

Post-Mid Term

SOLUTIONS

1. (b) : Let r be the radius, h be the height and l be the slant height of right circular cone.

Now, $h = 15$ cm, Diameter = 16 cm [Given]

$$\therefore r = \frac{16}{2} = 8 \text{ cm}$$

$$\text{Also, } l = \sqrt{r^2 + h^2} = \sqrt{8^2 + 15^2} = \sqrt{289} = 17 \text{ cm}$$

$$\therefore \text{Curved surface area of the cone} = \pi r l \\ = \pi \times 8 \times 17 = 136\pi \text{ cm}^2$$

2. (d) : Total number of cards = 52

Number of ace cards = 4

\therefore Number of favourable outcomes = $52 - 4 = 48$
(Non ace cards)

$$\therefore P(\text{not an ace card}) = \frac{48}{52} = \frac{12}{13}$$

3. (d) : We know that mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$\Rightarrow 8.1 = \frac{132 + 5k}{20} \quad [\sum f_i x_i = 132 + 5k, \sum f_i = 20 \text{ (Given)}]$$

$$\Rightarrow 132 + 5k = 162 \Rightarrow 5k = 30 \Rightarrow k = 6$$

4. We have, $p(x) = x^2 - p(x+1) - c = x^2 - px - p - c$

$$\therefore \alpha + \beta = p \text{ and } \alpha\beta = (-p - c)$$

Now, $(\alpha + 1)(\beta + 1) = 0$ [Given]

$$\Rightarrow \alpha\beta + \alpha + \beta + 1 = 0 \Rightarrow -p - c + p + 1 = 0 \Rightarrow c = 1$$

$$5. 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$$

6. Given, area of sector $OAPB = \frac{5}{12} \times \text{Area of circle}$

$$\Rightarrow \frac{x}{360^\circ} \times \pi r^2 = \frac{5}{12} \times \pi r^2 \Rightarrow x = 150^\circ$$

$$7. \text{ We have, } 3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$$

Here, $a = 3\sqrt{3}$, $b = 10$ and $c = \sqrt{3}$

$$\therefore \text{Discriminant } (D) = b^2 - 4ac = 10^2 - 4(3\sqrt{3})(\sqrt{3}) \\ = 100 - 36 = 64$$

8. Prime numbers which are less than 20 and starting from 2 are 2, 3, 5, 7, 11, 13, 17 and 19 i.e., 8 in number.

\therefore Number of favourable outcomes = 8

Total number of cards from 2 to 101 = 100

\therefore Total number of possible outcomes = 100

$\therefore P(\text{number on card is a prime number less than 20})$

$$= \frac{8}{100} = \frac{2}{25}$$

$$9. \text{ In } \triangle ABC, \frac{AP}{AB} = \frac{3}{5} \quad \dots(i)$$

$$\text{and } \frac{AQ}{AC} = \frac{6}{10} = \frac{3}{5} \quad \dots(ii)$$

$$\text{From (i) and (ii), we get } \frac{AP}{AB} = \frac{AQ}{AC} \Rightarrow PQ \parallel BC$$

(By converse of Thales theorem)

In $\triangle ABD$, $PQ \parallel BD$

$$\Rightarrow \frac{AP}{AB} = \frac{AR}{AD} \quad (\text{By Thales theorem})$$

$$\Rightarrow \frac{3}{5} = \frac{4.5}{AD} \Rightarrow AD = \frac{4.5 \times 5}{3} = 7.5 \text{ cm}$$

10. Tangents from an external point to a circle are equal in length.

Therefore,

$PA = PB \Rightarrow \triangle PAB$ is isosceles

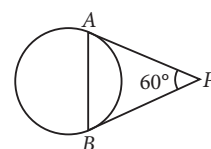
$$\Rightarrow \angle PAB = \angle PBA$$

In $\triangle APB$, $\angle PAB + \angle PBA + \angle APB = 180^\circ$

$$\Rightarrow 2\angle PAB = 180^\circ - 60^\circ = 120^\circ \Rightarrow \angle PAB = 60^\circ$$

$\Rightarrow \triangle PAB$ is an equilateral triangle.

Hence, $AB = 12$ cm.



11. Let the three parts which are in A.P. = $a - d$, a , $a + d$.

Now, sum of three parts = 69

$$\Rightarrow (a - d) + (a) + (a + d) = 69$$

$$\Rightarrow 3a = 69$$

$$\Rightarrow a = 23$$

Also, the product of two smaller parts = 483 (Given)

$$\Rightarrow (a - d) \times a = 483$$

...(i)

Substituting $a = 23$ in (i), we get

$$(23 - d) \times 23 = 483$$

$$\Rightarrow 23 - d = \frac{483}{23} = 21 \Rightarrow d = 23 - 21 = 2$$

Hence, the three parts of 69 are 21, 23, 25.

12.

No. of Accidents (x_i)	Frequency (f_i)	$f_i x_i$
0	46	0
1	x	x
2	y	$2y$
3	25	75
4	10	40
5	5	25
	$\Sigma f_i = 86 + x + y = 200$	$\Sigma f_i x_i = 140 + x + 2y$

Now, mean = 1.46 (Given)

$$\Rightarrow 1.46 = \frac{140 + x + 2y}{200}$$

$$\Rightarrow 292 = 140 + x + 2y$$

$$\Rightarrow x + 2y = 152 \quad \dots(i)$$

$$\text{Also, } 86 + x + y = 200 \Rightarrow x + y = 114 \quad \dots(ii)$$

Solving (i) and (ii), we get $x = 76$, $y = 38$.

13. Join OP . Draw $BQ \perp OP$, $OR \perp AH$ and $OS \perp BK$ produced BK to S .

In $\triangle ARO$ and $\triangle OQB$, $OA = OB$

[Radii of the same circle]

$$\angle ARO = \angle OQB = 90^\circ$$

$$\angle AOR = \angle OBQ \text{ [Corresponding angles as } RS \parallel QB]$$

$$\therefore 180^\circ - \angle ARO - \angle AOR = 180^\circ - \angle OQB - \angle OBQ$$

[By angle sum property]

$$\Rightarrow \angle OAR = \angle BOQ$$

$$\therefore \triangle ARO \cong \triangle OQB \quad \text{[By AAS congruence criterion]}$$

$$\Rightarrow AR = OQ \quad \text{[By CPCT]}$$

$$\text{Let } AR = OQ = x$$

$$\text{Now, } AH + BK = (x + RH) + (SK - x)$$

$$= x + OP + OP - x \quad [\because RH = SK = OP]$$

$$= 2OP = 2OA \quad [\because OP = OA = \text{Radius}]$$

$$= AB \quad \therefore AH + BK = AB$$

14. Clearly, one round of wire covers $4 \text{ mm} \left(= \frac{4}{10} \text{ cm} \right)$

of the surface of the cylinder and length of the cylinder is 24 cm .

$$\therefore \text{Number of rounds to cover } 24 \text{ cm} = \frac{24}{4/10} = 60$$

Diameter of the cylinder = 20 cm

\therefore Radius of the cylinder, $r = 10 \text{ cm}$

Length of wire required in completing one round = $2\pi r$
 $= (2\pi \times 10) \text{ cm} = 20\pi \text{ cm}$.

\therefore Length of wire required in covering the whole surface = Length of wire required in completing 60 rounds

$$= (20\pi \times 60) \text{ cm} = 1200\pi \text{ cm}.$$

$$\text{Radius of copper wire} = 2 \text{ mm} = \frac{2}{10} \text{ cm}$$

$$\therefore \text{Volume of wire} = \left(\pi \times \frac{2}{10} \times \frac{2}{10} \times 1200\pi \right) \text{ cm}^3 = 48\pi^2 \text{ cm}^3$$

$$\text{So, weight of wire} = (48\pi^2 \times 8.88) \text{ gm} = 426.24 \pi^2 \text{ gm}$$

15. The length of each side of a square lawn is 58 cm .

\therefore Length of the diagonal of the square

$$= 58\sqrt{2} \text{ cm}$$

$$\text{Radius of the circle} = 29\sqrt{2} \text{ cm}.$$

Let A be the area of one of the circular ends. Then,

A = Area of a segment of angle 90° in a circle of radius $29\sqrt{2} \text{ cm}$.

$$\Rightarrow A = \left\{ \frac{22}{7} \times \frac{90^\circ}{360^\circ} - \sin 45^\circ \cos 45^\circ \right\} \times (29\sqrt{2})^2 \text{ cm}^2$$

$$\left[\because \text{Area of minor segment} = \left\{ \frac{\pi\theta}{360} - \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right\} r^2 \right]$$

$$\Rightarrow A = \left(\frac{11}{14} - \frac{1}{2} \right) \times 29 \times 29 \times 2 \text{ cm}^2 = \frac{3364}{7} \text{ cm}^2$$

\therefore Area of the whole lawn = Area of the square + 2 (Area of a circular end)

$$= \left\{ 58 \times 58 + 2 \times \frac{3364}{7} \right\} \text{ cm}^2 = \left\{ 3364 + 2 \times \frac{3364}{7} \right\} \text{ cm}^2$$

$$= 3364 \left(1 + \frac{2}{7} \right) \text{ cm}^2 = 3364 \times \frac{9}{7} \text{ cm}^2 = 4325.14 \text{ cm}^2$$

