Straight Lines

EXAM DRILL

SOLUTIONS

1. **(a) :** Mid-point of $AB=\left(\frac{1+3}{2},\frac{4+0}{2}\right)=(2,2)$

Now, length of median through $C=\sqrt{(2-2)^2+(1-2)^2}$

$=\sqrt{0+1}=1$ unit

2. **(b) :** Let the coordinates of B be $(k, 0)$, then according to given condition, we have

$$\frac{0-(-3)}{k-5}=-2 \quad\Rightarrow\quad \frac{3}{k-5}=-2$$

$\Rightarrow\ 3=-2k+10 \Rightarrow 2k=7$

$\Rightarrow\ k=\frac{7}{2}$

Hence, coordinates of B are $\left(\frac{7}{2},0\right)$.

3. **(b) :** Slope of given line $y = x$ is 1

$\therefore$ Slope of perpendicular line $(m) = -1$

Now, equation of line which passes through (3, 2) and having slope '–1' is

$y-2=-1(x-3) \quad\Rightarrow\quad x+y=5.$

4. **(c) :** Equation of a line having intercepts $a, -b$ is

$$\frac{x}{a}-\frac{y}{b}=1 \quad\Rightarrow\quad y=\frac{bx}{a}-b$$

So, slope of this line *i.e.*, $m_1=\frac{b}{a}$.

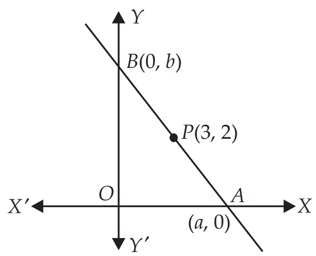
Also, equation of line having intercepts $b, -a$ is

$$\frac{x}{b}-\frac{y}{a}=1 \quad\Rightarrow\quad y=\frac{a}{b}x-a$$

Hence, slope of this line *i.e.*, $m_2=\frac{a}{b}$

Now, $\tan\theta=\frac{\frac{b}{a}-\frac{a}{b}}{1+\frac{a}{b}\cdot\frac{b}{a}}=\frac{b^2-a^2}{2ab}$

5. **(a) :** Since, the coordinates of the middle point is $P(3, 2)$



$\therefore\ 3=\frac{0+a}{2} \quad\Rightarrow\quad a=6$

Similarly, $2=\frac{0+b}{2} \Rightarrow b=4$

$\therefore$ Required equation of the line is

$\frac{x}{a}+\frac{y}{b}=1$ *i.e.*, $\frac{x}{6}+\frac{y}{4}=1$

$\Rightarrow\ 2x+3y=12$

6. The equation of the line passing through (2, – 3) and parallel to Y-axis is $x = 2$.

7. Let the equation of line be

$$\frac{x}{a}+\frac{y}{a}=1$$

or $\quad x+y=a$

Since, it passes through (2, 5), therefore we have

$2+5=a \Rightarrow a=7$

Hence, the required equation of line is $x + y = 7$

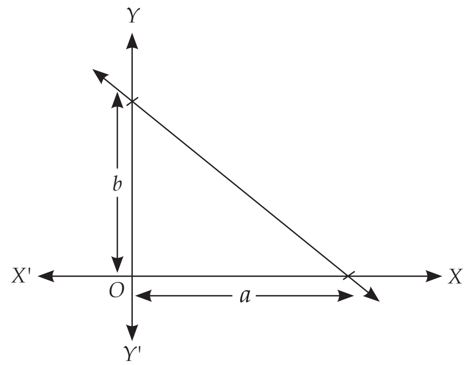
8. We have, $5x - 3y = 12$, for intersection with Y-axis put $x = 0$, we get

$-3y=12 \Rightarrow y=-4$

Thus, line cuts Y-axis at (0, – 4)

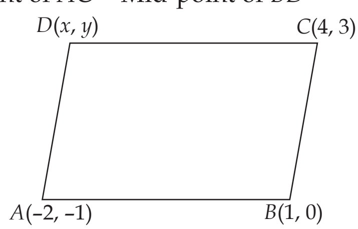
9. The area of the triangle formed by the given lines

$=\frac{1}{2}\times\text{base}\times\text{height}=\frac{1}{2}ab.$



10. Let $ABCD$ be the given parallelogram such that $A(-2, -1)$ $B(1, 0)$ and $C(4, 3)$.

Let the coordinates of fourth vertex be (x, y). Then, we get mid-point of AC = Mid-point of BD



$$\left(\frac{4-2}{2}, \frac{3-1}{2}\right) = \left(\frac{x+1}{2}, \frac{y+0}{2}\right)$$

$\Rightarrow \quad (1, 1) = \left(\frac{x+1}{2}, \frac{y}{2}\right)$

$\Rightarrow \frac{x+1}{2} = 1$ and $\frac{y}{2} = 1$

$\Rightarrow \quad x = 1$ and $y = 2$

Hence, the coordinates of point D are (1, 2).

11. Let the coordinates of third vertex be (x, y). Then,

$\frac{x+3-7}{3} = 2$ and $\frac{y-5+4}{3} = -1$

$\Rightarrow \quad x - 4 = 6$ and $y - 1 = -3 \Rightarrow x = 10$ and $y = -2$

Thus, the coordinates of the third vertex are (10, – 2).

12. The possible slopes of a line that makes equal angle with both axes are $m = 1$ and $m = -1$.

13. A line of negative slope makes an obtuse angle with the positive direction of X-axis in anticlockwise direction.

OR

The equation of line through a point (– 4, – 3) and parallel to X-axis is $y = -3$.

14. Since line cuts off equal intercepts.

$\therefore$ The intercepts along the X and Y-axes are a and a respectively.

$\therefore$ Equation of the line is $\frac{x}{a} + \frac{y}{a} = 1$ ...(i)

Since, the point (1, –2) lies on the line (i)

$\therefore \frac{1}{a} - \frac{2}{a} = 1 \quad \Rightarrow \quad a = -1$

Substituting $a = -1$ in (i), we get

$\frac{x}{-1} + \frac{y}{-1} = 1 \quad \Rightarrow \quad x + y = -1 \quad \Rightarrow \quad x + y + 1 = 0$ is the required equation of line.

15. **(i) :** Equation of BC is

$y - 1 = \frac{5-1}{4-2}(x-2)$

$\Rightarrow \quad y - 1 = 2x - 4 \Rightarrow 2x - y - 3 = 0$

(ii) Equation of AB is

$y - 5 = \frac{3-5}{-2-4}(x-4)$

$\Rightarrow \quad 3(y - 5) = x - 4$

$\Rightarrow \quad 3y - 15 = x - 4 \Rightarrow x - 3y + 11 = 0$

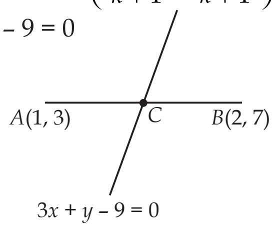
Let m_1 = Slope of AB = $\frac{3-5}{-2-4} = \frac{1}{3}$

and m_2 = Slope of BC = $\frac{5-1}{4-2} = 2 = 2$

Now, $\tan B = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{3} - 2}{1 + \frac{2}{3}}\right| = 1 \Rightarrow B = 45°$

16. Suppose the line $3x + y - 9 = 0$ divides the line segment joining $A(1, 3)$ and $B(2, 7)$ in the ratio $k : 1$ at point C. Then, the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$.

Also, point C lies on line $3x + y - 9 = 0$



$\Rightarrow \quad 3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0$

$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$

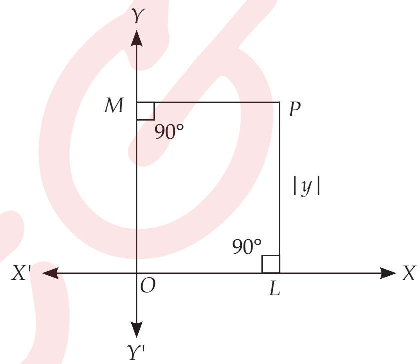
$\Rightarrow \quad 4k = 3 \Rightarrow k = 3/4$

$\therefore$ Required ratio is 3 : 4

17. Let $P(x, y)$ be the moving point.

Let $PL \perp X'X$

and $PM \perp Y'Y$



Given, $PL = 2PM$

$|y| = 2|x|$

$\Rightarrow \quad |y| = |2x|$

$\Rightarrow \quad y^2 = 4x^2$

Hence, the locus of P is $y^2 = 4x^2$

18. Let m be the slope of the line. Then,

$m = \tan 15° = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow \quad m = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

It is given that the line cuts an intercept of length 4 on the negative direction of Y-axis.

$\therefore \quad c = -4$

Substituting these values in $y = mx + c$, we get

$y = (2 - \sqrt{3})x - 4$ as the equation of the required line.

19. Assuming F along X-axis and K along Y-axis, we have two points (32, 273) and (212, 373) in XY-plane. By two-point form, the point (F, K) satisfies the equation

$$K - 273 = \frac{373 - 273}{212 - 32}(F - 32) \text{ or } K - 273 = \frac{100}{180}(F - 32)$$

or $\quad K = \frac{5}{9}(F - 32) + 273$

which is the required relation.

Also, when $K = 0$, we have

$$0 = \frac{5}{9}(F - 32) + 273$$

$$\Rightarrow \quad F - 32 = -273 \times \frac{9}{5}$$

$$F - 32 = -\frac{2457}{5} \Rightarrow F = \frac{-2297}{5}$$

20. Given equation of lines are

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{...(i)}$$

and $\frac{x}{a} - \frac{y}{b} = 1$ ...(ii)

$\therefore$ Slope of (i), $m_1 = \dfrac{-\left(\frac{1}{a}\right)}{\left(\frac{1}{b}\right)} = -\dfrac{b}{a}$

and slope of (ii), $m_2 = \dfrac{-\left(\frac{1}{a}\right)}{\left(-\frac{1}{b}\right)} = \dfrac{b}{a}$

Let θ be the angle between the given lines, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(\frac{-b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{-2b}{a}}{\frac{a^2 - b^2}{a^2}}\right| \Rightarrow \tan\theta = \frac{2ab}{a^2 - b^2}$$

Hence proved.

21. Given equation of line is,

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{...(i)}$$

Since perpendicular length from the origin on the line (i) is p

$$\Rightarrow \quad p = \left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| = \frac{ab}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \quad p^2 = \frac{a^2 b^2}{a^2 + b^2} \quad \text{... (ii)}$$

Since a^2, $2p^2$ and b^2 are in *A.P.*

$$\therefore \quad 4p^2 = a^2 + b^2 \Rightarrow \frac{4a^2b^2}{a^2 + b^2} = a^2 + b^2 \quad \text{(from (ii))}$$

$$\Rightarrow \quad 4a^2b^2 = a^4 + b^4 + 2a^2b^2 \Rightarrow \quad a^2 = b^2$$

Hence proved.

22. Let $A(5, 7)$, $B(6, 6)$ and $C(2, -2)$ be the vertices of the given triangle and let $P(x, y)$ be the circumcentre of this triangle.

Then, $PA^2 = PB^2 = PC^2$

Consider, $PA^2 = PB^2$

$\Rightarrow \quad (x - 5)^2 + (y - 7)^2 = (x - 6)^2 + (y - 6)^2$

$\Rightarrow \quad x^2 + 25 - 10x + y^2 + 49 - 14y = x^2 + 36 - 12x + y^2 + 36 - 12y$

$\Rightarrow \quad 2x - 2y + 2 = 0$

$\Rightarrow \quad x - y + 1 = 0$ ...(i)

Now, consider $PB^2 = PC^2$

$(x - 6)^2 + (y - 6)^2 = (x - 2)^2 + (y + 2)^2$

$\Rightarrow \quad x^2 + 36 - 12x + y^2 + 36 - 12y = x^2 + 4 - 4x + y^2 + 4 + 4y$

$\Rightarrow \quad -8x - 16y + 64 = 0$

$\Rightarrow \quad x + 2y - 8 = 0$ ...(ii)

On solving (i) and (ii), we get

$x = 2, y = 3$

Thus, the coordinates of circumcentre are $P(2, 3)$)

Now, circum-radius $PA = \sqrt{(2-5)^2 + (3-7)^2}$

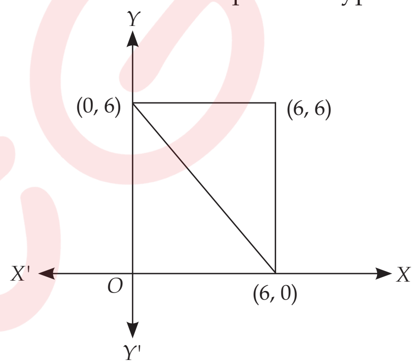
$= \sqrt{9 + 16} = 5$ units

23. Given vertices of a right angled triangle, right angled at (6, 6).

Clearly, coordinates of orthocentre are (6, 6)

[$\because$ Orthocentre is intersection of altitudes]

Also, circumcentre is the mid-point of hypotenuse.



$\therefore$ Required coordinates of circumcentre are

$$\left(\frac{0+6}{2}, \frac{6+0}{2}\right) = (3, 3)$$

Distance between circumcentre and orthocentre is

$$\sqrt{(6-3)^2 + (6-3)^2} = 3\sqrt{2} \text{ units.}$$

24. Let the coordinates of P be (x, y). Then,

$PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (x - 3)^2 + (y - 4)^2 = (x - 5)^2 + (y + 2)^2 \Rightarrow x - 3y - 1 = 0$

Now, Area of $\Delta PAB = 10$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10$$

$\Rightarrow \quad 6x + 2y - 26 = \pm 20$

$\Rightarrow \quad 6x + 2y - 46 = 0$ or, $6x + 2y - 6 = 0$

Solving $x - 3y - 1 = 0$ and $3x + y - 23 = 0$, we get, $x = 7$, $y = 2$.

Solving $x - 3y - 1 = 0$ and $3x + y - 3 = 0$, we get $x = 1$, $y = 0$.

Thus, the coordinates of P are (7, 2) or (1, 0).

OR

The vertices of the triangle are $A(-36, 7)$, $B(20, 7)$ and $C(0, -8)$. So we have,

$$a = BC = \sqrt{(0 - 20)^2 + (-8 - 7)^2} = 25$$

$$b = CA = \sqrt{(-36 - 0)^2 + (7 + 8)^2} = 39$$

$c = AB = \sqrt{(20+36)^2 + (7-7)^2} = 56$

We know that, incentre of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Where a, b and c are length of sides opposite to $\angle A$, $\angle B$ and $\angle C$ respectively.

$\therefore$ Coordinates of incentre O (say) are

$$\left(\frac{25(-36)+39(20)+56(0)}{25+39+56}, \frac{25(7)+39(7)+56(-8)}{25+39+56}\right)$$

$= (-1, 0)$

25. Let $A = (h, 0)$, $B = (0, k)$ and $P = (x, y)$

Given $AP = a$, $PB = b$

Now, $\frac{AP}{PB} = \frac{a}{b}$

Hence, P divides AB in the ratio $a : b$

$\therefore \quad x = \frac{a \cdot 0 + bh}{a+b} = \frac{bh}{a+b}$ ...(1)

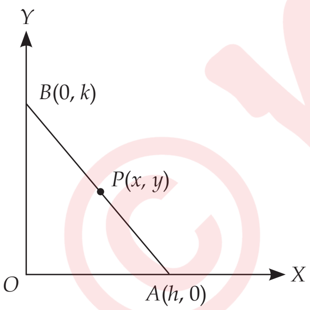
and $y = \frac{a \cdot k + b \cdot 0}{a+b} = \frac{ak}{a+b}$ ...(2)

Also, $AB^2 = OA^2 + OB^2$

$\therefore \quad (a+b)^2 = h^2 + k^2$

$= \frac{(a+b)^2 x^2}{b^2} + \frac{(a+b)^2 y^2}{a^2}$

[Putting the values of h and k from (1) and (2)]



or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Which is the required equation of the locus of P.

26. Let ABC be a triangle the coordinates of whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, where $x_1, x_2, x_3, y_1, y_2, y_3$ are integers. Let $\angle BAC = 30°$.

We have,

m_1 = Slope of BA

$\frac{y_1 - y_2}{x_1 - x_2}$,

and m_2 = Slope of $AC = \frac{y_1 - y_3}{x_1 - x_3}$

Now, $\angle BAC = 30°$

$\Rightarrow \quad \tan 30° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \left|\frac{\frac{y_1 - y_2}{x_1 - x_2} - \frac{y_1 - y_3}{x_1 - x_3}}{1 + \frac{y_1 - y_2}{x_1 - x_2} \times \frac{y_1 - y_3}{x_1 - x_3}}\right|$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \left|\frac{(y_1 - y_2)(x_1 - x_3) - (y_1 - y_3)(x_1 - x_2)}{(x_1 - x_2)(x_1 - x_3) + (y_1 - y_2)(y_1 - y_3)}\right|$

This is not possible as L.H.S. is an irrational number and R.H.S. is a rational number because $x_1, x_2, x_3, y_1, y_2, y_3$ are integers

OR

Let $P(2, 3)$, $Q(4, 1)$, $R(1, 2)$ and $S(4, 3)$ be the given points.

Equation of $PQ : y - 1 = \frac{3-1}{2-4}(x - 4)$

$\Rightarrow \quad y - 1 = \frac{-2}{2}(x - 4)$

$\Rightarrow \quad x + y - 5 = 0$ ...(i)

Equation of $SR : (y - 3) = \frac{2-3}{1-4}(x - 4)$

$\Rightarrow \quad y - 3 = \frac{1}{3}(x - 4)$

$\Rightarrow \quad 3y - 9 = x - 4$

$\Rightarrow \quad x - 3y + 5 = 0$ ...(ii)

Solving (i) and (ii), we get the point of intersection $\left(\frac{5}{2}, \frac{5}{2}\right)$.

Let $\left(\frac{5}{2}, \frac{5}{2}\right)$ divides RS in ratio $k : 1$, then $\frac{3k+2}{k+1} = \frac{5}{2}$

$\left[\because \frac{my_2 + ny_1}{m+n} = y\right]$

$\Rightarrow \quad 6k + 4 = 5k + 5$

$\Rightarrow \quad k = 1$

$\therefore$ Required ratio is $1 : 1$.

27. Let the line through $P(1, -7)$ meets the axes at $A(a, 0)$ and $B(0, b)$.

Given that $4AP - 3BP = 0$

$\Rightarrow \quad \frac{AP}{BP} = \frac{3}{4}$

$\therefore \quad P$ divides AB internally in the ratio $3 : 4$.

Thus coordinates of P are given by

$$P = \left(\frac{4a + 3(0)}{4+3}, \frac{4(0) + 3b}{4+3}\right)$$

$\Rightarrow \quad (1, -7) = \left(\frac{4a}{7}, \frac{3b}{7}\right)$

$\Rightarrow \quad \frac{4a}{7} = 1$ and $\frac{3b}{7} = -7$

$\Rightarrow \quad a = \frac{7}{4}$ and $b = \frac{-49}{3}$

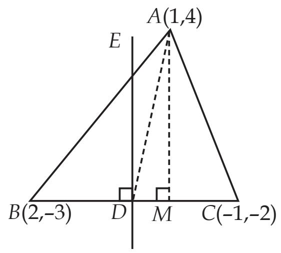
So, the equation of line will be

$$\frac{x}{\frac{7}{4}} + \frac{y}{\frac{-49}{3}} = 1$$

$\Rightarrow \quad 28x - 3y = 49$

28. (i) Let D be the mid-point of BC.

Then, $D=\left(\frac{2+(-1)}{2},\frac{-3+(-2)}{2}\right)$ *i.e.*, $D=\left(\frac{1}{2},-\frac{5}{2}\right)$



Slope of $AD=\frac{-\frac{5}{2}-4}{\frac{1}{2}-1}=\frac{-\frac{13}{2}}{-\frac{1}{2}}=13$

$\therefore$ The equation of the median AD is

$y-4=13(x-1)\Rightarrow 13x-y-9=0$

(ii) Let AM be the altitude through A, then $AM\perp BC$

Slope of $BC=\frac{-2-(-3)}{-1-2}=\frac{-2+3}{-3}=-\frac{1}{3}$

$\therefore$ Slope of the altitude $AM = 3$

$$\left(\because AM\perp BC \therefore m_2=-\frac{1}{m_1}\right)$$

$\therefore$ The equation of the altitude through A is

$y-4=3(x-1)$ or $3x-y+1=0$.

(iii) Since the right bisector of the side BC is perpendicular to BC.

$\therefore$ Slope of the right bisector of $BC = 3$.

Also the right bisector of BC passes through mid-point $D\left(\frac{1}{2},-\frac{5}{2}\right)$ of BC, therefore, its equation is

$y-\left(-\frac{5}{2}\right)=3\left(x-\frac{1}{2}\right)$ or $3x-y-4=0$.