Mid Term

SOLUTIONS

1. (d) : We have, $\tan^2\theta\,(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \cot^2\theta$ $\quad [\because\ 1 + \cot^2\theta = \text{cosec}^2\theta]$

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1$$

2. (c) : Mid-point of the line segment joining the points $(7, 7)$ and $(3, 5)$ = $\left(\frac{7+3}{2}, \frac{7+5}{2}\right)$ *i.e.*, $(5, 6)$.

Distance of $(5, 6)$ from $(1, 3)$

$$= \sqrt{(5-1)^2 + (6-3)^2} = \sqrt{25} = 5 \text{ units}$$

3. (c) : Since, $DE \parallel BC$ $\quad$ [Given]

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ $\quad$ [By B.P.T.]

$\Rightarrow \quad \frac{1}{DB} = \frac{2}{6} = \frac{1}{3} \Rightarrow DB = 3 \text{ cm}$

4. Let α, β be the roots of the equation $3x^2 + (2k + 1)x - (k + 5) = 0$

$\therefore \quad \alpha + \beta = \frac{-(2k+1)}{3}$ and $\alpha\beta = \frac{-(k+5)}{3}$

Now, $\alpha + \beta = \alpha\beta$ $\quad$ [Given]

$\Rightarrow \quad -(2k + 1) = -(k + 5) \Rightarrow k = 4$

5. Let AB be the given chord.

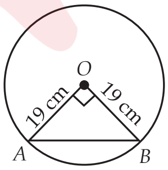
Then, $AB^2 = OA^2 + OB^2$

[By Pythagoras theorem]

$\Rightarrow \quad AB^2 = 19^2 + 19^2$

$\Rightarrow \quad AB^2 = 2(19)^2$

$\Rightarrow \quad AB = 19\sqrt{2}$ cm



6. If three terms are in A.P., then $13 - (2p + 1) = (5p - 3) - 13$

$\Rightarrow \quad 13 - 2p - 1 = 5p - 3 - 13 \Rightarrow 28 = 7p \Rightarrow p = 4$

7. Let $P(x, y)$ be the required point. Then,

$$x = \frac{3\times(-4)+2\times 6}{3+2} \text{ and } y = \frac{3\times 5+2\times 3}{3+2}$$

$\Rightarrow \quad x = 0$ and $y = \frac{21}{5}$

So, the coordinates of P are $(0, 21/5)$.

8. Given numbers are 1.08, 0.36 and 0.90.

H.C.F. of 108, 36 and 90 is 18.

$\therefore \quad$ H.C.F of given numbers = 0.18

9. Let $A(-5, 7)$ be the given point and let $P(0, y)$ be the required point on the y-axis. Then,

$PA = 13$ units

$\Rightarrow \quad PA^2 = 169$

$\Rightarrow \quad (0 + 5)^2 + (y - 7)^2 = 169$

$\Rightarrow \quad y^2 - 14y - 95 = 0 \Rightarrow (y - 19)(y + 5) = 0$

$\Rightarrow \quad y = 19$ or $y = -5$. Hence, the required points are $(0, 19)$ and $(0, -5)$.

10. L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$

$$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$[\because (a^3 + b^3) = (a + b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta = 1 =$ R.H.S.

$\therefore \quad$ L.H.S. = R.H.S.

11. Since, $AQ \parallel PR$ $\quad$ [Given]

$\therefore \quad \angle 1 = \angle 4$ and $\angle 2 = \angle 3$

Also $\angle 1 = \angle 2$ $\quad [\because PA = PQ]$

$\therefore \quad \angle 3 = \angle 4$

In ΔPQR and ΔPBR,

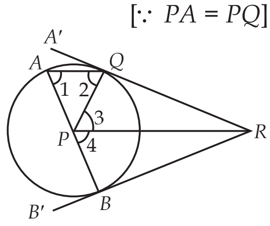
$PR = PR$ $\quad$ [Common]

$PQ = PB$ $\quad$ [Radii]

$\angle 3 = \angle 4$

$\therefore \quad \Delta PQR \cong \Delta PBR$ [By SAS]

$\Rightarrow \quad \angle PBR = \angle PQR$ [By CPCT]



But $\angle PQR = 90°$ $\quad [\because QR$ is a tangent and PQ is radius$]$

$\therefore \quad \angle PBR = 90°$

Thus, BR is a tangent at B.

12. $\tan\theta = -1 \Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = -1$

Now, $\frac{\sec\theta + \text{cosec}\,\theta}{\cos\theta - \sin\theta} = \frac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{\cos\theta - \sin\theta} = \frac{\frac{\sin\theta + \cos\theta}{\sin\theta\cdot\cos\theta}}{\cos\theta - \sin\theta}$

$$= \frac{-\cos\theta + \cos\theta}{\sin\theta \times \cos\theta \times (\cos\theta - \sin\theta)} \quad [\text{As } \sin\theta = -\cos\theta]$$

$$= \frac{0}{\sin\theta \times \cos\theta\,(\cos\theta - \sin\theta)} = 0$$

13. Let the fixed charges of taxi be ₹ x and the rate per km be ₹ y.

$\therefore \quad$ According to question

$x + 12y = 45$ $\quad$ ...(i)

$x + 20y = 73$ $\quad$ ...(ii)

Subtracting (i) from (ii), we get

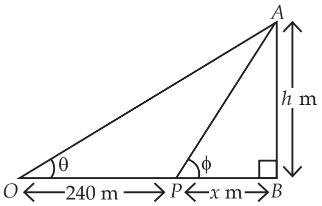
$8y = 28 \Rightarrow y = \frac{28}{8} = 3.5$

Now, substituting $y = 3.5$ in (i), we get

$x = 45 - 12 \times 3.5 = 45 - 42 = 3$

Hence, the fixed charges is ₹ 3 and rate per km is ₹ 3.5.

14. Let height of the lighthouse be h metres. Let O and P be the initial and final position of the person, making an angle of elevation θ and ϕ as shown in the figure. Let $PB = x$ metres



In right angled ΔOBA,

$\tan\theta = \frac{AB}{OB} \Rightarrow \frac{5}{12} = \frac{h}{240+x}$ ...(i) $\left[\because \tan\theta = \frac{5}{12} \text{(Given)}\right]$

In right angled ΔPBA,

$\tan\phi = \frac{AB}{PB} \Rightarrow \frac{3}{4} = \frac{h}{x}$ ...(ii) $\left[\because \tan\phi = \frac{3}{4} \text{ (Given)}\right]$

Dividing (i) by (ii), we get

$\frac{5}{12} \times \frac{4}{3} = \frac{h}{240+x} \times \frac{x}{h}$

$\Rightarrow \frac{5}{9} = \frac{x}{240+x} \Rightarrow 1200 + 5x = 9x$

$\Rightarrow 4x = 1200$

$\Rightarrow x = 300$

Putting $x = 300$ in (ii) we get, $h = \frac{3}{4} \times 300 = 225$

Hence height of the lighthouse is 225 metres.

15. Since $DEFG$ is a square.

$\therefore \angle BDG = 90° = \angle FEC$

Also, $DG = GF = FE = DE$ ...(i)

In ΔBAC and ΔBDG,

$\angle ABC = \angle DBG$ [Common]

$\angle BAC = \angle BDG$ [Each 90°]

$\therefore \Delta BAC \sim \Delta BDG$ [By AA similarity criterion]

$\Rightarrow \frac{AB}{BD} = \frac{AC}{DG} \Rightarrow \frac{AB}{AC} = \frac{BD}{DG}$ ...(ii)

In ΔBAC and ΔFEC,

$\angle ACB = \angle ECF$ [Common]

$\angle BAC = \angle FEC$ [Each 90°]

$\therefore \Delta BAC \sim \Delta FEC$ (By AA similarity criterion)

$\Rightarrow \frac{AB}{FE} = \frac{AC}{EC} \Rightarrow \frac{AB}{AC} = \frac{FE}{EC}$ ...(iii)

From (ii) and (iii), we get

$\frac{BD}{DG} = \frac{FE}{EC} \Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$ [Using (i)]

$\Rightarrow DE^2 = BD \times EC$