Straight Lines

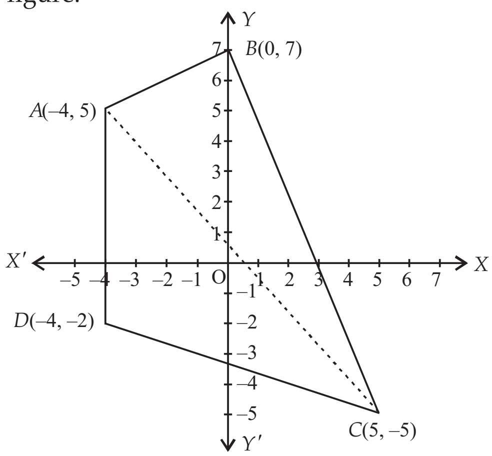
CHAPTER
10

NCERT FOCUS

SOLUTIONS

EXERCISE - 10.1

1. The figure of quadrilateral whose vertices are $A(-4, 5)$, $B(0, 7)$, $C(5, -5)$ and $D(-4, -2)$ is shown in the below figure.



Area of quadrilateral $ABCD$ = area of ΔABC + area of ΔADC ...(i)

Now, Area of $\Delta ABC = \frac{1}{2}|-4(7+5)-0(5+5)+5(5-7)|$

$= \frac{1}{2}|-4(12)+5(-2)| = \frac{1}{2}|-58| = 29$ sq. units

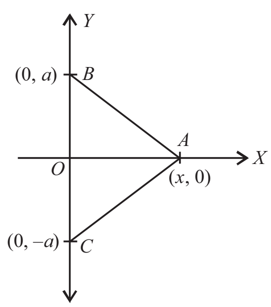
Also, Area of $\Delta ADC = \frac{1}{2}|-4(-2+5)-(-4)(5+5) + 5(5+2)|$

$= \frac{1}{2}|-4(3)+4(10)+5(7)| = \frac{1}{2}|-12+40+35|$

$= \frac{1}{2}|63| = \frac{63}{2}$ sq. units

$\therefore$ Area of quadrilateral $ABCD = 29 + \frac{63}{2} = \frac{121}{2}$ sq. units [From (i)]

2. Since, base of an equilateral triangle lies along y-axis.



So, $B(0, a)$ and $C(0, -a)$.

Let the third vertex be $A(x, 0)$

($\because$ ΔABC is an equilateral triangle and its base lies on y-axis)

$|BC| = |AB| = |AC| = 2a$

$\Rightarrow \sqrt{(x-0)^2+(0-a)^2} = 2a$

$\Rightarrow x^2 + a^2 = 4a^2 \Rightarrow x^2 = 3a^2 \Rightarrow x = \pm\sqrt{3}a$

$\therefore A = (\sqrt{3}a, 0)$ or $(-\sqrt{3}a, 0)$

Hence vertices of triangle are $(0, a)$, $(0, -a)$, and $(-\sqrt{3}a, 0)$ or $(0, a)$, $(0, -a)$ and $(\sqrt{3}a, 0)$.

3. We are given that co-ordinates of P is (x_1, y_1) and Q is (x_2, y_2).

Distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \text{... (1)}$$

(i) When PQ is parallel to y-axis then $x_1 = x_2$ from (1), we have $PQ = \sqrt{(x_1-x_1)^2+(y_2-y_1)^2} = |y_2 - y_1|$

(ii) When PQ is parallel to x-axis, then $y_1 = y_2$ from (1), we have $PQ = \sqrt{(x_2-x_1)^2+0} = |x_2 - x_1|$.

4. Let the point be $P(x, y)$. Since it lies on the x-axis $\therefore y = 0$ *i.e.*, required point be $(x, 0)$. Since, the required point is equidistant from points $A(7, 6)$ and $B(3, 4)$

$\Rightarrow PA = PB$

$\Rightarrow \sqrt{(x-7)^2+(0-6)^2} = \sqrt{(x-3)^2+(0-4)^2}$

$\Rightarrow \sqrt{x^2+49-14x+36} = \sqrt{x^2+9-6x+16}$

$\Rightarrow x^2 - 14x + 85 = x^2 - 6x + 25$

$\Rightarrow -14x + 6x = 25 - 85 \Rightarrow 8x = 60$

$\Rightarrow x = \frac{15}{2}$

$\therefore$ The required point is $\left(\frac{15}{2}, 0\right)$

5. We are given that $P(0, -4)$ and $B(8, 0)$.

Let A be the midpoint of PB, then

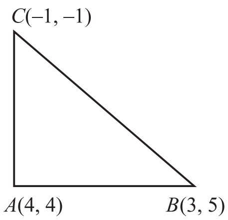
$$A = \left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$$

Slope of $OA = \frac{-2-0}{4-0} = \frac{-1}{2}$

($\because$ The line passes through origin).

6. Let $A(4, 4)$, $B(3, 5)$ and $C(-1, -1)$ be the vertices of ΔABC.

Let m_1 and m_2 be the slopes of AB and AC respectively.



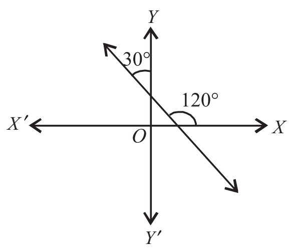
Then, $m_1 = \text{slope of } AB = \frac{5-4}{3-4} = -1$

$m_2 = \text{slope of } AC = \frac{-1-4}{-1-4} = \frac{-5}{-5} = 1$

$\therefore \quad m_1 m_2 = -1$

So, $AB \perp AC$ and therefore $\angle CAB = 90°$.
Hence, the given points are the vertices of a right angled triangle.

7. The given line makes an angle of 90° + 30° = 120° with the positive direction of x-axis.
Hence, $m = \tan 120° = -\sqrt{3}$.



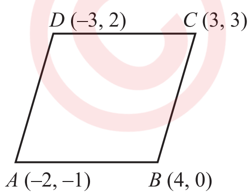
8. Let $A(x, -1)$, $B(2, 1)$ and $C(4, 5)$ be the given collinear points. Then by collinearity of A, B, C, we have slope of AB = slope of BC

$\Rightarrow \frac{1-(-1)}{2-x} = \frac{5-1}{4-2} \Rightarrow \frac{2}{2-x} = 2$

$\Rightarrow \quad 2 - x = 1 \Rightarrow x = 1$

Hence for $x = 1$ points A, B and C are collinear.

9. Let $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$ be the vertices of the given quadrilateral $ABCD$. Then,



Slope of $AB = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$

Slope of $DC = \frac{3-2}{3-(-3)} = \frac{1}{6}$

Slope of $BC = \frac{3-0}{3-4} = \frac{3}{-1} = -3$

Slope of $AD = \frac{2-(-1)}{-3-(-2)} = \frac{3}{-1} = -3$

Slope of AD = slope of $BC \Rightarrow AD \parallel BC$
Slope of AB = Slope of $CD \Rightarrow AB \parallel CD$
Hence, $ABCD$ is a parallelogram.

10. We are given that the points are $A(3, -1)$ and $B(4, -2)$

Slope of $AB = \frac{-2-(-1)}{4-3} = \frac{-2+1}{1} = -1$

$\Rightarrow$ $\tan\theta = -1$, where θ is the angle which AB makes with x-axis.
Now, $\tan\theta = -1 = -\tan 45° = \tan(180° - 45°) = \tan 135°$
$\Rightarrow \quad \theta = 135°$.

11. Let m_1 and m_2 be the slopes of two lines.

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Here, $\tan\theta = \frac{1}{3}$, $m_1 = m$, $m_2 = 2m$

$\frac{1}{3} = \left|\frac{m-2m}{1+m(2m)}\right| \Rightarrow \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right|$

$\Rightarrow \quad |1 + 2m^2| = 3|m| \Rightarrow 2|m|^2 - 3|m| + 1 = 0$

$\Rightarrow \quad 2|m|^2 - 2|m| - |m| + 1 = 0$

$\Rightarrow \quad (2|m| - 1)(|m| - 1) = 0$

$\Rightarrow |m| = \frac{1}{2}, 1 \Rightarrow m = \pm 1, \pm\frac{1}{2}$

Slope of the two lines are $1, 2; -1, -2; \frac{1}{2}, 1; \frac{-1}{2}, -1$.

12. A line passes through (x_1, y_1) and (h, k). Also, the slope of the line is m.

So, $m = \frac{k - y_1}{h - x_1} \Rightarrow (k - y_1) = m(h - x_1)$.

13. Let $A(h, 0)$, $B(a, b)$ and $C(0, k)$ be the given collinear points.

$\therefore$ Slope of AB = Slope of BC

$\Rightarrow \frac{b-0}{a-h} = \frac{k-b}{0-a} \Rightarrow \frac{b}{a-h} = \frac{b-k}{a}$

$\Rightarrow \quad ab = (a-h)(b-k)$

$\Rightarrow \quad ab = ab - ak - hb + hk \Rightarrow ak + hb = hk$

Dividing both sides by hk, we get $\frac{a}{h} + \frac{b}{k} = 1$.

14. Slope of line $AB = \frac{97-92}{1995-1985} = \frac{1}{2}$

Let the population in year 2010 be y, and co-ordinate of C be $(2010, y)$ then, slope of AB = slope of BC

$\Rightarrow \frac{1}{2} = \frac{y-97}{2010-1995} \Rightarrow \frac{1}{2} = \frac{y-97}{15}$

$\Rightarrow \quad 15 = 2y - 194 \Rightarrow 2y = 209 \Rightarrow y = 104.5$

Hence, the population in year 2010 will be 104.5 crores.

EXERCISE - 10.2

1. We know that the ordinate of each point on the x-axis is 0.
If $P(x, y)$ is any point on the x-axis, then $y = 0$.
$\therefore$ Equation of x-axis is $y = 0$.
Also, we know that the abscissa of each point on the y-axis is 0. If $P(x, y)$ is any point on the y-axis, then $x = 0$.
$\therefore$ Equation of y-axis is $x = 0$.

2. We know that the equation of a line with slope m and passing through the point (x_0, y_0) is given by

$(y - y_0) = m\,(x - x_0)$.

Here $m = \frac{1}{2}, x_0 = -4, y_0 = 3$

Hence, the required equation is $(y-3) = \frac{1}{2}(x+4)$

$\Rightarrow\ 2y-6 = x+4 \Rightarrow x - 2y + 10 = 0$

3. We know that the equation of a line with slope m and passing through the point (x_0, y_0) is given by $(y - y_0) = m(x - x_0)$

Here, slope $= m$, $x_0 = 0$, $y_0 = 0$

Required equation is $(y - 0) = m(x - 0)$

$\Rightarrow\ y = mx$.

4. We know that the equation of a line with slope m and passing through the point (x_0, y_0) is given by $(y - y_0) = m(x - x_0)$

Here, $m = \tan 75° = 2+\sqrt{3},\ x_0 = 2,\ y_0 = 2\sqrt{3}$

Hence, required equation is $(y - 2\sqrt{3}) = (2+\sqrt{3})(x-2)$

$\Rightarrow\ y - 2\sqrt{3} = 2x + \sqrt{3}x - 4 - 2\sqrt{3}$

$\Rightarrow\ (2+\sqrt{3})x - y - 4 = 0.$

5. We know that the equation of a line with slope m and passing through the point (x_0, y_0) is given by $(y - y_0) = m(x - x_0)$.

Here, $m = -2$, $x_0 = -3$, $y_0 = 0$

$y - 0 = -2\,(x + 3)\ \Rightarrow\ 2x + y + 6 = 0$

6. We know that the equation of line with slope m and passing through the point (x_0, y_0) is given by $(y - y_0) = m(x - x_0)$

Here, $m = \tan 30° = \frac{1}{\sqrt{3}}, x_0 = 0, y_0 = 2$

$(y-2) = \frac{1}{\sqrt{3}}(x - 0) \Rightarrow x - \sqrt{3}y + 2\sqrt{3} = 0.$

7. Let the given points be $A(-1, 1)$ and $B(2, -4)$.

We know that the equation of a line passing through the given points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Here, $x_1 = -1, y_1 = 1, x_2 = 2, y_2 = -4$

So, the equation of a line passing through the given points is

$$\frac{y-1}{\{x-(-1)\}} = \frac{(-4-1)}{2-(-1)} \Rightarrow \frac{y-1}{x+1} = \frac{-5}{3}$$

$\Rightarrow\ 3(y - 1) = -5(x + 1)\ \Rightarrow\ 5x + 3y + 2 = 0$

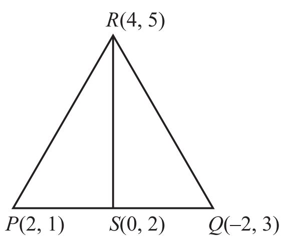
Hence, the required equation is $5x + 3y + 2 = 0$.

8. The vertices of ΔPQR are $P(2, 1)$, $Q(-2, 3)$ and $R(4, 5)$.

Let S be the midpoint of PQ.

So, $S = \left(\frac{-2+2}{2}, \frac{3+1}{2}\right) = (0, 2)$

Equation of median RS is



$$\frac{y-5}{x-4} = \frac{2-5}{0-4}$$

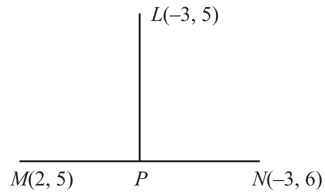
$$\Rightarrow\ \frac{y-5}{x-4} = \frac{-3}{-4}$$

$\Rightarrow\ 4y - 20 = 3x - 12$

$\Rightarrow\ 3x - 4y + 8 = 0$, which is the required equation.

9. Let $M(2, 5)$ and $N(-3, 6)$ be the end points of the given line segment.

Slope of $MN = \frac{6-5}{-3-2} = \frac{-1}{5}$



Since $LP \perp MN$

$\therefore$ Slope of LP is 5 and passing through $(-3, 5)$

Equation of line LP is

$(y - 5) = 5(x + 3)$

$\Rightarrow\ y - 5 = 5x + 15$

$\Rightarrow\ 5x - y + 20 = 0.$

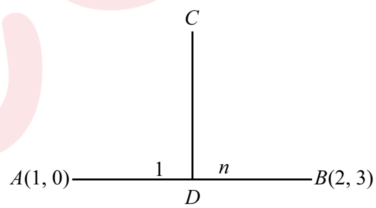
10. Let $A(1, 0)$ and $B(2, 3)$ be the given points and D divides the line segment in the ratio $1 : n$.

Coordinates of point D is $\left(\frac{n+2}{1+n}, \frac{3}{1+n}\right)$

Slope of line AB is $\frac{3-0}{2-1} = 3$

Since, $CD \perp AB$

$\therefore$ Slope of $CD = \frac{-1}{3}$



Required equation of line CD is

$$\left(y - \frac{3}{1+n}\right) = \frac{-1}{3}\left(x - \left(\frac{n+2}{1+n}\right)\right)$$

$$\Rightarrow\ \frac{1}{3}x + y - \left[\frac{n+11}{3(n+1)}\right] = 0$$

$\Rightarrow\ (1 + n)x + 3(1 + n)y = n + 11.$

11. Let the required line make intercepts 'a' on the x-axis and y-axis.

Then its equation is $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow\ x + y = a$ ...(i)

Since (i) passes through the point (2, 3), we have

$2 + 3 = a\ \Rightarrow\ a = 5$

So, required equation of the line is

$$\frac{x}{5} + \frac{y}{5} = 1 \Rightarrow x + y = 5.$$

12. Let the intercept made by the line on the x-axis be 'a' and intercepts made by the line on y-axis be $9 - a$

Then its equation is

$$\frac{x}{a} + \frac{y}{9-a} = 1$$

Since it passes through point (2, 2), we have $\frac{2}{a} + \frac{2}{9-a} = 1$

$\Rightarrow\ 2(9 - a) + 2a = a(9 - a)$

$\Rightarrow \quad 18 - 2a + 2a = 9a - a^2$

$\Rightarrow \quad 18 = 9a - a^2 \Rightarrow a^2 - 9a + 18 = 0$

$\Rightarrow \quad a^2 - 6a - 3a + 18 = 0$

$\Rightarrow \quad a(a-6) - 3(a-6) = 0 \Rightarrow a = 3, 6$

Now, if $a = 3 \Rightarrow b = 9 - 3 = 6$

and if $a = 6 \Rightarrow b = 9 - 6 = 3$

So, required equation is

$$\frac{x}{3} + \frac{y}{6} = 1 \text{ or } \frac{x}{6} + \frac{y}{3} = 1$$

i.e., $2x + y - 6 = 0$ or $x + 2y - 6 = 0$.

13. Here, $m = \tan\frac{2\pi}{3} = -\sqrt{3}$

The equation of the line passing through point (0, 2) is

$y - 2 = -\sqrt{3}(x - 0)$

$\Rightarrow \sqrt{3}x + y - 2 = 0$

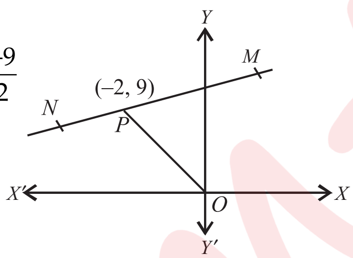
The slope of line parallel to $\sqrt{3}x + y - 2 = 0$ is $-\sqrt{3}$.

Since, it passes through (0, –2).

So, the equation of line is $y + 2 = -\sqrt{3}(x - 0)$

$\Rightarrow \sqrt{3}x + y + 2 = 0.$

14. Let $OP \perp MN$



Slope of $OP = \frac{9-0}{-2-0} = \frac{-9}{2}$

$\therefore$ Slope of $MN = \frac{2}{9}$

The equation of line is

$(y - 9) = \frac{2}{9}(x + 2)$

$\Rightarrow \quad 9y - 81 = 2x + 4 \quad \Rightarrow \quad 2x - 9y + 85 = 0$

15. Assuming L along x-axis and C along y-axis, we have two points (124.942, 20) and (125.134, 110). By two point form, the point (L, C) satisfies the equation

$$\frac{C - 20}{L - 124.942} = \left(\frac{110 - 20}{125.134 - 124.942}\right)$$

$\Rightarrow \quad C - 20 = \frac{90}{0.192}(L - 124.942)$

$\Rightarrow \quad 0.192(C - 20) = 90L - 11244.78$

$\Rightarrow \quad 0.192(C - 20) + 11244.78 = 90L$

$\Rightarrow \quad L = \frac{0.192}{90}(C - 20) + 124.942$

16. Assuming L (litres) along x-axis and R(rupees) along y-axis, we have two points (980, 14) and (1220, 16). By two point form, the point (L, R) satisfies the equation.

$$R - 14 = \left(\frac{16 - 14}{1220 - 980}\right)(L - 980)$$

$\Rightarrow \quad R - 14 = \left(\frac{2}{240}\right)(L - 980)$

$\Rightarrow \quad R - 14 = \left(\frac{1}{120}\right)(L - 980)$

Now, when $R = 17$, we have

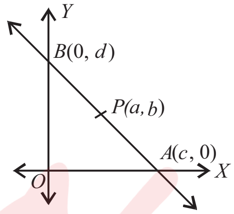
$17 - 14 = \left(\frac{1}{120}\right)(L - 980)$

$\Rightarrow \quad 360 = L - 980 \quad \Rightarrow \quad L = 980 + 360 = 1340$

Hence, the owner could sell 1340 litres of milk weekly at Rs. 17/litre.

17. Let the line AB makes intercepts c and d on the x-axis and y-axis respectively.

$\therefore \quad A(c, 0)$ and $B(0, d)$.



Let $P(a, b)$ be the midpoint of AB.

Then $\frac{c+0}{2} = a$ and $\frac{0+d}{2} = b$

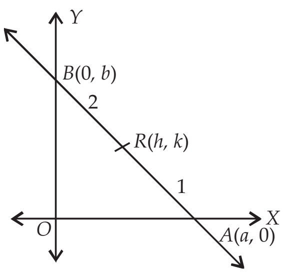
$\Rightarrow \quad c = 2a$ and $d = 2b$

So, the required equation is

$$\frac{x}{2a} + \frac{y}{2b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} = 2.$$

18. Let AB be the given line segment making intercepts a and b on the ,x-axis & y-axis respectively.

Then, the equation of line AB is $\frac{x}{a} + \frac{y}{b} = 1$



So, these points are $A(a, 0)$ and $B(0, b)$.

Now, $R(h, k)$ divides the line segment AB in the ratio 1 : 2.

$\therefore \quad \frac{1 \times 0 + 2 \times a}{1 + 2} = h$ and $\frac{1 \times b + 2 \times 0}{1 + 2} = k$

$\Rightarrow \quad a = \frac{3h}{2}$ and $b = 3k$

So, the required equation of the line is

$$\frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1 \Rightarrow \frac{2x}{3h} + \frac{y}{3k} = 1$$

19. Let the given points be $A(3, 0)$, $B(-2, -2)$ and $C(8, 2)$. Then, the equation of the line passing through A and B is

$\frac{y - 0}{x - 3} = \frac{-2 - 0}{-2 - 3}$

$\Rightarrow \quad \frac{y}{x - 3} = \frac{-2}{-5} \Rightarrow 5y = 2x - 6$

Clearly, the point $C(8, 2)$ satisfy the equation $2x - 5y - 6 = 0$.

$(\because \; 2(8) - 5(2) - 6 = 16 - 10 - 6 = 0)$

Hence, the given points lie on the same straight line whose equation is
$2x - 5y - 6 = 0$.

EXERCISE - 10.3

1. The equation of line is $12(x + 6) = 5(y - 2)$

$\Rightarrow \quad 12x + 72 = 5y - 10$

$\Rightarrow \quad 12x - 5y + 82 = 0 \qquad \text{...(i)}$

$\therefore$ Distance of the point (–1, 1) from the line (i)

$$= \frac{|12(-1) - 5(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} = \frac{65}{13} = 5 \text{ units.}$$

2. We have a equation of line $\frac{x}{3} + \frac{y}{4} = 1$, which can be written as $4x + 3y - 12 = 0$ ...(i)

Let $(a, 0)$ be the point on x-axis whose distance from line (i) is 4 units.

$$\Rightarrow \frac{|4 \times a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}} = 4 \Rightarrow \frac{|4a - 12|}{\sqrt{16 + 9}} = 4$$

$$\Rightarrow \frac{|4a - 12|}{\sqrt{25}} = 4 \Rightarrow \frac{|4a - 12|}{5} = 4$$

$\Rightarrow \quad |4a - 12| = 20 \Rightarrow 4a - 12 = \pm 20$

$\Rightarrow \quad 4a = 12 \pm 20 \Rightarrow a = 3 \pm 5$

$\Rightarrow \quad a = 3 + 5$ or $a = 3 - 5$

$\Rightarrow \quad a = 8$ or $a = -2$

Hence, the required points on the x-axis are (8, 0) and (–2, 0).

3. If lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then distance between parallel lines, $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

(i) Here, $A = 15$, $B = 8$, $C_1 = -34$, $C_2 = 31$

$$\therefore \quad d = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} = \frac{65}{17} \text{ units}$$

(ii) The line can be re-written as $lx + ly + p = 0$ and $lx + ly - r = 0$

Here $A = l$, $B = l$, $C_1 = p$, $C_2 = -r$

$$\therefore \quad d = \frac{|p + r|}{\sqrt{(l)^2 + (l)^2}} = \frac{p + r}{l\sqrt{2}} \text{ units}$$

4. We have given a point (2, 3), through which two lines are passing and intersects at an angle of 60°.

Let m be the slope of the other line.

$$\therefore \tan 60° = \left|\frac{m - 2}{1 + 2m}\right| \Rightarrow \pm\sqrt{3} = \frac{m - 2}{1 + 2m}$$

$$\Rightarrow \sqrt{3} = \frac{m - 2}{1 + 2m} \qquad \text{...(i)}$$

$$\text{or } -\sqrt{3} = \frac{m - 2}{1 + 2m} \qquad \text{...(ii)}$$

$\Rightarrow \sqrt{3} + 2\sqrt{3}m = m - 2$ or $\sqrt{3} + 2\sqrt{3}m = 2 - m$

$\Rightarrow \sqrt{3} + 2 = m - 2\sqrt{3}m$ or $2\sqrt{3}m + m = 2 - \sqrt{3}$

$$\Rightarrow m = \frac{\sqrt{3} + 2}{1 - 2\sqrt{3}} \text{ or } m = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}$$

Since, the line passes through (2, 3).

$\therefore$ Its equation is either

$$y - 3 = \frac{\sqrt{3} + 2}{1 - 2\sqrt{3}}(x - 2) \qquad \text{...(iii)}$$

$$\text{or } y - 3 = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}(x - 2) \qquad \text{...(iv)}$$

From (iii), we get $(y - 3)(1 - 2\sqrt{3}) = (\sqrt{3} + 2)(x - 2)$

$\Rightarrow y(1 - 2\sqrt{3}) - 3(1 - 2\sqrt{3}) = (\sqrt{3} + 2)x - 2(\sqrt{3} + 2)$

$\Rightarrow y(1 - 2\sqrt{3}) - (3 - 6\sqrt{3}) = (\sqrt{3} + 2)x - 2\sqrt{3} - 4$

$\Rightarrow 2\sqrt{3} + 4 - 3 + 6\sqrt{3} = (\sqrt{3} + 2)x + (2\sqrt{3} - 1)y$

$\Rightarrow (\sqrt{3} + 2)x + (2\sqrt{3} - 1)y - 8\sqrt{3} - 1 = 0 \qquad \text{...(v)}$

From (iv), we get

$\Rightarrow (y - 3)(1 + 2\sqrt{3}) = (2 - \sqrt{3})(x - 2)$

$\Rightarrow y(1 + 2\sqrt{3}) - 3(1 + 2\sqrt{3}) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$

$\Rightarrow -3 - 6\sqrt{3} + 2(2 - \sqrt{3}) = (2 - \sqrt{3})x - y(1 + 2\sqrt{3})$

$\Rightarrow -3 - 6\sqrt{3} + 4 - 2\sqrt{3} = (2 - \sqrt{3})x - y(1 + 2\sqrt{3})$

$\Rightarrow (2 - \sqrt{3})x - (1 + 2\sqrt{3})y + 8\sqrt{3} - 1 = 0 \qquad \text{... (vi)}$

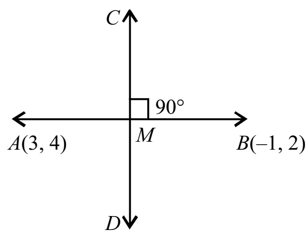
Thus (v) and (vi) are the equations of the required line.

5. Suppose the given points are A and B.

Let M be the mid point of AB.

$$\therefore \quad M = \left(\frac{3 + (-1)}{2}, \frac{4 + 2}{2}\right) = (1, 3)$$

$$\text{Slope of } AB = \frac{2 - 4}{-1 - 3} = \frac{-2}{-4} = \frac{1}{2}$$



$\therefore$ Slope of the right bisector of AB is –2.

Since the right bisector passes through $M(1, 3)$,

$\therefore$ The equation of the right bisector is $y - 3 = -2(x - 1)$

$\Rightarrow 2x + y - 5 = 0$

Hence, the required equation is $2x + y = 5$.

6. Given, the perpendicular from the origin to the line $y = mx + c$ meets it at the point (–1, 2)

$\therefore \quad 2 = m(-1) + c$

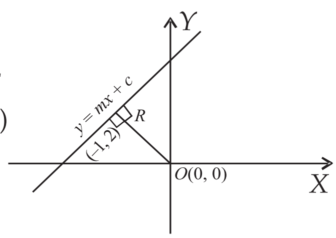
$\Rightarrow c - m = 2 \qquad \text{...(i)}$

Slope of the perpendicular

$$= \frac{2 - 0}{-1 - 0} = -2 \qquad \text{...(ii)}$$

Slope of the line

$y = mx + c$ is m



$\therefore \quad m \times (-2) = -1$

$\Rightarrow \quad m = \dfrac{1}{2}$ (From (ii))

Put the value of m in (i), we get

$c - \dfrac{1}{2} = 2 \Rightarrow c = 2 + \dfrac{1}{2} = \dfrac{5}{2}$.

7. Given, p and q are the lengths of perpendiculars from the origin to the lines $x \cos\theta - y \sin\theta = k \cos 2\theta$ and $x \sec\theta + y \operatorname{cosec}\theta = k$ respectively.

Here $p = \dfrac{|0\cdot\cos\theta - 0\sin\theta - k\cos 2\theta|}{\sqrt{(\cos\theta)^2 + (-\sin\theta)^2}}$

$\Rightarrow \quad p = \dfrac{|-k\cos 2\theta|}{1} \quad \Rightarrow \quad p = k\cos 2\theta$ ...(i)

and $q = \dfrac{|0\cdot\sec\theta + 0\cdot\operatorname{cosec}\theta - k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$

$= \dfrac{|-k|}{\sqrt{\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}}}$

$= k\cos\theta\cdot\sin\theta = \dfrac{k\sin 2\theta}{2}$ $[\because \sin 2\theta = 2\sin\theta\cdot\cos\theta]$

$\Rightarrow \quad 2q = k\sin 2\theta$ ...(ii)

Squaring and adding (i) and (ii), we get

$p^2 + 4q^2 = (k\cos 2\theta)^2 + (k\sin 2\theta)^2$

$\Rightarrow \quad p^2 + 4q^2 = k^2(\cos^2 2\theta + \sin^2 2\theta)$

$\Rightarrow \quad p^2 + 4q^2 = k^2$

Hence proved.

8. We have given a ΔABC with the vertices, A (2, 3), B (4, –1) and C (1, 2)

Clearly, slope of $BC = \dfrac{2-(-1)}{1-4} = \dfrac{2+1}{-3} = \dfrac{3}{-3} = -1$

$\therefore$ Equation of BC is

$y - (-1) = -1(x - 4)$

$\Rightarrow \quad y + 1 = -x + 4$

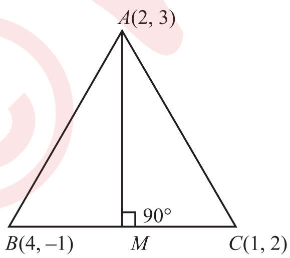
$\Rightarrow \quad x + y - 3 = 0$

Also, slope of $AM = 1$

$\therefore$ Equation of AM is

$y - 3 = 1(x - 2)$

$\Rightarrow \quad x - y + 1 = 0$



Length of perpendicular from $A(2, 3)$ on $BC = \dfrac{|2+3-3|}{\sqrt{1^2+1^2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$ units.

9. Given, p be the length of perpendicular from the origin to the line whose intercepts on the axes are a and b.

$\therefore$ Equation of the line is, $\dfrac{x}{a} + \dfrac{y}{b} = 1$

or $\quad bx + ay - ab = 0$ ... (i)

Since, p is the length of perpendicular from origin upon the line (i), we have

$p = \dfrac{|b\times 0 + a\times 0 - ab|}{\sqrt{b^2+a^2}} \Rightarrow p^2 = \dfrac{a^2b^2}{a^2+b^2}$

$\Rightarrow \dfrac{1}{p^2} = \dfrac{a^2+b^2}{a^2b^2} \Rightarrow \dfrac{1}{p^2} = \dfrac{a^2}{a^2b^2} + \dfrac{b^2}{a^2b^2} \Rightarrow \dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

NCERT MISCELLANEOUS EXERCISE

1. Given equation of a line is

$(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$ ... (i)

(a) The line (i) is parallel to x-axis if coefficient of $x = 0$

$\Rightarrow \quad k - 3 \Rightarrow k = 3$

($\because$ Any line parallel to x-axis is of the form $y = b$)

(b) Clearly the line (i) is parallel to y-axis if coefficient of $y = 0 \Rightarrow (4 - k^2) = 0 \Rightarrow k^2 = 4$, *i.e.* $k = \pm 2$.

($\because$ Any line parallel to y-axis is of the form $x = a$)

(c) Since the line (i) passes through the origin if (0, 0) lies on (i)

$\Rightarrow \quad 0 - 0 + k^2 - 7k + 6 = 0 \Rightarrow (k-6)(k-1) = 0$, *i.e.* $k = 6, 1$.

2. Suppose the intercepts be a and b

Given, Sum of intercepts = 1 *i.e.*, $a + b = 1$ ... (i)

and product of intercepts = – 6 *i.e.*, $ab = -6$ ... (ii)

From (i) & (ii), we get

$a(1-a) = -6 \Rightarrow a^2 - a - 6 = 0 \Rightarrow a = 3, -2$.

Case I : If $a = 3$, then $b = \dfrac{-6}{a} = \dfrac{-6}{3} = -2$

$\therefore$ Equation of the line is $\dfrac{x}{3} + \dfrac{y}{-2} = 1$ $\left[\because \dfrac{x}{a} + \dfrac{y}{b} = 1\right]$

$\Rightarrow \quad -2x + 3y = -6 \Rightarrow 2x - 3y - 6 = 0$

Case II : If $a = -2$, then $b = \dfrac{-6}{a} = \dfrac{-6}{-2} = 3$

Thus, equation of the line is $\dfrac{x}{-2} + \dfrac{y}{3} = 1$

$\Rightarrow \quad 3x - 2y = -6 \Rightarrow 3x - 2y + 6 = 0$.

3. Given line is $\dfrac{x}{3} + \dfrac{y}{4} = 1 \Rightarrow 4x + 3y - 12 = 0$ ... (i)

Let a point on y-axis be $(0, k)$

If its distance from line (i) is 4 units, then

$\left|\dfrac{4\times 0 + 3\times k - 12}{\sqrt{4^2+3^2}}\right| = 4 \Rightarrow |3k - 12| = 4\times 5 \Rightarrow 3k - 12 = \pm 20$

$\Rightarrow 3k = 12 \pm 20 \Rightarrow 3k = 12 + 20$ or $3k = 12 - 20$

$\Rightarrow \quad 3k = 32$ or $3k = -8 \Rightarrow k = \dfrac{32}{3}$ or $\dfrac{-8}{3}$.

$\therefore$ Required points are $\left(0, \dfrac{-8}{3}\right), \left(0, \dfrac{32}{3}\right)$

4. Let the points be $A(\cos\theta, \sin\theta)$, $B(\cos\phi, \sin\phi)$

Equation of the line AB is

$y - \sin\theta = \dfrac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$

$$\Rightarrow \quad y-\sin\theta = \frac{2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\phi-\theta}{2}\right)}{-2\sin\left(\frac{\phi+\theta}{2}\right)\sin\left(\frac{\phi-\theta}{2}\right)}(x-\cos\theta)$$

$$\Rightarrow \quad (y-\sin\theta) = -\frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\sin\left(\frac{\theta+\phi}{2}\right)}(x-\cos\theta)$$

$$\Rightarrow \quad y\sin\left(\frac{\theta+\phi}{2}\right)-\sin\theta\sin\left(\frac{\theta+\phi}{2}\right)$$

$$= -x\cos\left(\frac{\theta+\phi}{2}\right)+\cos\theta\cdot\cos\left(\frac{\theta+\phi}{2}\right)$$

$$\Rightarrow x\cos\left(\frac{\theta+\phi}{2}\right)+y\sin\left(\frac{\theta+\phi}{2}\right) - \left\{\cos\theta\cos\left(\frac{\theta+\phi}{2}\right)+\sin\theta\cdot\sin\left(\frac{\theta+\phi}{2}\right)\right\}=0$$

$$\Rightarrow \quad x\cos\left(\frac{\theta+\phi}{2}\right)+y\sin\left(\frac{\theta+\phi}{2}\right)-\cos\left(\theta-\left(\frac{\theta+\phi}{2}\right)\right)=0$$

[By using $\cos A\cdot\cos B + \sin A\cdot\sin B = \cos(A-B)$]

$$\Rightarrow \quad x\cos\left(\frac{\theta+\phi}{2}\right)+y\sin\left(\frac{\theta+\phi}{2}\right)-\cos\left(\frac{\theta-\phi}{2}\right)=0$$

Now, distance of the above line from the origin

$$= \frac{\left|0+0-\cos\left(\frac{\theta-\phi}{2}\right)\right|}{\sqrt{\cos^2\left(\frac{\theta+\phi}{2}\right)+\sin^2\left(\frac{\theta+\phi}{2}\right)}} = \left|\cos\left(\frac{\theta-\phi}{2}\right)\right|.$$

5. Given equation of lines are

$x-7y+5=0$ ... (i)

and $3x+y=0$ ... (ii)

On solving (i) and (ii), we get $x=\frac{-5}{22}, y=\frac{15}{22}$

$\therefore$ The point of intersection of the given lines is $\left(\frac{-5}{22},\frac{15}{22}\right)$.

Thus, equation of the line parallel to y-axis and drawn through the point $\left(\frac{-5}{22},\frac{15}{22}\right)$ is $x=\frac{-5}{22}$ which can be re-written as $22x+5=0$.

6. We have, $\frac{x}{4}+\frac{y}{6}=1$ ... (i)

Now, the slope of (i) $=\frac{-3}{2}$

Slope of any line perpendicular to line (i) is $\frac{2}{3}$.

$\therefore$ Equation of the line through (0, 6) and perpendicular to line (i) is $(y-6)=\frac{2}{3}(x-0) \Rightarrow 3y-18=2x \Rightarrow 2x-3y+18=0$.

7. The given equation of lines are

$y = m_1x + c_1$ ... (i)

$y = m_2x + c_2$ ... (ii)

$y = m_3x + c_3$ ... (iii)

On solving (i) and (ii), we get

$$x=\frac{-(c_2-c_1)}{m_2-m_1}, y=\frac{c_1m_2-c_2m_1}{m_2-m_1}$$

Hence, (i) and (ii) meets at $\left[\frac{-(c_1-c_2)}{(m_1-m_2)},\frac{(m_1c_2-m_2c_1)}{(m_1-m_2)}\right]$

Clearly (i), (ii) and (iii) lines are concurrent if the obtained point lies on (iii)

i.e., $\left[\frac{m_1c_2-m_2c_1}{m_1-m_2}\right]=m_3\left[\frac{-(c_1-c_2)}{(m_1-m_2)}\right]+c_3$

i.e., $\frac{(m_1c_2-m_2c_1)}{(m_1-m_2)}=\frac{m_3(-(c_1-c_2))}{(m_1-m_2)}+c_3$

i.e., $(m_1c_2-m_2c_1)=m_3(c_2-c_1)+c_3(m_1-m_2)$

$\Rightarrow \quad m_1c_2-m_2c_1=m_3c_2-m_3c_1+c_3m_1-c_3m_2$

$\Rightarrow \quad m_1c_2-m_2c_1-m_3c_2+m_3c_1-c_3m_1+c_3m_2=0$

$\Rightarrow \quad m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0.$

Hence proved.

8. The given equation of lines are

$4x+7y-3=0$ ... (i)

and $2x-3y+1=0$ ... (ii)

On solving (i) and (ii), we get

$$x=\frac{1}{13}, y=\frac{5}{13}$$

Thus, the lines (i) and (ii) meets at $\left(\frac{1}{13},\frac{5}{13}\right)$.

Let the required equation be $\frac{x}{a}+\frac{y}{b}=1$

Since, the equation passes through $\left(\frac{1}{13},\frac{5}{13}\right)$ and has equal intercepts on the axes.

$$\frac{1}{13a}+\frac{5}{13a}=1 \quad [\because a=b]$$

$$\Rightarrow \quad \frac{6}{13a}=1 \Rightarrow 6=13a \Rightarrow a=\frac{6}{13}.$$

$\therefore$ The required equation of line is $13(x+y)=6$.

9. The given equation of a line is $y=mx+c$ ... (i)

Slope of (i) is m.

Let n be the slope of the required line, then $\tan\theta=\left|\frac{n-m}{1+nm}\right|$

$$\Rightarrow \quad \tan\theta=\pm\left(\frac{n-m}{1+nm}\right)$$

Case I : When, $\tan\theta=\frac{n-m}{1+nm}$

Then, $\tan\theta + nm\tan\theta = n-m$

$\Rightarrow \quad nm\tan\theta - n = -m - \tan\theta \Rightarrow n=\frac{m+\tan\theta}{1-m\tan\theta}$

$\therefore$ Equation of the required line through the origin is $y - 0 = n\,(x - 0)$

i.e., $y = \left(\dfrac{m + \tan\theta}{1 - m\tan\theta}\right)x \Rightarrow \dfrac{y}{x} = \dfrac{m + \tan\theta}{1 - m\tan\theta}$ ... (ii)

Case II : When, $\tan\theta = -\left(\dfrac{n - m}{1 + mn}\right)$

Then, $\tan\theta + mn\tan\theta = -n + m$

$\Rightarrow n(1 + m\tan\theta) = m - \tan\theta \Rightarrow n = \dfrac{m - \tan\theta}{1 + m\tan\theta}$

Equation of the required line through the origin is $y - 0 = n(x - 0)$

i.e., $\dfrac{y}{x} = \dfrac{m - \tan\theta}{1 + m\tan\theta}$ ... (iii)

Hence, from (ii) and (iii), equation of line is $\dfrac{y}{x} = \dfrac{m \pm \tan\theta}{1 \mp m\tan\theta}$.
Hence proved.

10. The given equation of lines are

$3x - y + 1 = 0$ ... (i)

$x - 2y + 3 = 0$ ... (ii)

Clearly, slope of (i) is 3 and slope of (ii) is $\dfrac{1}{2}$. Since, we have given that both these lines are equally inclined to the line $y = mx + 4$, whose slope is m, then we must have

$$\left|\frac{m - 3}{1 + 3m}\right| = \left|\frac{m - \frac{1}{2}}{1 + m\cdot\frac{1}{2}}\right| \Rightarrow \left|\frac{m - 3}{1 + 3m}\right| = \left|\frac{2m - 1}{2 + m}\right|$$

$$\Rightarrow \frac{m - 3}{1 + 3m} = \pm\left(\frac{2m - 1}{2 + m}\right)$$

Now, we have two cases

Case I : $\dfrac{m - 3}{1 + 3m} = \dfrac{2m - 1}{2 + m}$

$\Rightarrow (m - 3)(2 + m) = (1 + 3m)(2m - 1)$

$\Rightarrow 2m + m^2 - 6 - 3m = 2m - 1 + 6m^2 - 3m$

$\Rightarrow -5m^2 - 5 = 0 \Rightarrow 5m^2 + 5 = 0$

$\Rightarrow m^2 = -1$, which is not possible.

Case II : $\dfrac{m - 3}{1 + 3m} = -\left(\dfrac{2m - 1}{2 + m}\right)$

$\Rightarrow (m - 3)(2 + m) = -(2m - 1)(1 + 3m)$

$\Rightarrow 2m + m^2 - 6 - 3m = -(2m + 6m^2 - 1 - 3m)$

$\Rightarrow 6m^2 - m - 1 + m^2 - 6 - m = 0 \Rightarrow 7m^2 - 2m - 7 = 0$

$$\Rightarrow m = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 7 \times (-7)}}{2 \times 7}$$

$$= \frac{2 \pm \sqrt{4 + 196}}{14} = \frac{2 \pm \sqrt{200}}{14} = \frac{2 \pm 10\sqrt{2}}{14}$$

Hence, $m = \dfrac{1 \pm 5\sqrt{2}}{7}$

11. The given equation of lines are

$9x + 6y - 7 = 0 \Rightarrow 3x + 2y - \dfrac{7}{3} = 0$ ...(i)

and $3x + 2y + 6 = 0$ ...(ii)

Let the equation of the line mid-way between the parallel lines (i) and (ii) be

$3x + 2y + \lambda = 0$ ...(iii)

Then, distance between (i) and (iii) = distance between (ii) and (iii)

$$\Rightarrow \frac{\left|\lambda + \frac{7}{3}\right|}{\sqrt{9 + 4}} = \frac{|\lambda - 6|}{\sqrt{9 + 4}} \Rightarrow \left|\lambda + \frac{7}{3}\right| = |\lambda - 6|$$

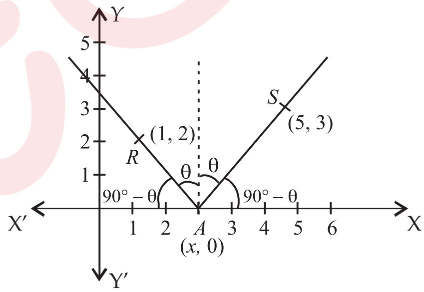
$$\Rightarrow \lambda + \frac{7}{3} = \pm(\lambda - 6)$$

$$\Rightarrow \lambda + \frac{7}{3} = \lambda - 6 \text{ (not possible)},\ \lambda + \frac{7}{3} = -\lambda + 6$$

$$\Rightarrow 2\lambda = 6 - \frac{7}{3} \Rightarrow 2\lambda = \frac{11}{3} \Rightarrow \lambda = \frac{11}{6}$$

Hence, the equation of the required line is $3x + 2y + \dfrac{11}{6} = 0$
i.e., $18x + 12y + 11 = 0$.

12. Since we know that both the rays reflected and incident are equally inclined to the normal at A.
Now, coordinates of $A = (x, 0)$.



If AR makes an angle θ with the normal at A, then it forms an angle $90° + \theta$ with the positive x-axis and AS forms an angle $90° - \theta$ with the positive x-axis.
Now, slope of $AR = \tan(90° + \theta) = -\cot\theta$
and slope of $AS = \tan(90° - \theta) = \cot\theta$

$\therefore$ Slope of AS + slope of $AR = 0$

$$\Rightarrow \frac{3 - 0}{5 - x} + \frac{0 - 2}{x - 1} = 0 \Rightarrow \frac{3}{5 - x} + \frac{-2}{x - 1} = 0$$

$\Rightarrow 3(x - 1) + (-2)(5 - x) = 0$

$\Rightarrow 3x - 3 - 10 + 2x = 0 \Rightarrow 5x - 13 = 0 \Rightarrow x = \dfrac{13}{5}$

$\therefore$ The co-ordinates of A are $\left(\dfrac{13}{5}, 0\right)$.

13. The given equation of line is

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta - 1 = 0$ ...(i)

Now, distance of (i) from the point $\left(\sqrt{a^2 - b^2}, 0\right)$

$$= \frac{\left|\frac{\sqrt{a^2 - b^2}}{a}\cos\theta - 1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2}}$$

And distance of (i) from the point $\left(-\sqrt{a^2 - b^2}, 0\right)$

$$= \frac{\left|\dfrac{-\sqrt{a^2-b^2}}{a}\cos\theta - 1\right|}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}}$$

Now, product of lengths of these two perpendiculars

$$= \frac{\left|\left(\dfrac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(\dfrac{-\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\right|}{\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|-\left(\dfrac{\sqrt{a^2-b^2}}{a}\cos\theta - 1\right)\left(\dfrac{\sqrt{a^2-b^2}}{a}\cos\theta + 1\right)\right|}{\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2}}$$

$$= \frac{\left|\left(\dfrac{a^2-b^2}{a^2}\right)\cos^2\theta - 1\right|}{\dfrac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2b^2}}$$

$$= \frac{|(a^2\cos^2\theta - b^2\cos^2\theta) - a^2| \times a^2b^2}{(b^2\cos^2\theta + a^2\sin^2\theta)a^2}$$

$$= \frac{|a^2(\cos^2\theta - 1) - b^2\cos^2\theta| \times b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$= \frac{|-a^2\sin^2\theta - b^2\cos^2\theta|b^2}{a^2\sin^2\theta + b^2\cos^2\theta}$$

$$= \left|\frac{-(a^2\sin^2\theta + b^2\cos^2\theta)}{(a^2\sin^2\theta + b^2\cos^2\theta)}\right| b^2 = |-1|\,b^2 = b^2$$

Hence proved.