house.
Hence, required ratio is 1 : 1.

(v) **(d) :** Mid-point of grocery store and electrician's shop is $\left(\frac{2-4}{2}, \frac{3-6}{2}\right)$, *i.e.*, $\left(-1, \frac{-3}{2}\right)$

Thus, the diagonals does not bisect each other
[$\because$ Mid-point are not same]
Hence, they form a quadrilateral.

19. **(i)** The coordinates of point A are $(9, 27)$, therefore its distance from x-axis = 27 units.

(ii) Coordinates of B and C are $(4, 19)$ and $(14, 19)$

$\therefore$ Required distance $= \sqrt{(14-4)^2 + (19-19)^2}$

$= \sqrt{10^2} = 10$ units

(iii) Coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively.

$\therefore$ Required distance $= \sqrt{(16-2)^2 + (6-6)^2}$

$= \sqrt{14^2} = 14$ units

(iv) Since the coordinates of F and G are $(2, 6)$ and $(16, 6)$ respectively therefore mid-point of FG is

$$\left(\frac{2+16}{2}, \frac{6+6}{2}\right) = (9, 6)$$

Thus, the mid-point of FG will lie on the line represented by $x = 9$.

(v) Coordinates of L and N are $(6, 4)$ and $(7, 1)$ respectively.

Length of $LN = \sqrt{(7-6)^2 + (1-4)^2}$

$= \sqrt{1+9} = \sqrt{10}$ units

$\Rightarrow$ Length of $MP = \sqrt{10}$ units

Now, perimeter of $LMPN = LN + LM + MP + NP$

$= \sqrt{10} + 6 + \sqrt{10} + 4 = (2\sqrt{10} + 10)$ units

[$\because$ $LM = 12 - 6 = 6$ units and $NP = 11 - 7 = 4$ units]

20. (i) Coordinates of Q are (9, 5).

$\therefore$ Distance of point Q from y-axis = 9 units

(ii) Coordinates of point U are (8, 2).

(iii) We have, $P(2, 5)$ and Q (9, 5)

$\therefore$ $PQ = \sqrt{(2-9)^2 + (5-5)^2} = \sqrt{49+0} = 7$ units

(iv) Point $A(x, y)$ is equidistant from R (3, 8) and T(3, 2).

$\therefore$ $AR = AT \Rightarrow AR^2 = AT^2$

$\Rightarrow$ $(x-3)^2 + (y-8)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow$ $y^2 + 64 - 16y = y^2 + 4 - 4y$

$\Rightarrow$ $16y - 4y = 64 - 4 \Rightarrow 12y = 60 \Rightarrow y = 5$

(v) Length of TU = 5 units and of TL = 2 units

$\therefore$ Perimeter of image of a rectangular face

= 2(5 + 2) = 14 units

21. We have, $P(x, y)$, $A(5, 1)$ and $B(-1, 5)$

Given, $AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow$ $(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$

$\Rightarrow$ $x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$

$\Rightarrow$ $-10x - 2y = 2x - 10y \Rightarrow -12x = -8y \Rightarrow 3x = 2y$

22. Point on the x-axis is in the form $(x, 0)$.

$\therefore$ $(x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$\Rightarrow$ $x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$\Rightarrow$ $-8x = 85 - 29 \Rightarrow x = -7$

$\therefore$ The required point is (– 7, 0).

23. Let $A(6, -2)$ and $B(-2, y)$ be the given points.

Length of the line segment AB = 10 units (Given)

$\Rightarrow$ $\sqrt{(-2-6)^2 + (y+2)^2} = 10$

$\Rightarrow$ $\sqrt{64 + y^2 + 4y + 4} = 10$

$\Rightarrow$ $y^2 + 4y + 68 = 100 \Rightarrow y^2 + 4y - 32 = 0$

$\Rightarrow$ $y^2 + 8y - 4y - 32 = 0 \Rightarrow y(y+8) - 4(y+8) = 0$

$\Rightarrow$ $(y+8)(y-4) = 0 \Rightarrow y = 4$ or $y = -8$

So, ordinate of B will be 4 or –8.

OR

The given points are $A(0, 2)$, $B(3, p)$ and $C(p, 5)$.

Since, A is equidistant from B and C.

$\therefore$ $AB = AC \Rightarrow AB^2 = AC^2$

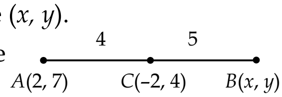
$\Rightarrow$ $(3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow$ $9 + p^2 + 4 - 4p = p^2 + 9 \Rightarrow 4 - 4p = 0$

$\Rightarrow$ $4p = 4 \Rightarrow p = 1$

24. Let the coordinates of B be (x, y).

Using section formula, we have coordinates of C are



$\left(\frac{4x+5(2)}{4+5}, \frac{4y+5(7)}{4+5}\right) = \left(\frac{4x+10}{9}, \frac{4y+35}{9}\right)$

Also, the coordinates of C are (–2, 4). (Given)

$\therefore$ $\frac{4x+10}{9} = -2$ and $\frac{4y+35}{9} = 4$

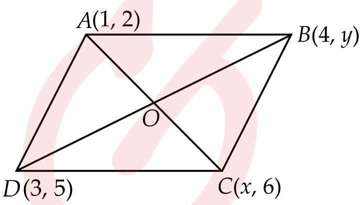
$\Rightarrow$ $4x + 10 = -18$ and $4y + 35 = 36$

$\Rightarrow$ $4x = -18 - 10$ and $4y = 36 - 35$

$\Rightarrow$ $4x = -28$ and $4y = 1$

$\Rightarrow$ $x = -7$ and $y = \frac{1}{4}$

25. (18) :



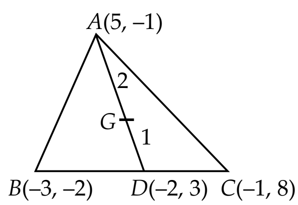
Coordinates of O is the mid-point of AC and BD

$\therefore$ $\frac{x+1}{2} = \frac{3+4}{2}$ and $\frac{6+2}{2} = \frac{y+5}{2}$

$\Rightarrow$ $x + 1 = 7$ and $8 = y + 5 \Rightarrow x = 6$ and $y = 3$

$\therefore$ $xy = 6 \times 3 = 18$

26. Let AD be the median from the vertex A of ΔABC. Then, D is the mid-point of BC. So, coordinates of D are $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$ i.e., (–2, 3).

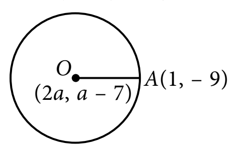


$\therefore$ $AD = \sqrt{(5+2)^2 + (-1-3)^2} = \sqrt{49+16} = \sqrt{65}$ units

Let G be the centroid of ΔABC.

Coordinates of G are $\left(\frac{5-3-1}{3}, \frac{-1-2+8}{3}\right) = \left(\frac{1}{3}, \frac{5}{3}\right)$.

27. (d) : Let O $(2a, a-7)$ be the centre and A (1, –9) be any point through which circle passes.



$\therefore$ $2(OA) = 10\sqrt{2}$

[$\because$ Diameter = 2 × radius]

$\Rightarrow$ $OA = 5\sqrt{2} \Rightarrow OA^2 = (5\sqrt{2})^2 = 50$

$\Rightarrow$ $(2a-1)^2 + (a-7+9)^2 = 50$

$\Rightarrow$ $4a^2 + 1 - 4a + a^2 + 4 + 4a = 50$

$\Rightarrow$ $5a^2 = 50 - 5 = 45 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$

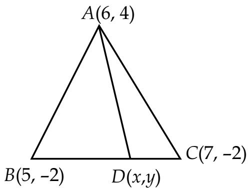
28. Using distance formula, we have

$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$ units

$AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$ units

$BC = \sqrt{(5-7)^2 + (-2+2)^2}$

$= \sqrt{4+0} = 2$ units



Now, $AB = AC \neq BC$.

So, ΔABC is an isosceles triangle.

Let D (x, y) be the mid-point of BC.

Using mid-point formula, we have

$(x, y) = \left(\frac{5+7}{2}, \frac{-2-2}{2}\right) = \left(\frac{12}{2}, \frac{-4}{2}\right) = (6, -2)$

$\therefore$ Coordinates of D are (6, –2)

$\therefore$ Length of median, $AD = \sqrt{(6-6)^2 + (-2-4)^2}$

$= \sqrt{0+36} = 6$ units

OR

Since C is equidistant from A and B.

$\therefore$ $AC = CB \Rightarrow AC^2 = CB^2$

$\Rightarrow$ $(3+2)^2 + (-1-3)^2 = (x+2)^2 + (8-3)^2$

$\Rightarrow$ $25 + 16 = x^2 + 4 + 4x + 25$

$\Rightarrow$ $x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow$ $x(x+6) - 2(x+6) = 0 \Rightarrow (x+6)(x-2) = 0$

$\Rightarrow$ $x = -6$ or $x = 2$

Now, using distance formula

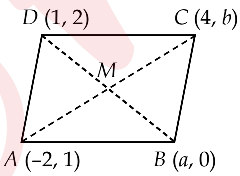
$BC = \sqrt{(-6+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

or $\sqrt{(2+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

$AB = \sqrt{(-6-3)^2 + (8+1)^2} = \sqrt{81+81} = 9\sqrt{2}$ units

or $\sqrt{(2-3)^2 + (8+1)^2} = \sqrt{1+81} = \sqrt{82}$ units

29. Let the given points are $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$.



We know, diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

$\Rightarrow \left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$

$\Rightarrow \left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right) \Rightarrow \frac{2}{2} = \frac{a+1}{2}$

$\Rightarrow$ $1 + a = 2$ $\Rightarrow$ $a = 1$ and $\frac{1+b}{2} = \frac{2}{2}$

$\Rightarrow$ $b + 1 = 2$ $\Rightarrow$ $b = 1$

Hence, $a = 1$, $b = 1$

Now, $AB = CD$ and $BC = AD$

($\because$ Opposite sides of a parallelogram are equal)

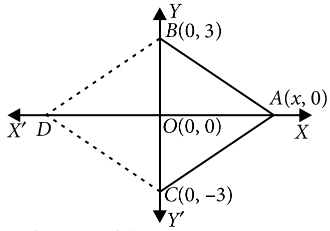
$\therefore$ $AB = CD = \sqrt{(-2-1)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$ units

and $BC = AD = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

30. $\because$ O is the mid-point of the base BC.

$\therefore$ Coordinates of point B are (0, 3)

So, $BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{6^2} = 6$ units.



Let the coordinates of point A be $(x, 0)$.

$\therefore$ $AB = \sqrt{(0-x)^2 + (3-0)^2}$

$= \sqrt{x^2 + 9}$

Also, $AB = BC$

($\because$ ΔABC is an equilateral triangle)

$\Rightarrow$ $\sqrt{x^2 + 9} = 6$ $\Rightarrow$ $x^2 + 9 = 36$

$\Rightarrow$ $x^2 = 27$ $\Rightarrow$ $x = \pm 3\sqrt{3}$

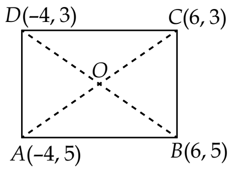
$\therefore$ Coordinates of point $A = (x, 0) = (3\sqrt{3}, 0)$

Since, $BACD$ is a rhombus.

$\therefore$ $AB = AC = CD = DB$

$\therefore$ Coordinates of point $D = (-3\sqrt{3}, 0)$

31. We have $ABCD$ is a rectangle, where AC and BD are its diagonal.



Now, $AC = \sqrt{[6-(-4)]^2 + (3-5)^2}$

$= \sqrt{(10)^2 + (-2)^2}$

$= \sqrt{100+4} = \sqrt{104}$ units

$BD = \sqrt{(-4-6)^2 + (3-5)^2} = \sqrt{(-10)^2 + (-2)^2}$

$= \sqrt{100+4} = \sqrt{104}$ units

$\Rightarrow$ $AC = BD$

Hence, diagonals of rectangle $ABCD$ are equal.

Let O is the mid-point of both AC and BD.

Using mid-point formula, we have

coordinates of O from $AC = \left(\frac{6+(-4)}{2}, \frac{3+5}{2}\right)$

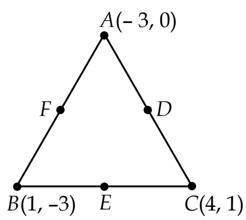
$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

Coordinates of O from $BD = \left(\frac{-4+6}{2}, \frac{3+5}{2}\right)$

$= \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

$\Rightarrow$ AC and BD bisect each other at O.

OR



Let D, E and F be the mid-points of the sides AC, BC and AB respectively.

Then the coordinates of D are

$\left(\frac{-3+4}{2}, \frac{0+1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

Coordinates of E are $\left(\frac{1+4}{2}, \frac{-3+1}{2}\right) = \left(\frac{5}{2}, \frac{-2}{2}\right) = \left(\frac{5}{2}, -1\right)$

Coordinates of F are $\left(\frac{-3+1}{2}, \frac{0-3}{2}\right) = \left(\frac{-2}{2}, \frac{-3}{2}\right) = \left(-1, \frac{-3}{2}\right)$

Using distance formula, lengths of medians are

$$\therefore\ AE=\sqrt{\left[\frac{5}{2}-(-3)\right]^2+(-1-0)^2}$$

$$=\sqrt{\left(\frac{11}{2}\right)^2+1}=\sqrt{\frac{121}{4}+1}=\sqrt{\frac{125}{4}}=\frac{5\sqrt{5}}{2}\text{ units}$$

$$BD=\sqrt{\left(\frac{1}{2}-1\right)^2+\left[\frac{1}{2}-(-3)\right]^2}=\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{7}{2}\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{49}{4}}=\sqrt{\frac{50}{4}}=\frac{\sqrt{50}}{2}\text{ units}$$

$$CF=\sqrt{(-1-4)^2+\left(\frac{-3}{2}-1\right)^2}=\sqrt{25+\frac{25}{4}}=\sqrt{\frac{100+25}{4}}$$

$$=\sqrt{\frac{125}{4}}=\frac{5\sqrt{5}}{2}\text{ units}$$

32. We have, points $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$.

$$\therefore\ SP=\sqrt{(at^2-a)^2+(2at-0)^2}=\sqrt{a^2t^4+a^2-2a^2t^2+4a^2t^2}$$

$$=\sqrt{a^2t^4+a^2+2a^2t^2}=\sqrt{(at^2+a)^2}=at^2+a \quad \text{...(i)}$$

$$SQ=\sqrt{\left(\frac{a}{t^2}-a\right)^2+\left(\frac{-2a}{t}-0\right)^2}=\sqrt{\frac{a^2}{t^4}+a^2-\frac{2a^2}{t^2}+\frac{4a^2}{t^2}}$$

$$=\sqrt{\frac{a^2}{t^4}+a^2+\frac{2a^2}{t^2}}=\sqrt{\left(\frac{a}{t^2}+a\right)^2}=\frac{a}{t^2}+a \quad \text{...(ii)}$$

Now, $\frac{1}{SP}+\frac{1}{SQ}=\frac{1}{at^2+a}+\frac{1}{\frac{a}{t^2}+a}$ [Using (i) and (ii)]

$=\frac{1}{at^2+a}+\frac{t^2}{a+at^2}=\frac{1+t^2}{a+at^2}=\frac{1+t^2}{a(1+t^2)}=\frac{1}{a}$, which is independent of t.

33. Let $P(1, 1)$, $Q(2, -3)$, $R(3, 4)$ be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC. Then

P is the mid-point of AB

$\Rightarrow\ \frac{x_1+x_2}{2}=1,\ \frac{y_1+y_2}{2}=1$

$\Rightarrow\ x_1+x_2=2$ and $y_1+y_2=2$ ...(1)

Q is the mid-point of BC

$\Rightarrow\ \frac{x_2+x_3}{2}=2,\ \frac{y_2+y_3}{2}=-3$

$\Rightarrow\ x_2+x_3=4$ and $y_2+y_3=-6$ ...(2)

R is the mid-point of AC

$\Rightarrow\ \frac{x_1+x_3}{2}=3$ and $\frac{y_1+y_3}{2}=4$

$\Rightarrow\ x_1+x_3=6$ and $y_1+y_3=8$ ...(3)

From (1), (2) and (3), we get

$x_1+x_2+x_2+x_3+x_1+x_3=2+4+6$

and $y_1+y_2+y_2+y_3+y_1+y_3=2-6+8$

$\Rightarrow\ x_1+x_2+x_3=6$ and $y_1+y_2+y_3=2$ ...(4)

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)=\left(\frac{6}{3}, \frac{2}{3}\right)=\left(2, \frac{2}{3}\right)$$

[Using (4)]

34. Let the vertices of an equilateral triangle are $A(-6, 5)$, $B(6, 5)$ and $C(x, y)$.

We know that, in an equilateral triangle all three sides are equal.

$\therefore\ AB=BC=CA \ \Rightarrow\ AB^2=BC^2=CA^2$

Consider, $AB^2=BC^2$

$\Rightarrow\ (6+6)^2+(5-5)^2=(x-6)^2+(y-5)^2$

$\Rightarrow\ 144+0=x^2+36-12x+y^2+25-10y$

$\Rightarrow\ x^2+y^2-12x-10y+61=144$

$\Rightarrow\ x^2+y^2-12x-10y=83$ ...(i)

Now, consider $AB^2=CA^2$

$\Rightarrow\ (6+6)^2+(5-5)^2=(x+6)^2+(y-5)^2$

$\Rightarrow\ 144=36+x^2+12x+25+y^2-10y$

$\Rightarrow\ x^2+y^2+12x-10y+61=144$

$\Rightarrow\ x^2+y^2+12x-10y=83$ ...(ii)

From (i) and (ii) we get

$x^2+y^2-12x-10y=x^2+y^2+12x-10y$

$\Rightarrow\ 12x+12x=0 \Rightarrow 24x=0 \Rightarrow x=0$

Puting $x=0$ in (i), we get

$0+y^2-12(0)-10y=83$

$\Rightarrow\ y^2-10y-83=0$

$$\therefore\ y=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(-83)}}{2\times 1}$$

$$\Rightarrow\ y=\frac{10\pm\sqrt{100+332}}{2}$$

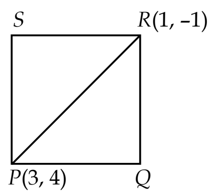
$$\Rightarrow\ y=\frac{10\pm\sqrt{432}}{2}$$

$$\therefore\ y=\frac{10+\sqrt{432}}{2}\text{ or } y=\frac{10-\sqrt{432}}{2}$$

Hence, the third vertex is $\left(0, \frac{10+\sqrt{432}}{2}\right)$ or $\left(0, \frac{10-\sqrt{432}}{2}\right)$.

OR

Let $PQRS$ be a square and let $P(3, 4)$ and $R(1, -1)$ be the given opposite angular points.



Let $Q(x, y)$ be the unknown vertex.

Since, all sides of square are equal.

$\therefore\ PQ=QR$

$\Rightarrow\ PQ^2=QR^2$

$\Rightarrow\ (x-3)^2+(y-4)^2=(x-1)^2+(y+1)^2$

$\Rightarrow\ x^2-6x+9+y^2-8y+16=x^2-2x+1+y^2+2y+1$

$\Rightarrow\ -6x-8y+25=-2x+2y+2$

$\Rightarrow\ -6x+2x-8y-2y=2-25$

$\Rightarrow \quad -4x - 10y = -23 \quad \Rightarrow \quad 4x + 10y = 23$

$\Rightarrow \quad x = \dfrac{23 - 10y}{4}$ ...(i)

In right angled triangle PQR, we have

$PQ^2 + QR^2 = PR^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (x-1)^2 + (y+1)^2 = (3-1)^2 + (4+1)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 + x^2 - 2x + 1 + y^2 + 2y + 1 = 4 + 25$

$\Rightarrow \quad 2x^2 + 2y^2 - 8x - 6y + 27 = 29$

$\Rightarrow \quad 2x^2 + 2y^2 - 8x - 6y = 2$

$\Rightarrow \quad x^2 + y^2 - 4x - 3y - 1 = 0$ ...(ii)

Substitute the value of x from (i) into (ii), we get

$$\left(\frac{23-10y}{4}\right)^2 + y^2 - 4\left(\frac{23-10y}{4}\right) - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529 + 100y^2 - 460y}{16}\right) + y^2 - 23 + 10y - 3y - 1 = 0$$

$$\Rightarrow \left(\frac{529 + 100y^2 - 460y}{16}\right) + y^2 + 7y - 24 = 0$$

$$\Rightarrow \frac{529 + 100y^2 - 460y + 16y^2 + 112y - 384}{16} = 0$$

$\Rightarrow \quad 116y^2 - 348y + 145 = 0$

$\Rightarrow \quad 4y^2 - 12y + 5 = 0$ [On dividing by 29]

$\Rightarrow \quad 4y^2 - 10y - 2y + 5 = 0$

$\Rightarrow \quad 2y(2y - 5) - 1(2y - 5) = 0$

$\Rightarrow \quad (2y - 5)(2y - 1) = 0$

$\Rightarrow \quad y = \dfrac{5}{2}$ or $y = \dfrac{1}{2}$

On putting $y = \dfrac{1}{2}$ in (i), we get

$$x = \frac{23 - 10\left(\frac{1}{2}\right)}{4} = \frac{23-5}{4} = \frac{18}{4} = \frac{9}{2}$$

On putting $y = \dfrac{5}{2}$ in (i) we get

$$x = \frac{23 - 10\left(\frac{5}{2}\right)}{4} = \frac{23-25}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Hence, the other two vertices are $\left(\dfrac{9}{2}, \dfrac{1}{2}\right)$ and $\left(\dfrac{-1}{2}, \dfrac{5}{2}\right)$.

35. Given, ΔAOB is a right angle triangle right angled at O and AB is hypotenuse and C is the mid-point of AB.

Let the coordinates of B and A are $(0, b)$ and $(a, 0)$ respectively.

$\Rightarrow \quad OA = a$ and $OB = b$

So, the coordinates of $C = \left(\dfrac{a+0}{2}, \dfrac{b+0}{2}\right) = \left(\dfrac{a}{2}, \dfrac{b}{2}\right)$

Now, $CO = \sqrt{\left(0 - \dfrac{a}{2}\right)^2 + \left(0 - \dfrac{b}{2}\right)^2}$

$$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2} \text{ units}$$

$$CA = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2} \text{ units}$$

$$CB = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

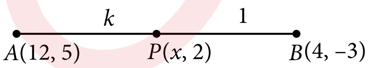
$$= \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2} \text{ units}$$

$\Rightarrow \quad CO = CA = CB$

Therefore, C is equidistant from the three vertices of ΔAOB.

OR

Let the point $P(x, 2)$ divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $k : 1$.



Using section formula, we have

coordinates of P are $\left(\dfrac{4k+12}{k+1}, \dfrac{-3k+5}{k+1}\right)$

Now, the coordinates of P are $(x, 2)$. (Given)

$\therefore \quad \dfrac{4k+12}{k+1} = x$ and $\dfrac{-3k+5}{k+1} = 2$

$\Rightarrow \quad -3k + 5 = 2k + 2 \Rightarrow 5k = 3 \Rightarrow k = 3/5$

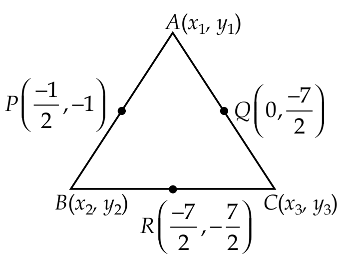
Substituting, $k = \dfrac{3}{5}$ in $\dfrac{4k+12}{k+1} = x$, we get

$$x = \frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1} = \frac{12 + 60}{3 + 5} = \frac{72}{8} = 9$$

Thus, the value of x is 9.

Also, the point P divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$ in the ratio $\dfrac{3}{5} : 1$ *i.e.*, $3 : 5$.

36. Let the coordinates of the vertices A, B and C of ΔABC be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.



As, $P\left(\dfrac{-1}{2}, -1\right)$ is the mid-point of AB.

$\therefore \quad \dfrac{x_1 + x_2}{2} = \dfrac{-1}{2}$ and $\dfrac{y_1 + y_2}{2} = -1$

$\Rightarrow \quad x_1 + x_2 = -1$ and $y_1 + y_2 = -2$ ...(i)

Point R is the mid-point of BC.

$\therefore \quad \frac{x_2 + x_3}{2} = \frac{-7}{2}$ and $\frac{y_2 + y_3}{2} = \frac{-7}{2}$

$\Rightarrow \quad x_2 + x_3 = -7$ and $y_2 + y_3 = -7$ ...(ii)

Point Q is the mid-point of AC.

$\therefore \quad \frac{x_1 + x_3}{2} = 0$ and $\frac{y_1 + y_3}{2} = \frac{-7}{2}$

$\Rightarrow \quad x_1 + x_3 = 0$ and $y_1 + y_3 = -7$ ...(iii)

Adding (i), (ii) and (iii), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = -1 - 7 + 0$ and $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = -2 - 7 - 7$

$\Rightarrow \quad 2(x_1 + x_2 + x_3) = -8$ and $2(y_1 + y_2 + y_3) = -16$

$\Rightarrow \quad x_1 + x_2 + x_3 = -4$ and $y_1 + y_2 + y_3 = -8$ ...(iv)

From (i) and (iv), we get

$-1 + x_3 = -4$ and $-2 + y_3 = -8$

$\Rightarrow \quad x_3 = -3$ and $y_3 = -6$

So, coordinates of C are $(-3, -6)$.

From (ii) and (iv) we get

$x_1 + (-7) = -4$ and $y_1 + (-7) = -8$

$\Rightarrow \quad x_1 = 3$ and $y_1 = -1$

$\therefore$ Coordinates of A are $(3, -1)$

From (iii) and (iv), we get

$x_2 + 0 = -4$ and $y_2 + (-7) = -8$

$\Rightarrow \quad x_2 = -4$ and $y_2 = -1$

$\therefore$ Coordinates of B are $(-4, -1)$.