Introduction to Trigonometry

CHAPTER 8

TRY YOURSELF SOLUTIONS

1. In ΔPQR, $RQ^2 = PR^2 + PQ^2 = 3^2 + 1^2 = 9 + 1 = 10$

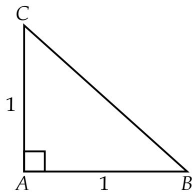
$\Rightarrow \quad RQ = \sqrt{10}$ cm

Now, $\cos R = \dfrac{PR}{RQ} = \dfrac{3}{\sqrt{10}}, \cos Q = \dfrac{PQ}{RQ} = \dfrac{1}{\sqrt{10}}$

$\cot Q = \dfrac{PQ}{PR} = \dfrac{1}{3}, \sec R = \dfrac{RQ}{PR} = \dfrac{\sqrt{10}}{3}$ and

$\tan R = \dfrac{PQ}{PR} = \dfrac{1}{3}$

2. In ΔABC, $BC^2 = AB^2 + AC^2$



$\Rightarrow \quad BC^2 = 1^2 + 1^2 = 2$

$\Rightarrow \quad BC = \sqrt{2}$ units

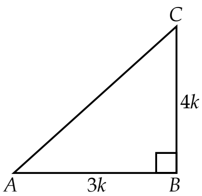
Now, $\sin B = \dfrac{AC}{BC} = \dfrac{1}{\sqrt{2}}$

$\cos B = \dfrac{AB}{BC} = \dfrac{1}{\sqrt{2}}, \tan B = \dfrac{AC}{AB} = \dfrac{1}{1} = 1,$

$\sec B = \dfrac{BC}{AB} = \dfrac{\sqrt{2}}{1} = \sqrt{2}, \operatorname{cosec} B = \dfrac{BC}{AC} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

and $\cot B = \dfrac{AB}{AC} = \dfrac{1}{1} = 1$

3. In ΔABC, $\tan A = \dfrac{BC}{AB} = \dfrac{4}{3}$.



Therefore, we have, $BC = 4k$ units
and $AB = 3k$ units
where k is a positive number.
By Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2 = (3k)^2 + (4k)^2 = 25k^2$

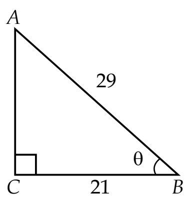
$\Rightarrow \quad AC = 5k$ units

Now, $\sin A = \dfrac{BC}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}, \ \cos A = \dfrac{AB}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5},$

$\cot A = \dfrac{AB}{BC} = \dfrac{3}{4}, \operatorname{cosec} A = \dfrac{AC}{BC} = \dfrac{5}{4}$

and $\sec A = \dfrac{AC}{AB} = \dfrac{5}{3}$.

4. In ΔACB, we have



$AC = \sqrt{AB^2 - BC^2} = \sqrt{(29)^2 - (21)^2}$

$= \sqrt{(29-21)(29+21)}$

$= \sqrt{(8)(50)} = \sqrt{400} = 20$ units

So, $\sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}, \cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$

(i) $\quad \cos^2\theta + \sin^2\theta = \left(\dfrac{21}{29}\right)^2 + \left(\dfrac{20}{29}\right)^2$

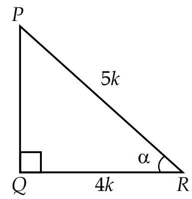
$$= \frac{21^2 + 20^2}{29^2} = \frac{441 + 400}{841} = 1$$

(ii) $\quad \cos^2\theta - \sin^2\theta$

$$= \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{(21+20)(21-20)}{29^2} = \frac{41}{841}$$

5. We have, $\sec\alpha = \dfrac{5}{4} = \dfrac{\text{Hypotenuse}}{\text{Base}}$

Let us draw a triangle PQR, right angled at Q such that $\angle PRQ = \alpha$, base $= QR = 4k$ units and hypotenuse $= PR = 5k$ units, where k is a positive number.



By Pythagoras theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow \quad 25k^2 = PQ^2 + 16k^2$

$\Rightarrow \quad PQ^2 = 25k^2 - 16k^2 = 9k^2 \Rightarrow PQ = 3k$ units

Now, $\tan\alpha = \dfrac{PQ}{QR} = \dfrac{3k}{4k} = \dfrac{3}{4}$

$$\therefore \quad \frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1-\frac{3}{4}}{1+\frac{3}{4}} = \frac{1}{7}$$

6. Given, $m\cot A = n$

$\Rightarrow \quad m \cdot \dfrac{1}{\tan A} = n \qquad \left[\because \cot A = \dfrac{1}{\tan A}\right]$

$\Rightarrow \quad \tan A = \dfrac{m}{n} \qquad \ldots\text{(i)}$

Now, $\dfrac{m\sin A - n\cos A}{n\cos A + m\sin A} = \dfrac{m \cdot \dfrac{\sin A}{\cos A} - n\dfrac{\cos A}{\cos A}}{n\dfrac{\cos A}{\cos A} + m \cdot \dfrac{\sin A}{\cos A}}$

[Dividing both numerator and denominator by $\cos A$]

$$= \frac{m\tan A - n}{n + m\tan A} = \frac{m\cdot\frac{m}{n} - n}{n + m\cdot\frac{m}{n}} \qquad \text{[From (i)]}$$

$$= \frac{\frac{m^2 - n^2}{n}}{\frac{n^2 + m^2}{n}} = \frac{m^2 - n^2}{m^2 + n^2}$$

7. We have, $3\tan A = 4 \Rightarrow \tan A = 4/3$

Now, $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}}$

$$= \sqrt{\frac{\frac{1}{\cos A} - \frac{1}{\sin A}}{\frac{1}{\cos A} + \frac{1}{\sin A}}} = \sqrt{\frac{\sin A - \cos A}{\sin A + \cos A}} = \sqrt{\frac{\tan A - 1}{\tan A + 1}}$$

[Dividing both numerator and denominator by $\cos A$]

$$= \sqrt{\frac{\frac{4}{3}-1}{\frac{4}{3}+1}} = \sqrt{\frac{\frac{1}{3}}{\frac{7}{3}}} = \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}} \qquad \left[\because \tan A = \frac{4}{3}\right]$$

Hence proved.

8. L.H.S $= \dfrac{\operatorname{cosec}^2\theta - \cos^2\theta}{\cot^2\theta} = \dfrac{\frac{1}{\sin^2\theta} - \cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}}$

$$= \frac{\frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta\sin^2\theta}{\cos^2\theta} = \sec^2\theta - \sin^2\theta = \text{R.H.S}$$

Hence proved

9. We have, $5\cos\theta = 7\sin\theta$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{7}{5} \qquad \ldots\text{(i)}$$

Now, $\dfrac{7\sin\theta + 5\cos\theta}{5\sin\theta + 7\cos\theta} = \dfrac{7+5\frac{\cos\theta}{\sin\theta}}{5+7\frac{\cos\theta}{\sin\theta}}$

[Dividing both numerator and denominator by $\sin\theta$]

$$= \frac{7+5\left(\frac{7}{5}\right)}{5+7\left(\frac{7}{5}\right)} \qquad \text{[Using (i)]}$$

$$= \frac{7+7}{5+\frac{49}{5}} = \frac{14}{\frac{74}{5}} = \frac{14\times 5}{74} = \frac{70}{74} = \frac{35}{37}$$

10. (i) We have, $\dfrac{4\cos^2 30^\circ - 5\sin 90^\circ}{6\sec^2 30^\circ + 2\cos 90^\circ}$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\right)^2 - 5\times 1}{6\left(\frac{2}{\sqrt{3}}\right)^2 + 2\times 0} = \frac{4\times\frac{3}{4} - 5}{6\times\frac{4}{3}} = \frac{3-5}{8} = \frac{-2}{8} = -\frac{1}{4}$$

(ii) We have, $\dfrac{\cos 30^\circ}{\sin^2 45^\circ} - \tan 60^\circ + 5\sin 0^\circ$

$$= \frac{\sqrt{3}/2}{\left(\frac{1}{\sqrt{2}}\right)^2} - \sqrt{3} + 5\times 0 = \frac{\sqrt{3}}{2}\times\frac{2}{1} - \sqrt{3} = 0$$

11. Here, L.H.S. $= \dfrac{1+\sin A}{\cos A}$

$$= \frac{1+\sin 60^\circ}{\cos 60^\circ} \qquad [\because A = 60^\circ]$$

$$= \frac{1+\sqrt{3}/2}{1/2} = 2+\sqrt{3}$$

Now, R.H.S. $= \dfrac{\cos A}{1-\sin A} = \dfrac{\cos 60^\circ}{1-\sin 60^\circ} \qquad [\because A = 60^\circ]$

$$= \frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

$\therefore$ L.H.S. = R.H.S.

12. We have, $\sin(A-B) = \dfrac{1}{2}$ and $\cos(A+B) = \dfrac{1}{2}$

We know that, $\sin 30^\circ = \dfrac{1}{2}$ and $\cos 60^\circ = \dfrac{1}{2}$

$\therefore$ $A - B = 30^\circ$ ...(i)

$A + B = 60^\circ$ ...(ii)

On adding (i) and (ii), we get

$2A = 90^\circ \Rightarrow A = 45^\circ$

On substituting the value of A in (i), we get

$45^\circ - B = 30^\circ \Rightarrow B = 15^\circ$

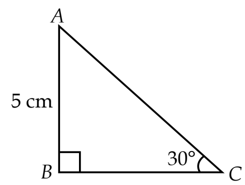
13. When $A = 10^\circ$, we have,

$$\frac{3\sin 3A + 2\cos(5A+10)^\circ}{\sqrt{3}\tan 3A + \operatorname{cosec}(5A-20)^\circ} = \frac{3\sin 30^\circ + 2\cos 60^\circ}{\sqrt{3}\tan 30^\circ + \operatorname{cosec} 30^\circ}$$

$$= \frac{3\left(\frac{1}{2}\right)+2\left(\frac{1}{2}\right)}{\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)+2} = \frac{\frac{3}{2}+1}{1+2} = \frac{\left(\frac{5}{2}\right)}{3} = \frac{5}{6}$$

14. We have, $\dfrac{\sin 90^\circ}{\tan 45^\circ} + \dfrac{1}{\sec 30^\circ} = \dfrac{1}{1} + \dfrac{1}{2/\sqrt{3}} = 1 + \dfrac{\sqrt{3}}{2}$

15. In ΔABC,



$\tan C = \dfrac{AB}{BC}$

$\Rightarrow \tan 30^\circ = \dfrac{5}{BC} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{5}{BC}$

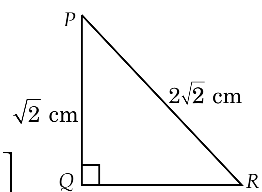
$\Rightarrow BC = 5\sqrt{3}$ cm

Now, $\sin 30^\circ = \dfrac{AB}{AC}$

$\Rightarrow \dfrac{1}{2} = \dfrac{5}{AC} \Rightarrow AC = 10$ cm

16. Given, $PQ = \sqrt{2}$ cm

and $PR = 2\sqrt{2}$ cm.



Clearly, $\sin R = \dfrac{PQ}{PR} = \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2}$

$\Rightarrow \sin R = \sin 30^\circ \quad \left[\because \sin 30^\circ = \dfrac{1}{2}\right]$

$\therefore$ $\angle R = 30^\circ$

Now, in ΔPQR, $\angle P + \angle Q + \angle R = 180^\circ$

[By angle sum property of triangle]

$\Rightarrow \angle P + 90^\circ + 30^\circ = 180^\circ \Rightarrow \angle P = 180^\circ - 120^\circ$

$\therefore$ $\angle P = 60^\circ$

17. We have, $\sec^2\theta - \dfrac{1}{\operatorname{cosec}^2\theta - 1}$

$= \sec^2\theta - \dfrac{1}{\cot^2\theta} \qquad [\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$= \sec^2\theta - \tan^2\theta \qquad \left[\because \tan\theta = \dfrac{1}{\cot\theta}\right]$

$= 1$

18. L.H.S. $= \sec A(1-\sin A)(\sec A + \tan A)$

$$= \left(\frac{1}{\cos A}\right)(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A} = \frac{1-\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1 = \text{R.H.S.}$$

19. L.H.S. = cot A + tan A = $\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$

$$= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin A} \cdot \frac{1}{\cos A} = \text{cosec}\, A \sec A$$

$$= \text{R.H.S.} \qquad \left[\because \ \text{cosec}\, A = \frac{1}{\sin A} \text{ and } \sec A = \frac{1}{\cos A}\right]$$

20. L.H.S = $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)} = \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{R.H.S.}$$

21. We have, $\sin^2\theta\,(1 + \cot^2\theta) = \sin^2\theta\,\text{cosec}^2\theta$

$$= \sin^2\theta \cdot \frac{1}{\sin^2\theta} = 1$$

22. We know that, cosec A = $\frac{1}{\sin A} = \frac{1}{\sqrt{1-\cos^2 A}}$

Also, tan A = $\frac{\sin A}{\cos A} = \frac{\sqrt{1-\cos^2 A}}{\cos A}$

And cot A = $\frac{1}{\tan A} = \frac{\cos A}{\sqrt{1-\cos^2 A}}$

23. Given, $\tan\theta = \frac{12}{5}$

$$\because \quad \sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{\frac{12}{5}}{\sqrt{1+\left(\frac{12}{5}\right)^2}} = \frac{\frac{12}{5}}{\sqrt{\frac{25+144}{25}}}$$

$$= \frac{\frac{12}{5}}{\sqrt{\frac{169}{25}}} = \frac{\frac{12}{5}}{\frac{13}{5}} = \frac{12}{13}$$

$$\text{Now, } \frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{13}}{\frac{13-12}{13}} = 25$$

24. $\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$$= \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Now, $3\sin^2\theta + 7\cos^2\theta$

$$= 3\left(\frac{1}{2}\right)^2 + 7\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} + \frac{21}{4} = \frac{24}{4} = 6$$