

1. (b) : We have, $\frac{\sin A}{\tan A} + \frac{\cos A \cot A}{\operatorname{cosec} A}$

$$= \frac{\sin A}{\frac{\sin A}{\cos A}} + \cos A \times \frac{\cos A}{\sin A} \times \sin A$$

$$\left[\because \tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}, \frac{1}{\operatorname{cosec} A} = \sin A \right]$$

$$= \cos A + \cos^2 A$$

2. (a) : We have, $\sin \theta = \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = 1 \Rightarrow \tan \theta = 1$$

$$\Rightarrow \tan \theta = \tan 45^\circ$$

$$\Rightarrow \theta = 45^\circ$$

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$[\because \tan 45^\circ = 1]$$

3. (c) : Given, $\sec(2x + 17)^\circ = \sqrt{2}$

$$\Rightarrow \sec(2x + 17)^\circ = \sec 45^\circ$$

$$[\because \sec 45^\circ = \sqrt{2}]$$

$$\Rightarrow 2x + 17 = 45 \Rightarrow 2x = 45 - 17$$

$$\Rightarrow 2x = 28 \Rightarrow x = 14$$

4. (b) : We have, $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \cdot \cos 4^\circ \cdot \dots \cdot \cos 100^\circ$

$$= \cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 90^\circ \dots \cos 100^\circ$$

$$= 0$$

$$[\because \cos 90^\circ = 0]$$

5. (b) : $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$

$$= \cos[90^\circ - (45^\circ + \theta)] - \cos(45^\circ - \theta) [\because \cos(90^\circ - \theta) = \sin \theta]$$

$$= \cos(45^\circ - \theta) - \cos(45^\circ - \theta) = 0$$

6. We have, $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

$$= \sin^2 \theta + \frac{1}{\sec^2 \theta} [\because 1 + \tan^2 \theta = \sec^2 \theta]$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$[\because \sec \theta = 1/\cos \theta]$$

$$= 1$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

7. Given, $\sin \theta = \frac{a}{b}$

$$\therefore \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \sqrt{1 - \left(\frac{a}{b}\right)^2} = \sqrt{1 - \frac{a^2}{b^2}} = \frac{\sqrt{b^2 - a^2}}{b}$$

8. $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta}$

$$= \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{2}{1 - \sin^2 \theta} = \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta$$

9. In $\triangle OPQ$, we have

$$OQ^2 = OP^2 + PQ^2$$

$$\Rightarrow (1 + PQ)^2 = OP^2 + PQ^2$$

$$\Rightarrow 1 + PQ^2 + 2PQ = OP^2 + PQ^2 \Rightarrow 1 + 2PQ = 7^2$$

$$\Rightarrow PQ = 24 \text{ cm and } OQ = 1 + PQ = 25 \text{ cm}$$

$$\therefore \sin Q = \frac{7}{25} \text{ and } \cos Q = \frac{24}{25}$$

10. L.H.S. = $\sqrt{\cos \alpha \operatorname{cosec} \beta - \cos \alpha \sin \beta}$

$$= \sqrt{\cos \alpha \operatorname{cosec}(90^\circ - \alpha) - \cos \alpha \sin(90^\circ - \alpha)}$$

$$[\text{Given } \alpha + \beta = 90^\circ]$$

$$= \sqrt{\cos \alpha \sec \alpha - \cos \alpha \cos \alpha} = \sqrt{1 - \cos^2 \alpha} = \sin \alpha = \text{R.H.S.}$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

11. $24 \cot A = 7 \Rightarrow \cot A = \frac{7}{24}$

$$\therefore \sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{7}{24}\right)^2}} = \frac{1}{\sqrt{1 + \frac{49}{576}}}$$

$$= \frac{1}{\sqrt{\frac{576 + 49}{576}}} = \frac{1}{\sqrt{\frac{625}{576}}} = \frac{1}{\left(\frac{25}{24}\right)} = \frac{24}{25}$$

12. Given, $x = b \sec^3 \theta$ and $y = a \tan^3 \theta$

$$\Rightarrow \sec^3 \theta = \frac{x}{b} \text{ and } \tan^3 \theta = \frac{y}{a} \quad \dots(i)$$

Now consider, $\left(\frac{x}{b}\right)^{2/3} - \left(\frac{y}{a}\right)^{2/3}$

$$= (\sec^3 \theta)^{2/3} - (\tan^3 \theta)^{2/3}$$

$$(\text{Using (i)})$$

$$= \sec^2 \theta - \tan^2 \theta = 1$$

13. We know that $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta)^3 = 1^3 \quad [\text{Cubing both the sides}]$$

$$\Rightarrow (\sin^2 \theta)^3 + (\cos^2 \theta)^3 + 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) = 1$$

$$\Rightarrow \sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta = 1$$

14. L.H.S. = $(\sin^4 \theta - \cos^4 \theta + 1) \operatorname{cosec}^2 \theta$

$$= [(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) + 1] \operatorname{cosec}^2 \theta$$

$$= (\sin^2 \theta - \cos^2 \theta + 1) \operatorname{cosec}^2 \theta \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= 2 \sin^2 \theta \operatorname{cosec}^2 \theta$$

$$[\because 1 - \cos^2 \theta = \sin^2 \theta]$$

$$= 2 = \text{R.H.S.}$$

15. We have, $2(\operatorname{cosec}^2\theta - 1)\tan^2\theta$
 $= 2(\cot^2\theta)\tan^2\theta$ [$\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta$]
 $= 2$ [$\because \tan\theta \cdot \cot\theta = 1$]

16. L.H.S. $= \frac{\tan^2 A(1 + \cot^2 A)}{(1 + \tan^2 A)} = \frac{\tan^2 A(\operatorname{cosec}^2 A)}{\sec^2 A}$
 $[\because 1 + \cot^2 A = \operatorname{cosec}^2 A; 1 + \tan^2 A = \sec^2 A]$
 $= \frac{\sin^2 A}{\cos^2 A} \cdot \frac{\cos^2 A}{\sin^2 A} = 1 = \text{R.H.S.}$

17. (i) In ΔAPQ , $\tan\theta = \frac{AQ}{PQ} = \frac{1.2}{1.6} = \frac{3}{4}$

(ii) In ΔPBQ , $\cot B = \frac{QB}{PQ} = \frac{3}{1.6} = \frac{15}{8}$... (1)

(iii) In ΔAPQ , $\tan A = \frac{PQ}{AQ} = \frac{1.6}{1.2} = \frac{4}{3}$... (2)

(iv) We have, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sec A = \sqrt{\left(\frac{4}{3}\right)^2 + 1}$ [Using (2)]
 $= \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

(v) Since, $\operatorname{cosec} B = \sqrt{\cot^2 B + 1}$
 $= \sqrt{\left(\frac{15}{8}\right)^2 + 1}$ [Using (1)]
 $= \frac{17}{8}$

18. $\because \Delta PQR$ is a right angled triangle.

$\therefore PR^2 + RQ^2 = PQ^2$
 $\Rightarrow PR^2 = (13)^2 - (12)^2 = 25 \Rightarrow PR = 5 \text{ cm}$

(i) (c) : $\cos\theta = \frac{QR}{PQ} = \frac{12}{13}$

(ii) (c) : $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$

(iii) (c) : $\tan\theta = \frac{PR}{RQ} = \frac{5}{12}$... (1)

$\therefore \frac{\tan\theta}{1 + \tan^2\theta} = \frac{\frac{5}{12}}{1 + \frac{25}{144}} = \frac{\frac{5}{12}}{\frac{169}{144}} = \frac{60}{169}$

(iv) (a) : $\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$ [Using (1)]

$\operatorname{cosec}\theta = \frac{PQ}{PR} = \frac{13}{5}$

$\therefore \cot^2\theta - \operatorname{cosec}^2\theta = \frac{144}{25} - \frac{169}{25} = -1$

(v) (b) : $\sin^2\theta + \cos^2\theta = 1$ (Using identity)

19. We have, $KL = 4 \text{ cm}$, $ML = 4\sqrt{3} \text{ cm}$, $KM = 8 \text{ cm}$

(i) (a) : $\tan M = \frac{KL}{LM} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan M = \tan 30^\circ \Rightarrow \angle M = 30^\circ$

(ii) (c) : $\tan K = \frac{ML}{KL} = \frac{4\sqrt{3}}{4} = \sqrt{3} = \tan 60^\circ$

$\Rightarrow \angle K = 60^\circ$

(iii) (b) (iv) (c)

(v) (a) : $\frac{\tan^2 45^\circ - 1}{\tan^2 45^\circ + 1} = \frac{(1)^2 - 1}{1^2 + 1} = \frac{0}{2} = 0$

20. We have, $AB = BC = 6\sqrt{2} \text{ m}$
and $AC = 12 \text{ m}$.

(i) $\because D$ is mid point of AC .

$\therefore AD = DC = 6 \text{ m}$

Now, $AB^2 = BD^2 + AD^2$ ($\because \Delta ABD$ is a right triangle)

$\Rightarrow BD^2 = (6\sqrt{2})^2 - 6^2 = 72 - 36 = 36$

$\Rightarrow BD = 6 \text{ m}$... (1)

(ii) In ΔABD , $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$

[Using (1)]

$\Rightarrow \sin A = \sin 45^\circ \Rightarrow \angle A = 45^\circ$

(iii) In ΔBDC , $\tan C = \frac{BD}{DC} = \frac{6}{6}$

[Using (1)]

$\Rightarrow \tan C = 1 = \tan 45^\circ \Rightarrow \angle C = 45^\circ$

(iv) $\sin A = \frac{1}{\sqrt{2}}$, $\cos C = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

(v) $\tan C = 1$, $\tan A = \tan 45^\circ = 1$

$\Rightarrow \tan^2 C + \tan^2 A = 1 + 1 = 2$

21. Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$ [$\because \tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan 45^\circ = 1$]

$\Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$ [$\because \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$]

$$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

22. Given that, $A = 60^\circ$ and $B = 30^\circ$

$$\therefore \cos A = \cos 60^\circ = \frac{1}{2}; \cos B = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{and } \cos(A + B) = \cos(60^\circ + 30^\circ) = \cos 90^\circ = 0$$

$$\text{Now, } \cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \neq 0$$

$$\therefore \cos(A + B) \neq \cos A + \cos B.$$

23. L.H.S. = $\operatorname{cosec}^2 60^\circ \sec^2 30^\circ \cos^2 0^\circ \sin 45^\circ \cot^2 60^\circ \tan^2 60^\circ$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 \left(\frac{2}{\sqrt{3}}\right)^2 (1)^2 \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{3}}\right)^2 (\sqrt{3})^2 = \frac{4}{3} \times \frac{4}{3} \times \frac{1}{\sqrt{2}} \times \frac{1}{3} \times 3$$

$$= \frac{16}{9} \times \frac{1}{\sqrt{2}} = \frac{8\sqrt{2}}{9} = \text{R.H.S.}$$

OR

We have, $\sin(A + B) = 1$

$$\Rightarrow \sin(A + B) = \sin 90^\circ \Rightarrow A + B = 90^\circ \quad \dots(i)$$

$$\text{Also, } \sin(A - B) = \frac{1}{2}$$

$$\Rightarrow \sin(A - B) = \sin 30^\circ \Rightarrow A - B = 30^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

$$A + B + A - B = 120^\circ \Rightarrow 2A = 120^\circ \Rightarrow A = 60^\circ$$

$$\text{From (i), we have } 60^\circ + B = 90^\circ \Rightarrow B = 30^\circ$$

24. We have, $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$
 $= k + \tan^2 \theta + \cot^2 \theta$

$$\Rightarrow \sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta$$

$$+ \cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta$$

$$= k + \tan^2 \theta + \cot^2 \theta$$

$$\Rightarrow \sin^2 \theta + \operatorname{cosec}^2 \theta + 2 + \cos^2 \theta + \sec^2 \theta + 2$$

$$= k + \tan^2 \theta + \cot^2 \theta$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + \operatorname{cosec}^2 \theta - \cot^2 \theta$$

$$+ \sec^2 \theta - \tan^2 \theta + 4 = k$$

$$\Rightarrow 1 + 1 + 1 + 4 = k \Rightarrow k = 7$$

25. Given, $2\sin^2 \theta - \cos^2 \theta = 2$

$$\Rightarrow 2\sin^2 \theta - (1 - \sin^2 \theta) = 2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow 2\sin^2 \theta + \sin^2 \theta - 1 = 2$$

$$\Rightarrow 3\sin^2 \theta = 3 \Rightarrow \sin^2 \theta = 1$$

$$\Rightarrow \sin \theta = 1 = \sin 90^\circ \quad [\because \sin 90^\circ = 1]$$

$$\therefore \theta = 90^\circ$$

26. In $\triangle OPQ$, we have $OQ^2 = OP^2 + PQ^2$

$$\Rightarrow (1 + PQ)^2 = OP^2 + PQ^2 \quad [\because OQ - PQ = 1]$$

$$\Rightarrow 1 + PQ^2 + 2PQ = OP^2 + PQ^2$$

$$\Rightarrow 1 + 2PQ = 7^2 \Rightarrow 2PQ = 48$$

$$\Rightarrow PQ = 24 \text{ cm and } OQ = 1 + PQ = 25 \text{ cm}$$

$$\text{So, } \sin Q = \frac{OP}{OQ} = \frac{7}{25} \text{ and } \cos Q = \frac{PQ}{OQ} = \frac{24}{25}$$

27. In $\triangle ABC$, $\tan A = \frac{BC}{AB} = 1$

$$\Rightarrow BC = AB$$

Let $AB = BC = k$ units

$\therefore$ By Pythagoras theorem, we have

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(k)^2 + (k)^2} = k\sqrt{2} \text{ units}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ and } \cos A = \frac{AB}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{So, } 2 \sin A \cos A = 2 \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) = 1$$

Hence verified.

28. We have, $\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$

So, we draw a $\triangle ABC$, right-angled at B such that

$BC = 1$ unit and $AC = \sqrt{10}$ units.

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (\sqrt{10})^2 = AB^2 + 1^2$$

$$\Rightarrow AB^2 = 10 - 1 = 9$$

$$\Rightarrow AB = \sqrt{9} = 3 \text{ units}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{1}{\sqrt{10}}, \cos A = \frac{AB}{AC} = \frac{3}{\sqrt{10}},$$

$$\tan A = \frac{BC}{AB} = \frac{1}{3}, \sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$$

$$\text{and } \cot A = \frac{1}{\tan A} = \frac{3}{1} = 3$$

29. Since, $\tan R = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow R = 30^\circ$

$$\text{Again, } \cos P = \frac{1}{2} = \cos 60^\circ \Rightarrow P = 60^\circ$$

(i) Now, $\cos P \cos R + \sin P \sin R$

$$= \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$(ii) \cos(P - R) = \cos(60^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

From (i) and (ii), we get

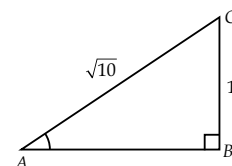
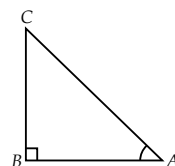
$$\cos(P - R) = \cos P \cos R + \sin P \sin R$$

30. Given, $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

Squaring both the sides, we get

$$(\cos \theta + \sin \theta)^2 = (\sqrt{2} \cos \theta)^2$$

$$\Rightarrow \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta = 2 \cos^2 \theta$$



$$\begin{aligned}
 &\Rightarrow 1 = 2\cos^2\theta - 2\cos\theta\sin\theta \\
 &\Rightarrow 1 - 2\cos^2\theta = -2\cos\theta\sin\theta \\
 &\Rightarrow 1 + 1 - 2\cos^2\theta = 1 - 2\cos\theta\sin\theta \\
 &\Rightarrow 2 - 2\cos^2\theta = \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta \\
 &\Rightarrow 2(1 - \cos^2\theta) = (\cos\theta - \sin\theta)^2 \\
 &\Rightarrow 2\sin^2\theta = (\cos\theta - \sin\theta)^2 \quad [\because \sin^2\theta + \cos^2\theta = 1] \\
 &\Rightarrow \cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta
 \end{aligned}$$

31. We have, $\sin\theta = \frac{24}{25}$

$$\Rightarrow \sin^2\theta = \left(\frac{24}{25}\right)^2 \Rightarrow 1 - \cos^2\theta = \frac{576}{625}$$

$$\Rightarrow \cos^2\theta = 1 - \frac{576}{625} = \frac{625 - 576}{625} = \frac{49}{625}$$

$$\Rightarrow \cos\theta = \frac{7}{25} \Rightarrow \sec\theta = \frac{25}{7}$$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{24/25}{7/25} = \frac{24}{7}$$

$$\therefore 100(\sec\theta + \tan\theta) = 100\left(\frac{25}{7} + \frac{24}{7}\right) = 100\left(\frac{49}{7}\right) = 700$$

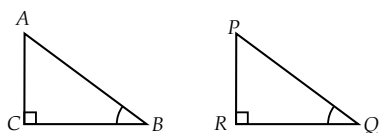
OR

$$\begin{aligned}
 \text{L.H.S.} &= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} \\
 &= \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta} \right] \\
 &= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cdot \cos^2\theta}} = \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}} \quad [\because \sin^2\theta + \cos^2\theta = 1] \\
 &= \frac{1}{\sin\theta \cdot \cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta} \quad [\because 1 = \sin^2\theta + \cos^2\theta] \\
 &= \frac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \frac{\cos^2\theta}{\sin\theta \cdot \cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\
 &= \tan\theta + \cot\theta \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta} \right] \\
 &= \text{R.H.S.}
 \end{aligned}$$

32. Let us consider two triangles ABC and PQR, right angled at C and R respectively such that $\sin B = \sin Q$.

We have, $\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

Then $\frac{AC}{AB} = \frac{PR}{PQ} \Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ... (i)



Now, by using Pythagoras theorem, we have

$$BC = \sqrt{AB^2 - AC^2} \text{ and } QR = \sqrt{PQ^2 - PR^2}$$

$$\begin{aligned}
 \text{So, } \frac{BC}{QR} &= \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} \\
 &= \frac{\sqrt{k^2 PQ^2 - k^2 PR^2}}{\sqrt{PQ^2 - PR^2}} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k \quad \dots (ii)
 \end{aligned}$$

From (i) and (ii), we have, $\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

So, $\triangle ACB \sim \triangle PRQ$ and therefore, $\angle B = \angle Q$.

33. Given, $\sin(A + C - B) = \frac{\sqrt{3}}{2}$ and $\cot(B + C - A) = \sqrt{3}$

$$\Rightarrow \sin(A + C - B) = \sin 60^\circ \text{ and } \cot(B + C - A) = \cot 30^\circ$$

$$\Rightarrow A + C - B = 60^\circ \quad \dots (i)$$

$$\text{and } B + C - A = 30^\circ \quad \dots (ii)$$

Adding (i) and (ii), we have $2C = 90^\circ \Rightarrow C = 45^\circ$

Putting this value of C in (i), we get $A - B = 15^\circ$... (iii)

Also, by angle sum property of a triangle

$$A + B + C = 180^\circ$$

$$\Rightarrow A + B = 135^\circ \quad \dots (iv)$$

Adding (iii) and (iv), we have

$$2A = 150^\circ \Rightarrow A = 75^\circ$$

From (iii), we have $B = 75^\circ - 15^\circ = 60^\circ$

OR

We are given that, $\operatorname{cosec}\theta = \frac{17}{12} \Rightarrow \sin\theta = \frac{12}{17}$

$$\therefore \cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{12}{17}\right)^2} = \frac{\sqrt{145}}{17}$$

$$\text{So, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{17} \times \frac{17}{\sqrt{145}} = \frac{12}{\sqrt{145}} \quad \dots (i)$$

$$\text{Now, } \sqrt{\frac{(\sec\theta - 1)(\sec\theta + 1)}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}} = \sqrt{\frac{(\sec^2\theta - 1)}{(\operatorname{cosec}^2\theta - 1)}}$$

$$= \sqrt{\frac{\tan^2\theta}{\cot^2\theta}} = \sqrt{\left(\frac{\tan\theta}{\cot\theta}\right)^2} = \frac{\tan\theta}{\cot\theta} = \tan\theta \times \tan\theta = \tan^2\theta$$

$$= \left(\frac{12}{\sqrt{145}}\right)^2 \quad [\text{Using (i)}]$$

$$= \frac{144}{145}$$

34. We have, $a \sin\theta = b \cos\theta$... (i)

Also, $a \sin^3\theta + b \cos^3\theta = \sin\theta \cos\theta$

$$\Rightarrow (a \sin\theta) \sin^2\theta + b \cos^3\theta = \sin\theta \cos\theta$$

$$\Rightarrow b \cos\theta \cdot \sin^2\theta + b \cos^3\theta = \sin\theta \cos\theta \quad [\text{Using (i)}]$$

$$\Rightarrow b \cos\theta [\sin^2\theta + \cos^2\theta] = \sin\theta \cos\theta$$

$$\Rightarrow b \cos\theta \times 1 = \sin\theta \cos\theta$$

$$\Rightarrow b = \sin \theta$$

From (i) and (ii), we have

$$a \cdot b = b \cos \theta \Rightarrow a = \cos \theta$$

Squaring and adding (ii) and (iii), we get

$$a^2 + b^2 = \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow a^2 + b^2 = 1$$

Hence proved.

35. Given, $m = \cos A - \sin A$, $n = \cos A + \sin A$

$$\text{Now, } \frac{m}{n} - \frac{n}{m} = \frac{m^2 - n^2}{mn}$$

$$= \frac{(\cos A - \sin A)^2 - (\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$$

$$\begin{aligned} \dots(ii) \quad & \cos^2 A + \sin^2 A - 2 \cos A \sin A - \cos^2 A \\ &= \frac{-\sin^2 A - 2 \sin A \cos A}{\cos^2 A - \sin^2 A} \\ \dots(iii) \quad &= \frac{-4 \sin A \cos A}{\cos^2 A - \sin^2 A} \end{aligned} \quad \dots(i)$$

Dividing numerator and denominator by $\sin A \cos A$, we get

$$\frac{\frac{-4}{\cos^2 A} - \frac{\sin^2 A}{\sin A \cos A}}{\frac{\sin A \cos A}{\sin A \cos A}} = \frac{-4}{\cot A - \tan A} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{m}{n} - \frac{n}{m} = \frac{-4 \sin A \cos A}{\cos^2 A - \sin^2 A} = \frac{-4}{\cot A - \tan A}$$

