Introduction to Trigonometry

CHAPTER 8

NCERT FOCUS

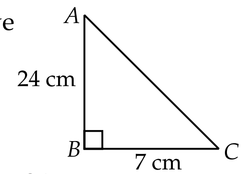
SOLUTIONS

EXERCISE - 8.1

1. In right angle ΔABC, we have

$AB = 24$ cm, $BC = 7$ cm

Using Pythagoras theorem, we have



$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = 24^2 + 7^2$

$= 576 + 49 = 625 = 25^2$

$\Rightarrow \quad AC = 25$ cm

(i) $\sin A = \dfrac{BC}{AC} = \dfrac{7}{25}, \ \cos A = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii) $\sin C = \dfrac{AB}{AC} = \dfrac{24}{25}, \ \cos C = \dfrac{BC}{AC} = \dfrac{7}{25}$

2. In right angle ΔPQR

Using Pythagoras theorem, we have

$QR^2 = PR^2 - PQ^2$

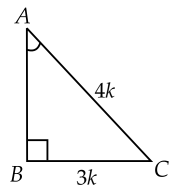
$\Rightarrow \quad QR^2 = 13^2 - 12^2 = (13 - 12)(13 + 12) = 1 \times 25 = 25$

$\therefore \quad QR = \sqrt{25} = 5$ cm

Now, $\tan P = \dfrac{QR}{PQ} = \dfrac{5}{12}, \ \cot R = \dfrac{QR}{PQ} = \dfrac{5}{12}$

$\therefore \quad \tan P - \cot R = \dfrac{5}{12} - \dfrac{5}{12} = 0$

3. In right angle ΔABC, we have



$\sin A = \dfrac{BC}{AC} = \dfrac{3}{4}$

Let $BC = 3k$ units and $AC = 4k$ units

Using Pythagoras theorem, we have

$AB^2 = AC^2 - BC^2$

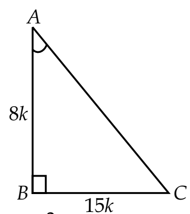
$= (4k)^2 - (3k)^2 = (4k - 3k)(4k + 3k) = k(7k) = 7k^2$

$\Rightarrow \quad AB = \sqrt{7k^2} = \sqrt{7}k$

$\therefore \quad \cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{7}k}{4k} = \dfrac{\sqrt{7}}{4}$

Also, $\tan A = \dfrac{BC}{AB} = \dfrac{3k}{\sqrt{7}k} = \dfrac{3}{\sqrt{7}}$

4. In right angle ΔABC, we have



$15 \cot A = 8 \Rightarrow \cot A = 8/15$

$\Rightarrow \quad \cot A = \dfrac{AB}{BC} = \dfrac{8}{15}$

Let $AB = 8k$ units and $BC = 15k$ units

Using Pythagoras theorem, we have

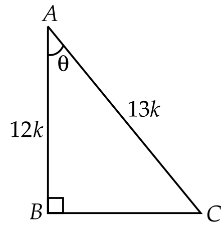
$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2 = (17k)^2$

$\Rightarrow \quad AC = \sqrt{(17k)^2} = 17k$

$\therefore \quad \sin A = \dfrac{BC}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$ and $\sec A = \dfrac{AC}{AB} = \dfrac{17k}{8k} = \dfrac{17}{8}$

5. Consider a right angled ΔABC with $\angle B = 90°$



Let $\angle A = \theta$ and $\sec \theta = 13/12$

$\Rightarrow \quad \dfrac{AC}{AB} = \dfrac{13}{12}$

Let $AC = 13k$ units and

$AB = 12k$ units

Using Pythagoras theorem, we have

$BC^2 = AC^2 - AB^2 \Rightarrow BC^2 = (13k)^2 - (12k)^2$

$= (13k - 12k)(13k + 12k) = k(25k) = 25k^2 = (5k)^2$

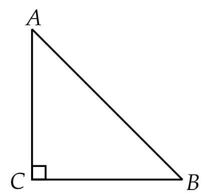
$\Rightarrow \quad BC = \sqrt{(5k)^2} = 5k$

$\therefore \quad \sin\theta = \dfrac{BC}{AC} = \dfrac{5k}{13k} = \dfrac{5}{13},$

$\cot\theta = \dfrac{AB}{BC} = \dfrac{12k}{5k} = \dfrac{12}{5}, \quad \tan\theta = \dfrac{BC}{AB} = \dfrac{5k}{12k} = \dfrac{5}{12}$

$\cos\theta = \dfrac{AB}{AC} = \dfrac{12k}{13k} = \dfrac{12}{13}, \quad \text{cosec}\,\theta = \dfrac{AC}{BC} = \dfrac{13k}{5k} = \dfrac{13}{5}$

6. Let us consider a right ΔABC, $\angle C = 90°$



Now, $\cos A = \dfrac{AC}{AB}$ and

$\cos B = \dfrac{BC}{AB}$

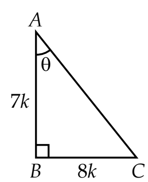
Since, $\cos A = \cos B$

$\therefore \quad \dfrac{AC}{AB} = \dfrac{BC}{AB} \Rightarrow AC = BC$

Now, in ΔABC, two sides AC and BC are equal.

$\therefore$ Their opposite angles are also equal. Hence, $\angle A = \angle B$

7. Let in right ΔABC, $\angle B = 90°$ and $\angle A = \theta$.



Given, $\cot\theta = \dfrac{7}{8} \Rightarrow \dfrac{AB}{BC} = \dfrac{7}{8}$

Now, let $AB = 7k$ units

and $BC = 8k$ units

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2$

$\Rightarrow \quad AC = \sqrt{49k^2 + 64k^2} = \sqrt{113k^2} = \sqrt{113}\,k$

$\therefore \quad \sin\theta = \dfrac{BC}{AC} = \dfrac{8k}{\sqrt{113}\,k} = \dfrac{8}{\sqrt{113}}$ and

$\cos\theta = \dfrac{AB}{AC} = \dfrac{7k}{\sqrt{113}\,k} = \dfrac{7}{\sqrt{113}}$

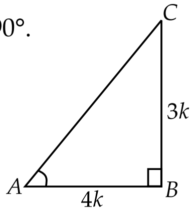
Now,

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

8. Let in a right-angled ΔABC, $\angle B = 90°$.



Now, $3 \cot A = 4$ (Given)

$\Rightarrow \cot A = \frac{4}{3} \Rightarrow \frac{AB}{BC} = \frac{4}{3}$

Now, let $AB = 4k$ units

and $BC = 3k$ units

Using Pythagoras theorem, we have,

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2$

$\Rightarrow AC = \sqrt{16k^2+9k^2} = \sqrt{25k^2} = \sqrt{(5k)^2} = 5k$ units

Now, $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

Also, $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, to check the given equation,

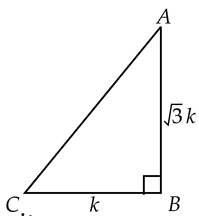
$$\text{L.H.S.} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

R.H.S. $= \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25} = \text{L.H.S.}$$

$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

9. In right ΔABC, $\angle B = 90°$



$\because \tan A = \frac{1}{\sqrt{3}}$ (Given)

$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Now, let $AB = \sqrt{3}\,k$ units and $BC = k$ units

Using Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (\sqrt{3}\,k)^2 + k^2$

$\Rightarrow AC = \sqrt{3k^2+k^2} = \sqrt{4k^2} = 2k$

Now, $\sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$, $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

Also, $\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$, $\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C$

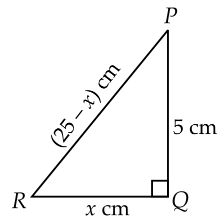
$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

10. In right ΔPQR, $\angle Q = 90°$

$PR + QR = 25$ cm and $PQ = 5$ cm

Let QR be x cm $\Rightarrow PR = (25 - x)$ cm

Using Pythagoras theorem, we have



$PR^2 = QR^2 + PQ^2$

$\Rightarrow (25 - x)^2 = x^2 + 5^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = \frac{-600}{-50} = 12$ *i.e.*, $QR = 12$ cm

$\Rightarrow PR = 25 - 12 = 13$ cm

Now, $\sin P = \frac{RQ}{RP} = \frac{12}{13}$, $\cos P = \frac{PQ}{RP} = \frac{5}{13}$ and

$\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

11. (i) **False**

$\because$ A tangent of an angle is the ratio of perpendicular to base which may be equal or unequal to each other.

(ii) **True**

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$ and hypotenuse is the greatest side of the triangle.

$\therefore$ $\cos A$ is always less than 1.

$\therefore$ $\frac{1}{\cos A}$ *i.e.*, $\sec A$ will always be greater than 1.

(iii) **False**

$\because$ 'cosine A' is abbreviated as 'cos A'.

(iv) **False**

$\because$ 'cot A' is a single and meaningful term whereas 'cot' alone has no meaning.

(v) **False**

$\because$ 4/3 is greater than 1 and $\sin\theta$ cannot be greater than 1.

EXERCISE - 8.2

1. (i) We have, $\sin 60° \cos 30° + \sin 30° \cos 60°$

$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\times\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) We have, $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2\times1 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) We have, $\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2+2\sqrt{3}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}\times\frac{1}{2(1+\sqrt{3})} = \frac{\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{2}}\times\frac{1-\sqrt{3}}{2(1+\sqrt{3})\times(1-\sqrt{3})}$$

$$= \frac{\sqrt{6}}{4}\times\frac{(1-\sqrt{3})}{1-3} = \frac{\sqrt{6}(1-\sqrt{3})}{4(-2)} = \frac{\sqrt{6}(\sqrt{3}-1)}{8}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) We have, $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{1+2}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1+2}{2}} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{3}{2}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{(3\sqrt{3})^2 + (4)^2 - 2\times 4\times 3\sqrt{3}}{(3\sqrt{3})^2-(4)^2} = \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) We have, $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}} = \frac{\frac{1}{12}(15+64-12)}{\frac{1+3}{4}} = \frac{\frac{1}{12}\times 67}{\frac{4}{4}} = \frac{67}{12}$$

2. (i) **(a)** : $\dfrac{2\tan 30°}{1+\tan^2 30°} = \dfrac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$

$$= \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{3}{\sqrt{3}}\times\frac{1}{2}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\times\sqrt{3}}{3\times 2} = \frac{\sqrt{3}}{2} = \sin 60°$$

(ii) **(d)** : $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} = \dfrac{1-(1)^2}{1+(1)^2} = \dfrac{1-1}{1+1} = \dfrac{0}{2} = 0$

(iii) **(a)** : When $A = 0°$, then

$\sin 2A = \sin 2(0°) = \sin 0° = 0$,

$2 \sin A = 2 \sin 0° = 2 \times 0 = 0$

i.e., $\sin 2A = 2\sin A$ for $A = 0°$

(iv) **(c)** : $\dfrac{2\tan 30°}{1-\tan^2 30°} = \dfrac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{2} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}}\times\sqrt{3} = \sqrt{3} = \tan 60°$$

3. We have,

$\tan 60° = \sqrt{3}$, $\tan 30° = \dfrac{1}{\sqrt{3}}$ …(i)

Also, $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \dfrac{1}{\sqrt{3}}$ …(ii)

From (i) and (ii), we get

$A + B = 60°$ …(iii)

and $A - B = 30°$ …(iv)

On adding (iii) and (iv), we get

$2A = 90° \Rightarrow A = 45°$

On subtracting (iv) from (iii), we get

$2B = 30° \Rightarrow B = 15°$

4. (i) **False :**

Let us take $A = 30°$ and $B = 60°$, then

L.H.S. = $\sin (30° + 60°) = \sin 90° = 1$

R.H.S.= $\sin 30° + \sin 60°$

$= \dfrac{1}{2}+\dfrac{\sqrt{3}}{2} = \dfrac{1+\sqrt{3}}{2}$, since $1 \neq \dfrac{1+\sqrt{3}}{2}$

∴ L.H.S. ≠ R.H.S.

(ii) **True :**

Since, the value of $\sin\theta$ increases from 0 to 1 as θ increases from 0° to 90°.

(iii) **False :**

Since, the value of $\cos\theta$ decreases from 1 to 0 as θ increases from 0° to 90°.

(iv) **False :**

Let us take $\theta = 30°$

$\sin 30° = \dfrac{1}{2}$ and $\cos 30° = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \sin 30° \neq \cos 30°$

(v) **True :**

We have, $\cot 0°$ = not defined

EXERCISE - 8.4

1. (i) $\sin A = \dfrac{1}{\text{cosec } A} = \dfrac{1}{\sqrt{\text{cosec}^2 A}} = \dfrac{1}{\sqrt{1+\cot^2 A}}$

(ii) $\sec A = \sqrt{\sec^2 A} = \sqrt{1+\tan^2 A}$

$$= \sqrt{1+\frac{1}{\cot^2 A}} = \sqrt{\frac{\cot^2 A + 1}{\cot^2 A}} = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

(iii) $\tan A = \dfrac{1}{\cot A}$

2. (i) $\sin A = \dfrac{\sin A}{1} = \dfrac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\tan A}{\sec A} = \frac{\sqrt{\tan^2 A}}{\sec A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(ii) $\cos A = \dfrac{1}{\sec A}$

(iii) $\tan A = \sqrt{\tan^2 A} = \sqrt{\sec^2 A - 1}$

(iv) $\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

(v) $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

3. (i) We have, $\sin 63° = \sin(90° - 27°) = \cos 27°$

$\Rightarrow \sin^2 63° = \cos^2 27°$

Similarly, $\cos^2 73° = \cos^2(90° - 17°) = \sin^2 17°$

$$\therefore \frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{\cos^2 27° + \sin^2 27°}{\cos^2 17° + \sin^2 17°} = 1$$

$[\because \cos^2 A + \sin^2 A = 1]$

(ii) We have, $\sin 25° = \sin(90° - 65°) = \cos 65°$

And $\cos 25° = \cos(90° - 65°) = \sin 65°$

$\therefore \sin 25° \cos 65° + \cos 25° \sin 65°$

$= \cos 65° \cos 65° + \sin 65° \sin 65° = (\cos 65°)^2 + (\sin 65°)^2$

$= \cos^2 65° + \sin^2 65° = 1$ $[\because \cos^2 A + \sin^2 A = 1]$

4. (i) **(b)** : We have, $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A) = 9(1) = 9$ $[\because \sec^2 A - \tan^2 A = 1]$

(ii) **(c)** : Here, $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= (1 + \tan\theta + \sec\theta)\left[1 + \frac{1}{\tan\theta} - \operatorname{cosec}\theta\right]$$

$$= (1 + \tan\theta + \sec\theta)\left[\frac{\tan\theta + 1 - \tan\theta\cdot\operatorname{cosec}\theta}{\tan\theta}\right]$$

$$= \frac{(1 + \tan\theta + \sec\theta)[\tan\theta + 1 - \sec\theta]}{\tan\theta}$$

$$\left[\because \tan\theta\cdot\operatorname{cosec}\theta = \frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta\right]$$

$$= \frac{(1 + \tan\theta)^2 - \sec^2\theta}{\tan\theta} = \frac{1 + \tan^2\theta + 2\tan\theta - \sec^2\theta}{\tan\theta}$$

$$= \frac{\sec^2\theta + 2\tan\theta - \sec^2\theta}{\tan\theta} \quad (\because 1 + \tan^2\theta = \sec^2\theta)$$

$$= \frac{2\tan\theta}{\tan\theta} = 2$$

(iii) **(d)** : We have, $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) = \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

(iv) **(d)** : Here, $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{1 + \tan^2 A}{1 + \dfrac{1}{\tan^2 A}}$

$$= \frac{1 + \tan^2 A}{\dfrac{\tan^2 A + 1}{\tan^2 A}} = (1 + \tan^2 A)\frac{\tan^2 A}{(1 + \tan^2 A)} = \tan^2 A$$

5. (i) L.H.S. $= (\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1 - \cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta} \quad [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$= \frac{(1 - \cos\theta)\times(1 - \cos\theta)}{(1 - \cos\theta)\times(1 + \cos\theta)} = \frac{1 - \cos\theta}{1 + \cos\theta} = \text{R.H.S.}$$

(ii) L.H.S. $= \dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A}$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1 + \sin A)\cos A}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1 + \sin A)\cos A}$$

$$= \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A} \quad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{2 + 2\sin A}{(1 + \sin A)\cos A} = \frac{2(1 + \sin A)}{\cos A(1 + \sin A)}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S.} \quad \left[\because \frac{1}{\cos A} = \sec A\right]$$

(iii) L.H.S. $= \dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{1 - \dfrac{\cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{1 - \dfrac{\sin\theta}{\cos\theta}} = \frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cdot\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cdot\cos\theta)}{\sin\theta\cdot\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(1 + \sin\theta\cdot\cos\theta)}{\sin\theta\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} + 1$$

$= 1 + \sec\theta \operatorname{cosec}\theta =$ R.H.S.

(iv) L.H.S. $= \dfrac{1 + \sec A}{\sec A} = \dfrac{1 + \dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$

$$= \frac{\dfrac{\cos A + 1}{\cos A}}{\dfrac{1}{\cos A}} = \frac{\cos A + 1}{\cos A}\times\frac{\cos A}{1} = \cos A + 1$$

$$= (1 + \cos A)\times\frac{(1 - \cos A)}{(1 - \cos A)}$$

[Multiplying and dividing by $(1 - \cos A)$]

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} \quad [\because 1 - \cos^2 A = \sin^2 A]$$

= R.H.S.

(v) L.H.S. = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$

[Dividing numerator and denominator by sin$A$]

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)(\cot A + \operatorname{cosec} A)}{(\cot A + 1 - \operatorname{cosec} A)(\cot A + \operatorname{cosec} A)}$$

[Multiplying and dividing by ($\cot A + \operatorname{cosec} A$)]

$$= \frac{[(\cot A + \operatorname{cosec} A) - 1](\cot A + \operatorname{cosec} A)}{[(\cot A - \operatorname{cosec} A) + 1](\cot A + \operatorname{cosec} A)}$$

$$= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{(\cot A - \operatorname{cosec} A)(\cot A + \operatorname{cosec} A) + (\cot A + \operatorname{cosec} A)}$$

$$= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{[\cot^2 A - \operatorname{cosec}^2 A] + (\cot A + \operatorname{cosec} A)}$$

$$= \frac{[\cot A + \operatorname{cosec} A - 1](\cot A + \operatorname{cosec} A)}{[-1 + \cot A + \operatorname{cosec} A]}$$

$[\because \cot^2 A - \operatorname{cosec}^2 A = -1]$

= $\cot A + \operatorname{cosec} A$ = R.H.S.

(vi) L.H.S. = $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

[Multiplying and dividing by $\sqrt{(1+\sin A)}$]

$$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}}$$

$$= \frac{\sqrt{(1+\sin A)^2}}{\sqrt{\cos^2 A}} \quad [\because 1 - \sin^2 A = \cos^2 A]$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= $\sec A + \tan A$ = R.H.S.

(vii) L.H.S. = $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$$= \frac{\sin\theta[(\sin^2\theta + \cos^2\theta) - 2\sin^2\theta]}{\cos\theta[2\cos^2\theta - (\sin^2\theta + \cos^2\theta)]}$$

$$= \frac{\sin\theta[\cos^2\theta - \sin^2\theta]}{\cos\theta[\cos^2\theta - \sin^2\theta]} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{R.H.S.}$$

(viii) L.H.S. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$

$[\because \sin A \operatorname{cosec} A = 1$ and $\sec A \cos A = 1]$

$= 1 + \operatorname{cosec}^2 A + \sec^2 A + 4 \quad [\because \sin^2 A + \cos^2 A = 1]$

$= 5 + (1 + \cot^2 A) + (1 + \tan^2 A)$

$= 7 + \cot^2 A + \tan^2 A$ = R.H.S.

(ix) L.H.S. = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right) = \frac{\cos^2 A \sin^2 A}{\sin A \cos A}$$

$[\because 1 - \sin^2 A = \cos^2 A$ and $1 - \cos^2 A = \sin^2 A]$

$= \sin A \cos A$

$$= \frac{\sin A \cos A}{1} = \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A} \quad [\because 1 = \sin^2 A + \cos^2 A]$$

$$= \frac{\frac{\sin A \cos A}{\sin A \cos A}}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}}$$

[Dividing num. and den. by $\sin A \cos A$]

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\tan A + \cot A} = \text{R.H.S.}$$

(x) We have, $\dfrac{1+\tan^2 A}{1+\cot^2 A} = \dfrac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$

$$= \frac{1+\tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}} = \frac{1+\tan^2 A}{1} \times \frac{\tan^2 A}{1+\tan^2 A} = \tan^2 A \quad \text{...(i)}$$

Also, $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)^2$$

$$= \left(\frac{(1-\tan A)}{1} \times \frac{-\tan A}{(1-\tan A)}\right)^2 = (-\tan A)^2 = \tan^2 A \quad \text{...(ii)}$$

$\therefore$ From (i) and (ii), we get

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$$