Mid Term

SOLUTIONS

1. **(d) :** We have, $\tan^2\theta\,(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \cot^2\theta$ [$\because$ $1 + \cot^2\theta = \text{cosec}^2\theta$]

$= \tan^2\theta \times \dfrac{1}{\tan^2\theta} = 1$

2. **(c) :** Mid-point of the line segment joining the points (7, 7) and (3, 5) = $\left(\dfrac{7+3}{2}, \dfrac{7+5}{2}\right)$ *i.e.*, (5, 6).

Distance of (5, 6) from (1, 3)

$= \sqrt{(5-1)^2 + (6-3)^2} = \sqrt{25} = 5$ units

3. **(c) :** Since, $DE \parallel BC$ [Given]

$\therefore$ $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ [By B.P.T.]

$\Rightarrow$ $\dfrac{1}{DB} = \dfrac{2}{6} = \dfrac{1}{3} \Rightarrow DB = 3$ cm

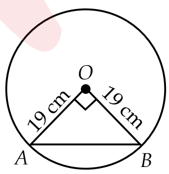
4. Let α, β be the roots of the equation
$3x^2 + (2k + 1)x - (k + 5) = 0$

$\therefore$ $\alpha + \beta = \dfrac{-(2k+1)}{3}$ and $\alpha\beta = \dfrac{-(k+5)}{3}$

Now, $\alpha + \beta = \alpha\beta$ [Given]

$\Rightarrow$ $-(2k + 1) = -(k + 5)$ $\Rightarrow$ $k = 4$

5. Let AB be the given chord.



Then, $AB^2 = OA^2 + OB^2$

[By Pythagoras theorem]

$\Rightarrow$ $AB^2 = 19^2 + 19^2$

$\Rightarrow$ $AB^2 = 2(19)^2$

$\Rightarrow$ $AB = 19\sqrt{2}$ cm

6. If three terms are in A.P., then $13 - (2p + 1) = (5p - 3) - 13$

$\Rightarrow$ $13 - 2p - 1 = 5p - 3 - 13 \Rightarrow 28 = 7p \Rightarrow p = 4$

7. Let $P(x, y)$ be the required point. Then,

$x = \dfrac{3\times(-4) + 2\times 6}{3+2}$ and $y = \dfrac{3\times 5 + 2\times 3}{3+2}$

$\Rightarrow$ $x = 0$ and $y = \dfrac{21}{5}$

So, the coordinates of P are (0, 21/5).

8. Given numbers are 1.08, 0.36 and 0.90.

H.C.F. of 108, 36 and 90 is 18.

$\therefore$ H.C.F of given numbers = 0.18

9. Let $A(-5, 7)$ be the given point and let $P(0, y)$ be the required point on the y-axis. Then,

$PA = 13$ units

$\Rightarrow$ $PA^2 = 169$

$\Rightarrow$ $(0 + 5)^2 + (y - 7)^2 = 169$

$\Rightarrow$ $y^2 - 14y - 95 = 0 \Rightarrow (y - 19)(y + 5) = 0$

$\Rightarrow$ $y = 19$ or $y = -5$. Hence, the required points are (0, 19) and (0, – 5).

10. L.H.S. $= \dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$

$= \dfrac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$

[$\because$ $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$]

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta = 1 =$ R.H.S.

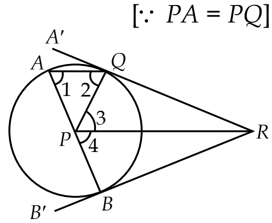
$\therefore$ L.H.S. = R.H.S.

11. Since, $AQ \parallel PR$ [Given]

$\therefore$ $\angle 1 = \angle 4$ and $\angle 2 = \angle 3$

Also $\angle 1 = \angle 2$ [$\because$ $PA = PQ$]

$\therefore$ $\angle 3 = \angle 4$



In ΔPQR and ΔPBR,

$PR = PR$ [Common]

$PQ = PB$ [Radii]

$\angle 3 = \angle 4$

$\therefore$ $\Delta PQR \cong \Delta PBR$ [By SAS]

$\Rightarrow$ $\angle PBR = \angle PQR$ [By CPCT]

But $\angle PQR = 90°$ [$\because$ QR is a tangent and PQ is radius]

$\therefore$ $\angle PBR = 90°$

Thus, BR is a tangent at B.

12. $\tan\theta = -1$ $\Rightarrow$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = -1$

Now, $\dfrac{\sec\theta + \text{cosec}\,\theta}{\cos\theta - \sin\theta} = \dfrac{\dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}}{\cos\theta - \sin\theta} = \dfrac{\dfrac{\sin\theta + \cos\theta}{\sin\theta\cdot\cos\theta}}{\cos\theta - \sin\theta}$

$= \dfrac{-\cos\theta + \cos\theta}{\sin\theta \times \cos\theta \times (\cos\theta - \sin\theta)}$ [As $\sin\theta = -\cos\theta$]

$= \dfrac{0}{\sin\theta \times \cos\theta\,(\cos\theta - \sin\theta)} = 0$

13. Let the fixed charges of taxi be ₹ x and the rate per km be ₹ y.

$\therefore$ According to question

$x + 12y = 45$ ...(i)

$x + 20y = 73$ ...(ii)

Subtracting (i) from (ii), we get

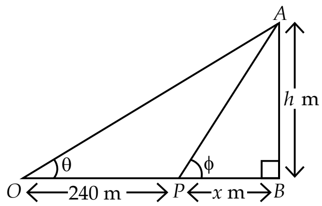
$8y = 28 \Rightarrow y = \frac{28}{8} = 3.5$

Now, substituting $y = 3.5$ in (i), we get

$x = 45 - 12 \times 3.5 = 45 - 42 = 3$

Hence, the fixed charges is ₹ 3 and rate per km is ₹ 3.5.

14. Let height of the lighthouse be h metres. Let O and P be the initial and final position of the person, making an angle of elevation θ and ϕ as shown in the figure. Let $PB = x$ metres



In right angled ΔOBA,

$$\tan\theta = \frac{AB}{OB} \Rightarrow \frac{5}{12} = \frac{h}{240+x} \quad ...(i) \quad \left[\because \tan\theta = \frac{5}{12} \text{ (Given)}\right]$$

In right angled ΔPBA,

$$\tan\phi = \frac{AB}{PB} \Rightarrow \frac{3}{4} = \frac{h}{x} \quad ...(ii) \quad \left[\because \tan\phi = \frac{3}{4} \text{ (Given)}\right]$$

Dividing (i) by (ii), we get

$$\frac{5}{12} \times \frac{4}{3} = \frac{h}{240+x} \times \frac{x}{h}$$

$$\Rightarrow \frac{5}{9} = \frac{x}{240+x} \Rightarrow 1200 + 5x = 9x$$

$\Rightarrow 4x = 1200$

$\Rightarrow x = 300$

Putting $x = 300$ in (ii) we get, $h = \frac{3}{4} \times 300 = 225$

Hence height of the lighthouse is 225 metres.

15. Since $DEFG$ is a square.

$\therefore \angle BDG = 90° = \angle FEC$

Also, $DG = GF = FE = DE$ ...(i)

In ΔBAC and ΔBDG,

$\angle ABC = \angle DBG$ [Common]

$\angle BAC = \angle BDG$ [Each 90°]

$\therefore \Delta BAC \sim \Delta BDG$ [By AA similarity criterion]

$$\Rightarrow \frac{AB}{BD} = \frac{AC}{DG} \Rightarrow \frac{AB}{AC} = \frac{BD}{DG} \quad ...(ii)$$

In ΔBAC and ΔFEC,

$\angle ACB = \angle ECF$ [Common]

$\angle BAC = \angle FEC$ [Each 90°]

$\therefore \Delta BAC \sim \Delta FEC$ (By AA similarity criterion)

$$\Rightarrow \frac{AB}{FE} = \frac{AC}{EC} \Rightarrow \frac{AB}{AC} = \frac{FE}{EC} \quad ...(iii)$$

From (ii) and (iii), we get

$$\frac{BD}{DG} = \frac{FE}{EC} \Rightarrow \frac{BD}{DE} = \frac{DE}{EC} \quad \text{[Using (i)]}$$

$\Rightarrow DE^2 = BD \times EC$