Pre-Mid Term

SOLUTIONS

1. **(c) :** 95 = 5 × 19 and 152 = 2 × 2 × 2 × 19

∴ HCF (95, 152) = 19

2. **(b) :** Since, α and β are the zeroes of $f(x) = x^2 + x + 1$

∴ α + β = –1 and αβ = 1

Now, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-1}{1}=-1$

3. **(d) :** Since, 1 is a root of the equation $ay^2 + ay + 3 = 0$

∴ $a(1)^2 + a(1) + 3 = 0$

⇒ $2a = -3$ ⇒ $a=\frac{-3}{2}$

4. Common difference, $d = a_2 - a_1 = \frac{1-6b}{2b}-\frac{1}{2b}$

$=\frac{1-6b-1}{2b}=\frac{-6b}{2b}=-3$

5. $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

∴ Exponent of 2 in the prime factorisation of 144 = 4

6. Let two consecutive positive integers be x and $x + 1$. It is given that the product of two consecutive positive integers is 240.

∴ $x(x+1) = 240 \Rightarrow x^2 + x - 240 = 0$, which is the required quadratic equation.

7. $y = 0$ and $y = -5$ are parallel lines, hence they have no common solution.

8. Let first term = a and common difference = d.

According to the question, $5 \times a_5 = 10 \times a_{10}$

⇒ $5(a + 4d) = 10(a + 9d)$

⇒ $5a + 20d = 10a + 90d$ ⇒ $a = -14d$

Now, $a_{15} = a + 14d \Rightarrow a_{15} = -14d + 14d = 0$

9. 576 = 2 × 2× 2 × 2 × 2 × 2 × 3 × 3

448 = 2 × 2 × 2 × 2 × 2 × 2 × 7

∴ HCF of 576 and 448 = 64

∴ Number of sections = $\frac{576}{64}+\frac{448}{64}=9+7=16$

10. We have, $2qx^2 - (2q - p^2)x - p^2 = 0$

⇒ $2qx^2 - 2qx + p^2x - p^2 = 0$

⇒ $2qx(x - 1) + p^2(x - 1) = 0$

⇒ $(2qx + p^2)(x - 1) = 0$

⇒ $(2qx + p^2) = 0$ or $(x - 1) = 0$

⇒ $x=-\frac{p^2}{2q}$ or $x = 1$

11. We have, $x+\frac{6}{y}=6$ ...(i)

$3x-\frac{8}{y}=5$ ...(ii)

Multiplying (i) by 3, we get

$3x+\frac{18}{y}=18$ ...(iii)

Subtracting (ii) from (iii), we get

$\frac{1}{y}(18+8)=13$ ⇒ $\frac{26}{y}=13$ ⇒ $y=\frac{26}{13}=2$

Putting $y = 2$ in (i), we get

$x+\frac{6}{2}=6 \Rightarrow x+3=6$ ⇒ $x = 6 - 3 = 3$

Hence, $x = 3$ and $y = 2$.

12. Let a = first term and d = common difference

Now, $a_{12} = a + (12 - 1)d$ [∵ $a_n = a + (n - 1)d$]

⇒ $-13 = a + 11d$ [Given, $a_{12} = -13$] ...(i)

Also, $S_4=\frac{4}{2}[2a+(4-1)d]$

⇒ $24 = 2(2a + 3d)$ [Given, $S_4 = 24$]

⇒ $2a + 3d = 12$ ...(ii)

Multiplying (i) by 2 and subtracting from (ii), we get

$3d - 22d = 12 - (-26)$ ⇒ $-19d = 38 \Rightarrow d = -2$

Putting the value of d in (i), we get

$a = -13 - 11d = -13 - 11 \times (-2) = -13 + 22 = 9$

∴ $S_{10}=\frac{10}{2}[2a + (10 - 1)d]$

$= 5[2 \times 9 + 9 \times (-2)] = 5(18 - 18) = 0$

13. Let $\sqrt{3}$ be a rational number.

∴ $\sqrt{3}=\frac{a}{b}$, where a and b are co-prime integers, $b \neq 0$.

Squaring both sides, we get $3=\frac{a^2}{b^2}$.

Multiplying with b on both sides, we get $3b=\frac{a^2}{b}$

Now, LHS = 3 × b = Integer

and RHS $=\frac{a^2}{b}=\frac{\text{Integer}}{\text{Integer}}=$ Rational number

Since, LHS ≠ RHS

∴ Our supposition is wrong.

⇒ $\sqrt{3}$ is an irrational number.

Let $15+17\sqrt{3}$ be a rational number

$\therefore$ $15+17\sqrt{3}=\frac{a}{b}$

$\Rightarrow$ $17\sqrt{3}=\frac{a}{b}-15$

$\Rightarrow$ $\sqrt{3}=\frac{a-15b}{17b}$, which is a contradiction because $\sqrt{3}$ is an irrational number and $\frac{a-15b}{17b}$ is a rational number.

$\therefore$ Our supposition is wrong and hence $15+17\sqrt{3}$ is irrational.

14. Let denominator of a fraction = x

$\Rightarrow$ Numerator = $x-2$

$\therefore$ Original fraction $=\frac{x-2}{x}$ ...(i)

On adding 1 to numerator as well as denominator, the fraction becomes $\frac{x-2+1}{x+1}$ *i.e.*, $\frac{x-1}{x+1}$. ...(ii)

According to question,

$$\frac{x-2}{x}+\frac{x-1}{x+1}=\frac{19}{15}\Rightarrow\frac{(x-2)(x+1)+x(x-1)}{x(x+1)}=\frac{19}{15}$$

$$\Rightarrow\frac{x^2-2x+x-2+x^2-x}{x^2+x}=\frac{19}{15}$$

$$\Rightarrow\frac{2x^2-2x-2}{x^2+x}=\frac{19}{15}$$

$\Rightarrow$ $30x^2-30x-30=19x^2+19x$

$\Rightarrow$ $11x^2-49x-30=0$ $\Rightarrow 11x^2-55x+6x-30=0$

$\Rightarrow$ $11x(x-5)+6(x-5)=0\Rightarrow(x-5)(11x+6)=0$

$\Rightarrow$ $x=5,-\frac{6}{11}$

$\therefore$ $x=5$ (Rejecting fractional value)

Thus, original fraction $=\frac{5-2}{5}=\frac{3}{5}$.

15. $2x^2+12\sqrt{2}\,x+35=2x^2+7\sqrt{2}\,x+5\sqrt{2}\,x+35$

$=\sqrt{2}\,x[\sqrt{2}\,x+7]+5[\sqrt{2}\,x+7]$

$=[\sqrt{2}\,x+7][\sqrt{2}\,x+5]$

$\therefore$ Zeroes are $\frac{-5}{\sqrt{2}},\frac{-7}{\sqrt{2}}$. So zeroes of other polynomial are $\frac{-10}{\sqrt{2}},\frac{-14}{\sqrt{2}}$

Sum of zeroes $=\frac{-24}{\sqrt{2}}$ and product $=\frac{140}{2}=70$

$\therefore$ New polynomial is $x^2-(\text{sum})\,x+\text{product}$

$=x^2+\frac{24}{\sqrt{2}}x+70$ *i.e.*, $\sqrt{2}x^2+24x+70\sqrt{2}$