

Coordinate Geometry

CHAPTER 7



TRY YOURSELF

SOLUTIONS

1. We have, $P(-6, 3)$ and $Q(-1, -4)$

Using distance formula, we get

$$PQ = \sqrt{(-1+6)^2 + (-4-3)^2} = \sqrt{25+49} = \sqrt{74} \text{ units}$$

2. We have, $P(a+b, a-b)$ and $Q(a-b, -a-b)$

$$\begin{aligned} \therefore PQ &= \sqrt{[(a-b)-(a+b)]^2 + [(-a-b)-(a-b)]^2} \\ &= \sqrt{(a-b-a-b)^2 + (-a-b-a+b)^2} \\ &= \sqrt{4b^2 + 4a^2} = 2\sqrt{a^2 + b^2} \text{ units} \end{aligned}$$

3. Let $P(x, -2)$ and $Q(3, 2)$ be the given points.

Then, we have $PQ = 5$

[Given]

$$\therefore \sqrt{(3-x)^2 + (2+2)^2} = 5$$

$$\Rightarrow (3-x)^2 + 16 = 25$$

[Squaring both sides]

$$\Rightarrow x^2 + 9 - 6x + 16 = 25$$

$$\Rightarrow x^2 - 6x = 0 \Rightarrow x(x-6) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 6$$

4. Using distance formula, we have

$$AB = \sqrt{(6-3)^2 + (0-5)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$BC = \sqrt{(1-6)^2 + (-3-0)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$$CD = \sqrt{(-2-1)^2 + (2+3)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$DA = \sqrt{(3+2)^2 + (5-2)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$$\text{Diagonal, } AC = \sqrt{(1-3)^2 + (-3-5)^2}$$

$$= \sqrt{4+64} = \sqrt{68} \text{ units}$$

$$\text{and Diagonal, } BD = \sqrt{(-2-6)^2 + (2-0)^2}$$

$$= \sqrt{64+4} = \sqrt{68} \text{ units}$$

Since, $AB = BC = CD = DA$ i.e., all the sides of given quadrilateral are equal in length and also, $AC = BD$ i.e., diagonal are of same length.

Therefore, the given points are the vertices of a square $ABCD$.

Hence proved.

5. Using distance formula, we have

$$AB = \sqrt{(2+2)^2 + (-1-3)^2} = \sqrt{16+16} = 4\sqrt{2} \text{ units}$$

$$BC = \sqrt{(4-2)^2 + (-3+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$\text{and } CA = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = 6\sqrt{2} \text{ units}$$

$$\text{Now, as } AB + BC = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2} \text{ units}$$

$$\text{i.e., } AB + BC = CA$$

$\therefore$ A, B and C are collinear points.

6. Let $A(3, 2)$, $B\left(4, \frac{5}{2}\right)$ and $C(5, 3)$ are given points.

Using distance formula, we have

$$AB = \sqrt{(4-3)^2 + \left(\frac{5}{2}-2\right)^2} = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \text{ units}$$

$$BC = \sqrt{(5-4)^2 + \left(3-\frac{5}{2}\right)^2} = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \text{ units}$$

$$\text{and } AC = \sqrt{(5-3)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$\text{Now, as } AB + BC = \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2} = \sqrt{5} \text{ units} = AC$$

$\therefore$ The given points are collinear.

Hence proved.

7. Given, point $P(x, y)$ is equidistant from the points $A(6, 1)$ and $B(1, 6)$

$$\therefore AP = PB \Rightarrow AP^2 = PB^2$$

$$\Rightarrow (x-6)^2 + (y-1)^2 = (x-1)^2 + (y-6)^2$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 36 - 12y$$

$$\Rightarrow -10x = -10y \Rightarrow x = y$$

Hence proved.

8. Given, $A(a+b, b-a)$ and $B(a-b, a+b)$ are equidistant from $P(x, y)$.

$$\therefore AP = PB \Rightarrow (AP)^2 = (PB)^2$$

$$\Rightarrow [x - (a+b)]^2 + [y - (b-a)]^2 = [x - (a-b)]^2 + [y - (a+b)]^2$$

$$\Rightarrow x^2 + (a+b)^2 - 2(a+b)x + y^2 + (b-a)^2 - 2(b-a)y$$

$$= x^2 + (a-b)^2 - 2(a-b)x + y^2 + (a+b)^2 - 2(a+b)y$$

$$\Rightarrow -2[(a+b)x + (b-a)y] = -2[(a-b)x + (a+b)y]$$

$$\Rightarrow (a+b)x + (b-a)y = (a-b)x + (a+b)y$$

$$\Rightarrow (a+b-a+b)x = (a+b-b+a)y$$

$$\Rightarrow 2bx = 2ay \text{ or } bx = ay$$

Hence proved.

9. Let P be $(0, y)$, which is equidistant from $A(4, 8)$ and $B(-6, 6)$.

$$\therefore AP = PB \Rightarrow AP^2 = PB^2$$

$$\Rightarrow (4-0)^2 + (8-y)^2 = (0+6)^2 + (y-6)^2$$

$$\Rightarrow 16 + 64 + y^2 - 16y = 36 + y^2 + 36 - 12y$$

$$\Rightarrow 80 - 16y = 72 - 12y$$

$$\Rightarrow 4y = 8 \Rightarrow y = 2$$

$\therefore$ Coordinates of P are $(0, 2)$.

$$\begin{aligned}\text{Now, } AP &= \sqrt{(4-0)^2 + (8-2)^2} \\ &= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \text{ units}\end{aligned}$$

10. Let the point $P(0, y)$ divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$ in the ratio $k : 1$.

Using section formula, we have coordinates of P are

$$\left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1} \right).$$

Since, x -coordinate of P is zero

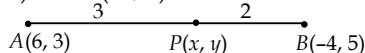
$$\therefore \frac{-k+5}{k+1} = 0 \Rightarrow -k+5=0 \Rightarrow k=5$$

Hence, the point P divides the line segment in the ratio $5 : 1$.

$$\text{Also, } y\text{-coordinate of } P = \frac{-4(5)-6}{5+1} = \frac{-20-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$$

$$\therefore \text{Coordinates of } P \text{ are } \left(0, \frac{-13}{3} \right).$$

11. Let $P(x, y)$ divides the line segment joining the points $A(6, 3)$ and $B(-4, 5)$ in the ratio $3 : 2$



Using section formula, we have

$$\begin{aligned}(x, y) &= \left(\frac{3 \times (-4) + 2 \times 6}{3+2}, \frac{3 \times 5 + 2 \times 3}{3+2} \right) \\ &= \left(\frac{-12+12}{5}, \frac{15+6}{5} \right) = \left(0, \frac{21}{5} \right)\end{aligned}$$

12. Let $P(-3, p)$ divides the line segment joining the points $A(-5, -4)$ and $B(-2, 3)$ in the ratio $k : 1$.

Using section formula,

$$\text{Coordinates of } P \text{ are } \left(\frac{-2k-5}{k+1}, \frac{3k-4}{k+1} \right)$$

But, coordinates of P are given as $(-3, p)$.

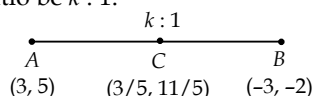
$$\therefore \frac{-2k-5}{k+1} = -3 \text{ and } \frac{3k-4}{k+1} = p$$

$$\Rightarrow -2k-5 = -3k-3 \Rightarrow k=2$$

$$\text{Now, } p = \frac{3 \times 2 - 4}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

Hence, the ratio is $2 : 1$ and $p = \frac{2}{3}$

13. Let the ratio be $k : 1$.



Using section formula, we have coordinates of C are

$$\left(\frac{-3k+3}{k+1}, \frac{-2k+5}{k+1} \right)$$

But, coordinates of C are given as $\left(\frac{3}{5}, \frac{11}{5} \right)$.

$$\therefore \frac{-3k+3}{k+1} = \frac{3}{5} \text{ and } \frac{-2k+5}{k+1} = \frac{11}{5}$$

$$\Rightarrow -15k+15 = 3k+3 \text{ and } -10k+25 = 11k+11$$

$$\Rightarrow -18k = -12 \text{ and } -21k = -14 \Rightarrow k = \frac{2}{3}$$

Hence, the ratio is $\frac{2}{3} : 1$ i.e., $2 : 3$.

14. Let coordinates of the point A be (x, y) and O is the mid-point of AB .

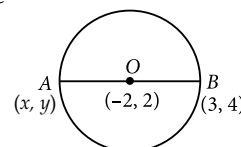
Using mid-point formula, we have

$$-2 = \frac{x+3}{2} \text{ and } 2 = \frac{y+4}{2}$$

$$\Rightarrow -4 = x+3 \text{ and } 4 = y+4$$

$$\Rightarrow x = -7 \text{ and } y = 0$$

$\therefore$ Coordinates of A are $(-7, 0)$.



15. Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram, $ABCD$ taken in order.

$\therefore$ The diagonals of parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{-1+2}{2}, \frac{0+2}{2} \right) = \left(\frac{3+x}{2}, \frac{1+y}{2} \right)$$

$$\Rightarrow \left(\frac{1}{2}, 1 \right) = \left(\frac{3+x}{2}, \frac{1+y}{2} \right) \Rightarrow \frac{3+x}{2} = \frac{1}{2} \text{ and } \frac{1+y}{2} = 1$$

$$\Rightarrow 3+x=1 \text{ and } 1+y=2 \Rightarrow x=-2 \text{ and } y=1$$

Hence, the coordinates of fourth vertex is $(-2, 1)$

16. Let D , E and F be the mid-point of the sides BC , CA and AB respectively. Then, the coordinates of D , E and F are

$$D\left(\frac{5+3}{2}, \frac{3-1}{2} \right) = D(4, 1),$$

$$E = \left(\frac{3+7}{2}, \frac{-1-3}{2} \right) = E(5, -2)$$

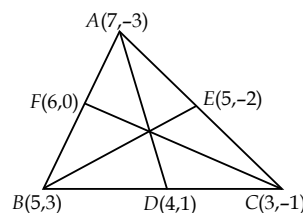
$$\text{and } F\left(\frac{7+5}{2}, \frac{-3+3}{2} \right) = F(6, 0)$$

$$\therefore AD = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = 5 \text{ units;}$$

$$BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0+25} = 5 \text{ units and}$$

$$CF = \sqrt{(6-3)^2 + (0+1)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$\therefore$ Lengths of medians of $\triangle ABC$ are 5 units, 5 units and $\sqrt{10}$ units.



17. Let $D(2, -1)$ and $E(0, -1)$ be the mid-point of AB and AC respectively.

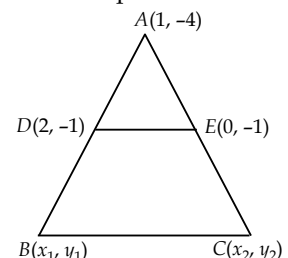
Let coordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively.

$\therefore$ Mid-point of AB

$$= \left(\frac{x_1+1}{2}, \frac{y_1-4}{2} \right)$$

$$\Rightarrow (2, -1) = \left(\frac{x_1+1}{2}, \frac{y_1-4}{2} \right)$$

$$\Rightarrow 2 = \frac{x_1+1}{2} \text{ and } -1 = \frac{y_1-4}{2}$$



(Given)

$$\Rightarrow 4 = x_1 + 1 \text{ and } -2 = y_1 - 4 \Rightarrow x_1 = 3 \text{ and } y_1 = 2$$

$\therefore$ Coordinates of B are $(3, 2)$.

$$\text{Now, mid-point of } AC = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2} \right)$$

$$\Rightarrow (0, -1) = \left(\frac{1+x_2}{2}, \frac{-4+y_2}{2} \right) \quad (\text{Given})$$

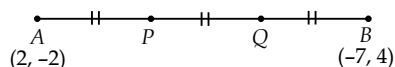
$$\Rightarrow 0 = \frac{1+x_2}{2} \text{ and } -1 = \frac{-4+y_2}{2}$$

$$\Rightarrow x_2 + 1 = 0 \text{ and } -2 = -4 + y_2 \Rightarrow x_2 = -1 \text{ and } y_2 = 2$$

$\therefore$ Coordinates of C are $(-1, 2)$.

$$\text{Now, mid-point of } BC = \left(\frac{3-1}{2}, \frac{2+2}{2} \right) = (1, 2).$$

18. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ divides AB into three equal parts.



$\therefore$ P divides AB in the ratio $1 : 2$

Using section formula, we have

$$(x_1, y_1) = \left(\frac{-7+4}{1+2}, \frac{4-4}{1+2} \right) = \left(\frac{-3}{3}, 0 \right) = (-1, 0)$$

$\therefore$ Coordinates of P are $(-1, 0)$

Also, Q divides AB in the ratio $2 : 1$.

Using section formula, we have

$$(x_2, y_2) = \left(\frac{-14+2}{2+1}, \frac{8-2}{2+1} \right) = \left(\frac{-12}{3}, \frac{6}{3} \right) = (-4, 2)$$

$\therefore$ Coordinates of Q are $(-4, 2)$.

19. Let $G(x, y)$ be the centroid of triangle.

$$\therefore (x, y) = \left(\frac{-2+4+4}{3}, \frac{3-3+5}{3} \right) = \left(\frac{6}{3}, \frac{5}{3} \right) = \left(2, \frac{5}{3} \right)$$

20. Let G be the centroid of triangle.

$$\therefore \text{Centroid of } \triangle ABC (G) = \left(\frac{-1+0-5}{3}, \frac{3+4+2}{3} \right) = \left(\frac{-6}{3}, \frac{9}{3} \right) = (-2, 3)$$

Since, G lies on the median $x - 2y + k = 0$

So, coordinates of G must satisfy the equation of median.

$$\therefore -2 - 2 \times 3 + k = 0 \Rightarrow -2 - 6 + k = 0 \Rightarrow k = 8$$

21. Let $C(x, y)$ be the third vertex.

$$\text{Given, centroid of } \triangle ABC, (G) = \left(\frac{5}{3}, -\frac{1}{3} \right)$$

$$\Rightarrow \left(\frac{3-2+x}{3}, \frac{2+1+y}{3} \right) = \left(\frac{5}{3}, -\frac{1}{3} \right)$$

$$\Rightarrow \left(\frac{1+x}{3}, \frac{3+y}{3} \right) = \left(\frac{5}{3}, -\frac{1}{3} \right)$$

$$\Rightarrow \frac{1+x}{3} = \frac{5}{3} \text{ and } \frac{3+y}{3} = -\frac{1}{3}$$

$$\Rightarrow 1+x = 5 \text{ and } 3+y = -1$$

$$\Rightarrow x = 4 \text{ and } y = -4$$

Hence, coordinates of C are $(4, -4)$.

