

Pre-Mid Term

SOLUTIONS

1. (c) : $95 = 5 \times 19$ and $152 = 2 \times 2 \times 2 \times 19$

\therefore HCF (95, 152) = 19

2. (b) : Since, α and β are the zeroes of $f(x) = x^2 + x + 1$

$\therefore \alpha + \beta = -1$ and $\alpha\beta = 1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-1}{1} = -1$

3. (d) : Since, 1 is a root of the equation $ay^2 + ay + 3 = 0$

$\therefore a(1)^2 + a(1) + 3 = 0$

$\Rightarrow 2a = -3 \Rightarrow a = \frac{-3}{2}$

4. Common difference, $d = a_2 - a_1 = \frac{1-6b}{2b} - \frac{1}{2b}$
 $= \frac{1-6b-1}{2b} = \frac{-6b}{2b} = -3$

5. $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

\therefore Exponent of 2 in the prime factorisation of 144 = 4

6. Let two consecutive positive integers be x and $x + 1$.
 It is given that the product of two consecutive positive integers is 240.

$\therefore x(x + 1) = 240 \Rightarrow x^2 + x - 240 = 0$, which is the required quadratic equation.

7. $y = 0$ and $y = -5$ are parallel lines, hence they have no common solution.

8. Let first term = a and common difference = d .

According to the question, $5 \times a_5 = 10 \times a_{10}$

$\Rightarrow 5(a + 4d) = 10(a + 9d)$

$\Rightarrow 5a + 20d = 10a + 90d \Rightarrow a = -14d$

Now, $a_{15} = a + 14d \Rightarrow a_{15} = -14d + 14d = 0$

9. $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$448 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7$

\therefore HCF of 576 and 448 = 64

\therefore Number of sections = $\frac{576}{64} + \frac{448}{64} = 9 + 7 = 16$

10. We have, $2qx^2 - (2q - p^2)x - p^2 = 0$

$\Rightarrow 2qx^2 - 2qx + p^2x - p^2 = 0$

$\Rightarrow 2qx(x - 1) + p^2(x - 1) = 0$

$\Rightarrow (2qx + p^2)(x - 1) = 0$

$\Rightarrow (2qx + p^2) = 0$ or $(x - 1) = 0$

$\Rightarrow x = -\frac{p^2}{2q}$ or $x = 1$

11. We have, $x + \frac{6}{y} = 6$... (i)

$3x - \frac{8}{y} = 5$... (ii)

Multiplying (i) by 3, we get

$3x + \frac{18}{y} = 18$... (iii)

Subtracting (ii) from (iii), we get

$\frac{1}{y}(18 + 8) = 13 \Rightarrow \frac{26}{y} = 13 \Rightarrow y = \frac{26}{13} = 2$

Putting $y = 2$ in (i), we get

$x + \frac{6}{2} = 6 \Rightarrow x + 3 = 6 \Rightarrow x = 6 - 3 = 3$

Hence, $x = 3$ and $y = 2$.

12. Let a = first term and d = common difference

Now, $a_{12} = a + (12 - 1)d$ [$\because a_n = a + (n - 1)d$]
 $\Rightarrow -13 = a + 11d$ [Given, $a_{12} = -13$] ... (i)

Also, $S_4 = \frac{4}{2}[2a + (4 - 1)d]$

$\Rightarrow 24 = 2(2a + 3d)$ [Given, $S_4 = 24$]

$\Rightarrow 2a + 3d = 12$... (ii)

Multiplying (i) by 2 and subtracting from (ii), we get

$3d - 22d = 12 - (-26) \Rightarrow -19d = 38 \Rightarrow d = -2$

Putting the value of d in (i), we get

$a = -13 - 11d = -13 - 11 \times (-2) = -13 + 22 = 9$

$\therefore S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$= 5[2 \times 9 + 9 \times (-2)] = 5(18 - 18) = 0$

13. Let $\sqrt{3}$ be a rational number.

$\therefore \sqrt{3} = \frac{a}{b}$, where a and b are co-prime integers, $b \neq 0$.

Squaring both sides, we get $3 = \frac{a^2}{b^2}$.

Multiplying with b on both sides, we get $3b = \frac{a^2}{b}$

Now, LHS = $3 \times b$ = Integer

and RHS = $\frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}}$ = Rational number

Since, LHS \neq RHS

\therefore Our supposition is wrong.

$\Rightarrow \sqrt{3}$ is an irrational number.

Let $15 + 17\sqrt{3}$ be a rational number

$$\therefore 15 + 17\sqrt{3} = \frac{a}{b}$$

$$\Rightarrow 17\sqrt{3} = \frac{a}{b} - 15$$

$\Rightarrow \sqrt{3} = \frac{a-15b}{17b}$, which is a contradiction because $\sqrt{3}$ is an irrational number and $\frac{a-15b}{17b}$ is a rational number.

\therefore Our supposition is wrong and hence $15 + 17\sqrt{3}$ is irrational.

14. Let denominator of a fraction = x

$$\Rightarrow \text{Numerator} = x - 2$$

$$\therefore \text{Original fraction} = \frac{x-2}{x} \quad \dots(i)$$

On adding 1 to numerator as well as denominator, the fraction becomes $\frac{x-2+1}{x+1}$ i.e., $\frac{x-1}{x+1}$. $\dots(ii)$

According to question,

$$\frac{x-2}{x} + \frac{x-1}{x+1} = \frac{19}{15} \Rightarrow \frac{(x-2)(x+1) + x(x-1)}{x(x+1)} = \frac{19}{15}$$

$$\Rightarrow \frac{x^2 - 2x + x - 2 + x^2 - x}{x^2 + x} = \frac{19}{15}$$

$$\Rightarrow \frac{2x^2 - 2x - 2}{x^2 + x} = \frac{19}{15}$$

$$\Rightarrow 30x^2 - 30x - 30 = 19x^2 + 19x$$

$$\Rightarrow 11x^2 - 49x - 30 = 0 \Rightarrow 11x^2 - 55x + 6x - 30 = 0$$

$$\Rightarrow 11x(x-5) + 6(x-5) = 0 \Rightarrow (x-5)(11x+6) = 0$$

$$\Rightarrow x = 5, -\frac{6}{11}$$

$\therefore x = 5$ (Rejecting fractional value)

$$\text{Thus, original fraction} = \frac{5-2}{5} = \frac{3}{5}.$$

$$\mathbf{15.} \quad 2x^2 + 12\sqrt{2}x + 35 = 2x^2 + 7\sqrt{2}x + 5\sqrt{2}x + 35$$

$$= \sqrt{2}x[\sqrt{2}x + 7] + 5[\sqrt{2}x + 7]$$

$$= [\sqrt{2}x + 7][\sqrt{2}x + 5]$$

\therefore Zeroes are $\frac{-5}{\sqrt{2}}, \frac{-7}{\sqrt{2}}$. So zeroes of other polynomial

$$\text{are } \frac{-10}{\sqrt{2}}, \frac{-14}{\sqrt{2}}$$

$$\text{Sum of zeroes} = \frac{-24}{\sqrt{2}} \text{ and product} = \frac{140}{2} = 70$$

\therefore New polynomial is $x^2 - (\text{sum})x + \text{product}$

$$= x^2 + \frac{24}{\sqrt{2}}x + 70 \text{ i.e., } \sqrt{2}x^2 + 24x + 70\sqrt{2}$$

